NEW PERSPECTIVES ON LEARNING AND INSTRUCTION

Transformation of Knowledge Through Classroom Interaction

Edited by Baruch Schwarz, Tommy Dreyfus, and Rina Hershkowitz

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Routledge
Taylor & Francis Group
LONDON AND NEW YORK

First published 2009
by Routledge

2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN

Simultaneously published in the USA and Canada
by Routledge
270 Madison Ave, New York, NY 10016

Routledge is an imprint of the Taylor & Francis Group, an informa business

This edition published in the Taylor & Francis e-Library, 2009.

To purchase your own copy of this or any of Taylor & Francis or Routledge's
collection of thousands of eBooks please go to www.eBookstore.tandf.co.uk.

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication Data
Transformation of knowledge through classroom interaction / edited
by Baruch Schwarz, Tommy Dreyfus, and Rina Hershkowitz.
p. cm. – (New perspectives on learning and instruction)
1. Interaction analysis in education. 2. Classroom environment.
3. Learning. I. Schwarz, Baruch. II. Dreyfus, Tommy.
III. Hershkowitz, Rina.
LB1034.T73 2009
371.102'2–dc22 2008043876

ISBN 0-203-87927-9 Master e-book ISBN

ISBN10: 0-415-49224-6 (hbk)
ISBN10: 0-415-49225-4 (pbk)
ISBN10: 0-203-87927-9 (ebk)

ISBN13: 978-0-415-49224-9 (hbk)
ISBN13: 978-0-415-49225-6 (pbk)
ISBN13: 978-0-203-87927-6 (ebk)

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Introduction

Classrooms are among the most familiar learning contexts in Western society. Their impact on society has been formidable. However, besides definitive successes such as democratizing access to knowledge, the constraints that classrooms arise to learning have engendered many criticisms. One constraint concerns the socio-spatial structure of classrooms that leads teachers to function as isolated individual practitioners. A bureaucratic and institutional constraint concerns the temporal structure of discrete lessons and short time sequences of work, punctuated by tests and exams. Another constraint is motivational: Attaining grades is a main motive for school work which leads to classification of students into categories such as weak, competent, passive, etc. These constraints and many others have moulded recurrent learning practices such as lectures administered by the teacher, teacher-led plenary discussions, or drill-and-practice individual activities.

The current research literature on classroom interactions takes into consideration the complexity of classroom constraints and practices. Many of the perspectives adopted function as snapshots that help understanding the processes involved in classroom interaction: the *process-product perspective* helps correlating between teachers' actions and students' further outcomes; the *ethnographic perspective* helps comprehending how classroom practices enact and build culture; the *discourse-analysis* perspective focuses on how classroom practices and discursive events shape each other. These and other perspectives generally take the classroom context as a given in which the changes that may occur leave this context almost intact.

This book is not characterized by one perspective shared by all contributors. Unity comes from the fact that researchers belong to a common adventure – changing school practices and norms. This adventure is moved by societal ideals of reason and equity. Contexts for learning are not immutable givens. Rather, the classroom context is the result of a programmatic design: researchers are interested in implementing certain practices or instilling certain norms. Practices instigated include small group collaborative problem solving, non-intrusive guidance (also called moderation) in group discussion, or collective argumentation, to cite a few. By doing so, all authors share a *critical perspective* that aims at supporting change at all levels (institutional, curricular, communicative, material).

The authors also adopt a common dialogic approach in their striving for change that denies a radical deconstructive stance. Typical dialogic actions include the instigation of new evaluation practices to instil new norms (at the institutional level), the negotiation of new contents (at the curricular level), the implementation of interactive practices (at the communicative level), and the elaboration of new (technological) tools that support productive constructions. The theoretical tenets of most of the contributors stem from socio-cultural psychology as well as from constructivism. They often refer to *activity theory* to trace changes that involve all the levels cited above among researchers, designers, teachers and students.

We chose the term *transformation* for the title of the book to indicate that the changes most of the contributors describe have a historical dimension. Transformation concerns tools as well as individual or collective outcomes. If the English language had permitted it, we would have labelled what we study and are engaged to foster as 'transformation of knowing' (instead of 'transformation of knowledge'). The terms that fuel our quest for tracing and fostering transformation in classroom interaction include *actions*, *shared understanding*, *intersubjectivity*, *argumentation* and especially *succession of activities*. The transformation concerns both the community and the individual; we focus on changes of (communal) practices and of identity.

The transformations we attempt to trace encompass moments of unequal importance, and the authors seek to identify the most crucial ones (designated differently by different authors – constructions, epistemological discontinuities, breakdowns between intra- and intersubjective processes, knowledge creation and so on); these impinge on further actions or on further successive activities. Our focus on transformation in special moments of classroom interactions as well as in successions of activities necessitates the adoption of a multi-level of analysis (micro, meso and macro). The coordination of these different levels is not easy since it is natural at a micro-level to analyse discursive events, but at a macro-level other perspectives are necessary. The multilevel invites then a multiplicity of methods that are not always compatible. Methodological efforts are thus necessary to resolve tensions between, for example, construction of knowledge within activities and learning processes in successive activities, or the relations between shared and individual understandings.

The dialogic approach that characterizes most of the contributions necessitates researchers to engage in activities with teachers, students, designers, educators and, in some cases, policy makers. The researchers are then insiders and, as such, consider the specificity of the transformations which are bonded to the norms of discourse, knowledge structures and objectives of the domain taught. The focus on mathematics and science classrooms helps, we hope, in studying in-depth specificities, for example, classroom interactions in which participants engage in mathematical abstraction or scientific hypothesizing.

The book is organized in five parts. The first part provides contributions on construction of knowledge. Schwarz, Dreyfus and Hershkowitz (Chapter 1) present the nested epistemic actions model for studying mathematical abstraction in classroom contexts. This chapter has a practical character in the sense that it provides tools for describing the emergence of abstraction and its consolidation. The authors stress the utility of this model to describe what they consider fundamental in mathematics classrooms, a construction through vertical reorganization of previous constructs. This process that the authors call 'abstraction in context' is driven by a need. It becomes known through three basic epistemic actions (recognizing, building-with and constructing) whose emergence involves interactions between tasks and activities.

Tiberghien (Chapter 2) proposes and illustrates a theoretical framework to characterize high school physics classrooms by establishing relationships between what is taught and what is learned. She defines the fundamental idea of 'taught knowledge' as the researcher's (re)construction of joint productions of the teacher and the students in a physics classroom to trace students' evolution of performances before and after mechanics teaching sequences through relations established between taught knowledge and students' activities in the classroom. Tiberghien analyses at different levels of analysis how 'conventional meanings' are interwoven in classroom activities.

While the first two chapters mainly encompass micro- and meso-levels, Yerushalmi (Chapter 3) adopts a macro-level perspective. She uses the notion of epistemological discontinuity as a design principle that indicates expected difficulties in long-term learning. She exemplifies the design of an algebra curriculum at a point of discontinuity to demonstrate why the use of technology has the potential to introduce strengths that make the 'unnatural' look 'natural'. However, Yerushalmi suggests that known discontinuities are generally stable because they indicate an epistemological gap. She then suggests that identifying critical discontinuities is crucial for studying student constructions of knowledge and analysing classroom guided inquiry supported by technology.

Hakkarainen and Paavola (Chapter 4) propose a useful metaphor for learning, the *trialogical learning* metaphor, which is more appropriate than the acquisition (or knowledge construction) and the participation metaphors in activities in which students collaborate to construct an outcome, especially when this outcome is material, reused in following activities as an artefact, and when the collaboration is mediated by technological tools. They exemplify three cases in which the trialogical metaphor usefully portrays gains of individuals and groups in joint activities.

In their commentary on the first part, Kidron and Monaghan compare different theories underlying the chapters. They establish links to some additional theories including the theory of didactic situations. They then point out the importance of considering epistemology, pedagogy and teaching, and need and anticipation in theories on knowledge construction, and show how the four chapters fare on this account. They similarly look at how the four chapters deal with the quandary of individual versus social construction of knowledge. And they end by asking to what extent these chapters together contribute to a consistent wider understanding and analysis of knowledge construction.

The second part of the book is dedicated to guidance in construction of knowledge. In the three chapters of this part, the authors stress that the role of guidance is delicate to play since it can easily undermine understandings constructed by individuals or by groups. On the other hand, they see unguided group work as clearly insufficient for learning in science or mathematics classrooms. The question is then under what conditions can expertise and group activity be productively combined.

Howe (Chapter 6) explores how groups can be guided through the principles of hypothesis testing in a fashion that does not compromise the benefits that group interaction can have for domain understanding. She shows that knowledge gain is optimized when groups discuss and reach consensus about causal factors, and these consensual positions subsequently become the subject of guided hypothesis testing. Her findings have educational significance as well as theoretical implications: The inherently egalitarian, socially constructed and tool-like function of consensus challenges Piagetian as well as Vygotskian tenets of learning and development.

Sohmer, Michaels, O'Connor and Resnick (Chapter 7) consider the direct roles (as opposed to Howe who focuses on indirect roles) teachers play in science classrooms. They consider the complexity of what they call the amalgam of tasks, talk and tools in the guided construction of knowledge. Talk, with particular norms and forms, is recognized as supporting rigorous academic learning in ethno-linguistically and socio-economically diverse classrooms. But to lead to construction of knowledge, talk should be 'accountable' to bring students from all backgrounds into productive conversation. In the ideal discussion-based classroom community, students have the right to speak and the obligation to explicate their reasoning, providing warranted evidence for their claims so that others can understand and critique their arguments. In science classrooms, teachers are accountable not only to the class community but to rationality and scientific norms, a fact that necessitates using tasks and tools with the help of which synergy between these poles can be obtained.

In Chapter 8, Hoppe, de Groot and Hever provide a design perspective to guidance: the help teachers provide in their guidance involves interactive and cooperative technologies. For example, they afford smooth information flow between different media and representations; they also include archiving and retrieval functions which reach out beyond the single classroom and facilitate reuse, sharing and exchange of results between teachers and students from different learning groups; they bridge between different conditions of learning, such as individual, small group or large community activities as well as between synchronous and asynchronous settings. Hoppe, de Groot and Hever are aware that such an integrative design does not necessarily lead to actual integration. Strong motivation is required from the teachers that undergo adaptations and changes of their teaching styles and pedagogical beliefs. The authors show the synergy between teachers, designers and researchers for supporting effective guidance.

The third part of the book delves into the learning mechanisms involved in argumentation, the main vehicle for reasoning in classroom. The two contributions, by Baker (Chapter 9) and Asterhan and Schwarz (Chapter 10), both clarify notions that are generally integrated according to a too-general social and cultural approach to learning and development. Baker shows the gaps that may arise between inter-subjective and intra-subjective rationalities when students engage in discussions on scientific issues. Cooperative learning is generally considered to be drawing continuities between the actions that help students to communicate meanings in collaborative settings and the psychological mechanisms involved during and after such collaborations. Baker shows a detailed example of the degree of tension and the potential breakdowns between the inter-subjective and intra-subjective rationalities. These tensions originate from an inevitable encounter between two timescales of development (ontogenetic and microgenetic) but is necessary for learning to emerge beyond each participant in a concrete situation. Baker pledges for new theoretical efforts to bridge between dialectical and dialogical theories of collaborative learning. Asterhan and Schwarz initiate an effort in this very direction by reporting on two different settings that eventually led dyads that discussed an issue on evolutionary theory to promote their conceptual understanding. Asterhan and Schwarz show that eliciting both monological and dialogical argumentation led to conceptual learning. Through scrutiny of exemplary dialogues, Asterhan and Schwarz show that processes that govern collaborative learning are not monolithic. They show that one should differentiate between dialectical collaborative and adversarial argumentation: while for both cases, different perspectives are brought forward, challenged and possibly drawn back, aggressiveness impairs conceptual learning.

The efforts to discern between communication and learning processes (both in Baker and Asterhan and Schwarz chapters) are challenged by Sfard (Chapter 11) through the lens of the 'Commognitive Theory' that she recently developed. By her description readers are able to come close to the main elements of this theory and even to reflect upon their potential in describing and interpreting 'Transformation of Knowledge through Classroom Interaction'.

In the contribution that closes this part, Wegerif (Chapter 12) invites his colleagues Andriessen, Boero and Forman to join him in a dialogue about the place of dialogue in education. Ideas are raised one after the other by the authors of the chapter, who have a unified goal to understand, clarify and formulate the essence of a human dialogue. Understandably, form and content are here unified and dialogue is explained as well as exemplified: the diverging or converging ideas of the different participants interweave together in harmony through conservation of autonomy. This way of discovering layers of thinking and ideas, through the accumulative steps of a dialogic discourse, demonstrates so well the strength of this genre of human communication.

The fourth part of the book deals with methodologies to study transformation of knowledge in classroom interaction. Saxe, Gearhart, Shaughnessy, Earnest, Cremer, Sitabkhan, Platas and Young (Chapter 13) recognize the emergence and travel of ideas in the process of classroom life as basic to understanding learning and teaching in inquiry-oriented classrooms and as fundamental to understanding the guided construction of knowledge in classroom communities. Their chapter presents a methodological approach and associated empirical techniques for the study of the travel of ideas in classrooms. The method involves micro-, onto- and socio-genetic strands of analysis in which form and function are tracked within and between activities. These strands provide a method for understanding the interplay between the constructive activities of the individual and the shifting organization of communicative interactions of the collective. The empirical techniques support data collection efforts guided by the analytic strands. This approach, applicable to any classroom community, provides ways of gaining insight into opportunities to learn as well as insights into their dynamics of the shifting organization of collective mathematical representations over the course of lessons.

Cobb, Gresalfi and Ho (Chapter 14) focus on a methodology to document the identities that students develop in mathematics classrooms. According to Cobb and colleagues, this methodology is useful for supporting and understanding students' identification with classroom mathematical activity as well as their development of significant mathematical ideas. The interpretive scheme proposed focuses in particular on the constructs of normative identity and personal identity. The analyses presented document the forms of agency that students can legitimately exercise in particular classrooms, together with how authority is distributed and thus to whom students are accountable, and what they are accountable for mathematically. Cobb and colleagues place the interpretive scheme in a broader theoretical context by discussing its relation to alternative approaches that analyse the identities that students are developing across longer timescales, in the process taking account of issues of race, ethnicity and culture.

After methodological tools for studying the travel of ideas and the development of identity, Nathan, Kim and Eilam (Chapter 15) propose a methodology for studying intersubjectivity in socially distributed activities in mathematics classrooms. Through the analysis of video-based data, Nathan and colleagues could identify convergent and divergent forms of intersubjectivity. At a meso-level of analysis, they scrutinize what role the teacher and students play in creating and operating within common ground, what role objects and representations play in creating intersubjectivity and how intersubjectivity structures discourse. At a macro-level, they identify how intersubjectivity supports dialogic interaction and contributes to achieving the participants' overall goals.

This part of the book is completed by a commentary chapter by Bikner-Ahsbahs and Williams (Chapter 16), in which they try to link and even to combine the contributions by Saxe, Cobb, Nathan and their colleagues to ask whether they do not miss something crucial. They first point at the interest of the students, its fluctuations and its origins. Their quest for intensity and for flow of mathematical activity goes deep into the gist of knowledge transformation in classroom interaction.

Finally, in Part 5 of the book, three scientists interweave their individual rich perspectives and experience in the area, and pull together several of the issues discussed in the book. Hershkowitz (Chapter 17) reflects on the flexible boundaries between theory, methodology and methodological tools in research on models of transformation of knowledge in the classroom, upon the research main goals. The dynamic character of these relations points at a genuine scientific enterprise in which the framework is always challenged by new observations.

Although the perspectives adopted in this book are extremely diverse and involve great complexity, Roth (Chapter 18) feels that the picture that emerges from the different analyses is still incomplete. He points at important shortcomings. For example, he notices that although most of the authors agree with a cultural-historical activity theory direction, the studies do not articulate the specificities of Western contexts (cultural and historical) implicit in the experimentations. Also, Roth complains about the lack of reference to the role of emotion in transformation of knowledge.

Ludvigsen (Chapter 19) provides an integrative chapter to answer the central question with which all contributors have come to grips: how do different types of interactions contribute to students' participation and understanding of concepts and conceptual systems in domains like mathematics and science? Ludvigsen recalls the multi-layered character of human cognition to adopt the methodology proposed by Saxe and colleagues to evaluate the scientific value of the contributions. His reflection on the different contributions through micro-, onto- and socio-genetic lenses points at strengths and weaknesses of the contributions, and by such validates the importance of this methodology to understand transformation of knowledge in classroom interaction.

Due to the complexity involved in the study of the transformation of knowledge in classroom interaction, different researchers have so far pursued theoretical, empirical and design-based directions in parallel. However, one of the basic features of classroom activity, the fact that activities are successive, has forced researchers in each of those directions to consider learning extremely seriously. The book then concretizes a converging interest of researchers from different backgrounds to understand their own domain. The first part, which is dedicated to construction of knowledge, encompasses consolidation in the long run, the second part about guidance stresses a dialogical developmental approach to teaching in which knowledge of students, the actions of the teacher and instructional materials are incessantly redefined. The researchers involved in argumentation, dialogism and learning broaden their scope on learning, which was so far based on a collaborative setting, to consider more deeply the relations between inter- and intra-processes. Such developments, such cross-boundary directions demand new methodological efforts to include different scales of time, kinds of activities and theoretical perspectives encompassing individuals, groups and technologies to understand how learning can emerge in the classroom. The authors of this book have shown, we hope, how each perspective has been broadened and how new methodologies can cope with this enlargement.

The workshop from which this book was born took place on 5–8 February 2007 at the Hebrew University of Jerusalem. We would like to thank the following institutions for contributing to the organization of the workshop: The Israel Science Foundation, the Hebrew University, the Tel-Aviv University, the Weizmann Institute, as well as two projects funded by the European Community, the Argunaut project (IST-2005027728) and the KP-Lab project (IST-2005027490).

Baruch Schwarz, Tommy Dreyfus and Rina Hershkowitz
Israel, September 2008

Part 1

Construction of knowledge in classroom interaction

Chapter 1

The nested epistemic actions model for abstraction in context

Baruch B. Schwarz, Tommy Dreyfus and Rina Hershkowitz

In this chapter we study mathematical abstraction in educational contexts. Our purpose is to observe the act of abstracting throughout successive activities in mathematics classrooms. To do so, we operationally define abstraction in a way which is general enough to encompass empirical and theoretical aspects, we elaborate a methodological tool for observing the emergence of abstraction in successive activities, and we refine the theory of abstraction through empirical analyses in classrooms. The tool was originally inspired by Freudenthal's idea of vertical mathematization, as was our preliminary, vague definition of mathematical abstraction. This definition stirred the design of activities with potential for abstraction but needed to be refined. The nested epistemic actions model constitutes such a refinement, based on our empirical studies as well as theoretical considerations on mathematical abstraction by others. We show that the model is adequate to identify and trace abstraction and its consolidation in successive activities in numerous contexts, including work by individuals, small and large groups in unguided and guided activities with and without technological tools.

A story from a classroom

The students sit in threes and fours, some sharing a joke, some chatting idly, one or two just staring into space. Two seem to be engaged in a furious argument about something terribly important. One student raises his hand, eager to be noticed. The teacher wanders through the classroom and passes him by, as though she had not seen him. They all have the same worksheet; some of them write on it, some keep it idly on their laps, and some examine it intently. It links them all together, despite their wildly different reactions, from studiousness to indifference, from enthusiasm to nervy attention. A researcher is present, and so is the designer of the learning unit, to examine its impact on the students. The teacher pauses momentarily next to a group of three whose heads are close together, poring over the page – they seem to have hit on an answer! She nods, and asks them something; they look up and think for a moment, before one of them offers a response. The teacher crosses the room to the designer, smiling broadly. 'You did it!' her smile seems to say, they are with you, they are engaged, they love it!

The story we would like to tell is about three girls who carry out the activity on this worksheet (Figure 1.1): Yevgenia, Chen and Shany are used to working as a

Activity 3, Problem 1

1a. Yossi and Ruti roll 2 white dice. They decide that Ruti wins if the number of dots is the same on the two dice, and Yossi wins if they are different.
Do you think that the game is fair? Explain!

1b. The rule of the game is changed. Yossi wins if the dice show consecutive numbers: Do you think the game is fair?

1c. How many possible outcomes are there when rolling two dice?

1d. Suppose Yossi and Ruti play with one red die and one white die. Will this change the answers for 1a, 1b and 1c?

Activity 3, Problem 2

This time we observe the difference between the larger number of dots and smaller number of dots on the two dice. (If the number of dots is the same on the two dice, the difference is 0.)

2a. Make a hypothesis about whether all differences have the same probability. Explain!

2b. How many possible differences are there?

Figure 1.1 Activity 3, problems 1 and 2.

group in their class. Their story relates to the *construction* of the *order principle* in probability, namely that in a two-dimensional sample space (2d SS) the events (a,b) and (b,a) are different, each one of them having the same probability as (a,a). Order becomes an issue when the group discusses problem 1b (Figure 1.1). After the girls agree that there are 12 pairs in the SS ('6 possibilities for each die gives 12 events'), that Yossi's odds are 5 out of 12, and Ruti's odds are 6 out of 12, Yevgenia raises the issue of order:

YEV 56: One moment . . . something does not seem logical to me . . .
YEV 59: No, no, no . . . it can be different, every die is 10 . . . he [Yossi] still remains with the same 10: (1,2) and (2,1).
CHEN 60: So . . . what did we do? (1,2) is one and (3,2) is one . . .
SHANY 61: Or the opposite, on the second die 2 dots. This is one die and this is a second die.

And soon thereafter:

YEV 71: It is clear that the game is not fair, it can be that on one die you get 1 and on the second 2, or the opposite 2 and 1.
CHEN 72: And then it will be 10 to 12? [Meaning 10 out of 12]
SHANY 73: We also have to reverse the dice?

Yevgenia recognizes that order is an issue, Chen joins her to calculate the probability, and Shany asks for clarifications. At the end of a discussion about whether there are 10 or only 5 simple events of successive numbers, Yevgenia decides:

YEV 88: Yes, there are a total of 5 pairs of successive numbers.

After Shany repeats that Yossi has five possibilities and Yevgenia adds that Ruti has six, Chen again raises the order principle but Yevgenia resolutely rejects it:

CHEN 92: But it is possible to reverse it!
YEV 93: No! It's impossible to reverse it!
RES 94: What do you say Chen?
CHEN 95: It's possible to reverse the dice 1 and 2, and 2 and 1.
YEV 96: It will not change anything! It will not change anything!

Yevgenia who had previously raised the issue (turns 56, 59), is now sure that order is not important. She clearly accounts for having changed her mind:

YEV 98: I totally change my mind, it will not change anything. (2,1) or (1,2) are the same successive numbers! It will not change anything. They are the same successive numbers! The same successive numbers! I was wrong before and I totally changed my mind!

The girls next discuss whether the dice are rolled together or one after the other. Then Yevgenia starts summing up, Chen tries to object and Shany allies with Yevgenia, her dominant peer:

YEV 109: It does not make any difference.
CHEN 111: If Yossi gets 2, and she gets 1…
YEV 112: Even if it comes out the opposite it will not make any difference.
SHANY 113: Yevgenia is right, it does not make any difference if it comes out (1,2) or (2,1).
CHEN 114: I don't know, maybe he [Yossi] has more chances…
YEV 115: He doesn't! He doesn't! He has only the successive numbers from 1 to 6.
CHEN 117: [to the researcher] What shall I do? 10 out of 12 or 5 out of 12?
YEV 118: This is the end … thanks Chen … we haven't yet read the next question.

Chen seems to progress in her understanding of the order principle, and tries to get help from the researcher. Yevgenia, who exhibits confidence in denying the importance of order, closes the discussion and moves to the next question. Shany supports Yevgenia. When turning to problem 1c (Figure 1.1), Yevgenia counts 21 possible pairs, thus showing consistency in denying the importance of order. But Chen is still worried:

CHEN 153: [to the researcher] (1,2) and (2,1) are different outcomes?
RES 154: Ask your friends, you must decide if they are different.

CHEN 156: [to her peers] (1,2) and (2,1) are different outcomes?
YEV 157: Yes. No! I counted this as one outcome.

Chen then writes down that there are 21 pairs, as agreed. But her doubts emerge again in problem 1d (Figure 1.1) where distinguishable dice are considered:

SHANY 177: It doesn't make any difference, right? The colour of the die doesn't matter?
CHEN 178: For sure there is a difference!
YEV 179: How come?!
CHEN 189: I still don't understand what it will be: 5 [out of] 12, or 10 [out of] 12?

At the end of the lesson, they turn to problem 2 (Figure 1.1) and agree on six possible differences with decreasing probabilities; in particular, they agree that the probability to roll a difference of 0 is higher than that for a difference of 1. In the following lesson, a computer simulation challenges their claims by showing that the probability for difference 0 is lower than that for difference 1. Then the teacher presents a table for representing the differences (problem 2b, Figure 1.1) for all possible pairs in 2d SS (problem 1c, Figure 1.1), and instigates a whole class discussion on the issue of order. For this purpose she colours her dice, one in yellow and the other in green, and asks the students to mark in yellow (green) all the cells in the table representing a difference in which the number of dots on the yellow (green) die is bigger (Figure 1.2). In the discussion, the class concludes that each cell in the table represents a different simple event.

Henceforth, the three girls count dice pairs with the same number of dots in reverse order as different, when working as a group or individually. Chen

Yellow

Green

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Figure 1.2 The table constructed by the teacher to solve problem 2.

becomes dominant in group work, and seems happy to provide explanations to Shany who begins to collaborate with her. Yevgenia steps back, and works by herself. As requested in the worksheet, they return to problems 1a and 1b (Figure 1.1). For problem 1b, Chen asserts 'Look, in this game Ruti wins if the number of dots is equal and Yossi wins if the numbers on the dice are consecutive. You have to complete the table'. She inscribes ✓ marks in table cells to indicate Ruti as winner and ✗ marks for Yossi. Later she turns to Shany to summarize: 'The probability for Yossi to win is 10/36 and for Ruti 6/36.' Working alone, Yevgenia produces a similar table, showing the information for both problems 1a and 1b, and writes the correct results for problem 1a.

The researcher confronted them with their initial answers that the SS contains only 21 events. Chen explains their initial thinking on order, and seems happy to have eventually been shown right:

RES 388: You wrote 21 [before]; now what is the number of all the possible outcomes?
CHEN 389: Now 36!
RES 390: Then why did you think 21?
SHANY 391: Because we thought (1,1) (1,2) (1,3)...
RES 392: What was the problem there? What understanding do you have now?
CHEN 393: That (1,2) and (2,1) are not the same! It is a different event!

Yevgenia, who was so dominant before, remained quiet but the researcher addressed her:

RES 394: What is the probability in game 1a and what is the probability in game 1b?
YEV 395: In any case Ruti is losing!
RES 396: What is the probability in the second game?
YEV 397: Ruti 6/36 and Yossi 10/36.
RES 398: Where is his chance to win bigger, in the first or in the second?
YEV 399: In the first. Come on, let's go....

It seems that Yevgenia understands her mistake, but refuses to admit it and smoothly adapts to the idea of 36 events in the SS (Yev 397). We saw above that Shany seemed to agree with her peers. But does this agreement imply any learning? What we saw in the class is subject to possible interpretations that are difficult to confirm; it is often difficult to make solid inferences about individual interpretations and learning on the basis of participation in classroom interactions. But additional data may restrict possible interpretations. And indeed, in one of the post-tests, each girl was interviewed individually on a similar question. Data are not displayed due to space limitation. We will only say here that each girl produced a different table; each table served its purpose, showing that all three girls were fluent with the order principle.

This seems an interesting story: After becoming aware of the issue of order, the three girls agree at first that order does not matter. The agreement proves to be fragile, since Chen is not really convinced and repeatedly questions it. The three girls engage in collective argumentation on the importance of order. However, Yevgenia imposes her opinion on her peers. Shany follows Yevgenia, but Chen opposes her whenever an opportunity occurs. When the teacher constructs a table as a basis for a general discussion, it becomes clear that Chen was right. Yevgenia becomes quite silent and works alone, leaving Chen to lead the collaboration with Shany. Finally, in their post-tests, all three students appear to have capitalized on what emerged in their interaction.

The above tasks form part of a sequence of activities that constitutes a 10-lesson introductory probability unit. Given our aim to understand more about the learning processes involved, we observed stories such as the one of Yevgenia, Chen and Shany. But as researchers, we needed more than stories. We needed a theoretical framework with efficient notions and a clear language to capture stories on learning processes in mathematical classrooms. The specificity of the mathematics was clear. The principle we considered appeared differently in different contexts. The students' explanations underwent transformations that appeared to support them in reaching a mathematically valuable understanding. This is what we, as mathematics educators, mean intuitively by a process of *mathematical abstraction*. The time has come for us, as researchers, to be more explicit about what constitutes mathematical abstraction and to find ways to trace it as it occurs in mathematics classrooms.

Studying abstraction in classrooms

Scientists often produce thoughtful ideas by reflecting on their experience. Freudenthal is one of them. Freudenthal (1991) stressed the idea of mathematics as a human activity during which students are given opportunities to re-invent mathematics. He labelled this process *mathematization*. Mathematics educators who joined Freudenthal in what is today known as 'the Freudenthal Institute' distinguished between horizontal and vertical mathematization (Treffers 1987). In horizontal mathematization, students 're-invent' mathematical tools to organize and solve problems in a real-life situation. Vertical mathematization relates to reorganization within mathematics, for instance, by finding shortcuts and discovering connections between concepts and strategies and then applying these discoveries. As will be shown below, vertical mathematization suits ideas expressed theoretically on abstraction. Our story seems to exemplify vertical mathematization: In 388–393, Chen, with some help from Shany and the researcher, is reflecting on the group's way of thinking. They thought that there are only 21 simple events in the sample space (SS), because they did not take order into account. Now, they know that (1,2) and (2,1) are different events, and are able to conclude, by mathematical means, that there are 36 events in the SS. Chen and Shany's reflection provides evidence that the 36-event SS has been constructed by reorganization of their existing knowledge to include order as a principle, which they had just constructed. Constructing the order principle was a process that lasted throughout the lesson. We have evidence of Chen's construct towards the end of the lesson, and of Shany's and Yevgenia's from the post-test. Constructing the order principle included a lengthy collaborative period, during which the students recognized previous constructs, such as what constitutes an event in 2d SS (e.g. 60, 61), and built solutions to problems and answers to questions with these previous constructs (e.g. 88). Recognizing previous constructs and building with them were necessary for, and contributed to the construction of, the order principle. Such actions were thus nested in the constructing action.

We can identify in our story three stages. The second stage exemplifies vertical mathematization through a constructing action. It is preceded by a first stage, in which a need arises to clarify the relationship between the events (1,2) and (2,1); this need is built into the task design, hence evident for all three students (e.g. 59, 73, and 95). It arises when they first discuss problem 1b and constitutes a need for a new construct, thus leading up to the constructing action. The constructing action is followed by reflections on and further uses of the new constructs. For example, the queries of the researcher in 390 and 392 lead Chen to become aware of and articulate a formal link between two new constructs, the order principle and the 36-event sample space. This increasing awareness indicates a third stage, namely consolidation. The consolidation stage is indefinite in time and our story only shows its beginning. Every further use of the new constructs, whether in reflection, or problem solving, or in further constructing actions, is likely to contribute to consolidating the new constructs, and to serve as evidence that they are part of the students' knowledge.

In the main section of this chapter, we will articulate our framework, *Abstraction in Context* (AiC), and elaborate the three stages, the epistemic actions, and the idea of nesting. We will do this by generalizing and abstracting from many cases. In the preceding sections, we present the mathematical context for which the AiC framework is appropriate, and the theoretical grounds on which AiC grew.

Mathematical contexts for the study of abstraction

The AiC framework is suitable for investigating learning in situations that offer potential for vertical reorganization of knowledge and for consolidation of the ensuing constructs. In this section, we specify what this means in terms of the nature of the mathematical knowledge and in terms of the standards on which the design of learning situations that serve as context for AiC research is based.

Vertical reorganization is necessarily based on prior constructs that are to be reorganized and thus serve as bricks in the process of constructing. Ideally, these bricks are not only reorganized but also integrated and interwoven, thus adding a layer of depth to the learner's knowledge, and giving expression to the composite nature of the mathematics. The potential for consolidation implies opportunities to capitalize on the new constructs repeatedly, and to turn them into bricks for further construction. This requires a sequence of problem situations, in which each construct includes 'pockets' of past constructs on one hand, and is itself a potential component for new constructs, on the other.

Our approach is largely empirical. It took shape in the course of research that accompanied curriculum development, when questions arose such as 'What has been learned and consolidated, and how?'. Our curriculum standards are formulated here succinctly (for a more detailed treatment, see Hershkowitz et al. 2002).

Standards

S1 Inquiry (observing, hypothesizing, generalizing and checking) is a desirable mathematical activity.
S2 Mathematical activity should be driven by the goals of understanding and convincing.
S3 Proving is not only the central tool for providing evidence that a statement is true, but should also support understanding of why it is true.
S4 Mathematical activity should take place in situations that are meaningful for the students.
S5 Mathematical activity must stem from previous knowledge (including intuitive knowledge).
S6 Mathematical activity should be largely reflective.
S7 Mathematical languages (notation systems) foster the consolidation of mathematical knowledge; it should be introduced to students when they feel the need for it.
S8 Technical manipulation is not a goal in itself, but a mean to do mathematics.

Further reflection on our curricular efforts led us to make explicit a last standard that was self-evident for us:

S9 Students' activity should take place in different social and mediating settings: individual and collaborative problem solving and reflective activities, teacher-led discussions, all possibly mediated by technologies and various other tools.

These standards were implemented by means of explicit or implicit design guidelines for learning units. The design guidelines for the probability unit from which the activities in the above story were taken are listed elsewhere (Hadas, Hershkowitz & Ron, in press).They resemble those adopted by leading educators sensitive to the present zeitgeist in mathematics education. Let us exemplify some of these processes, as they occur in 'our story'.

The problem situations on which the students are asked to work are meaningful to them (S4). In each of them, the students are first asked to hypothesize (S1, see problems 1a, 1b and 2a, Figure 1.1). While discussing problem 1b, Yevgenia (56, 59) becomes aware of the possibility of the issue of order, and her peers soon follow (Chen 60 and Shany 61). Is it intuition which leads them (S5)? It is hard to tell. But Yevgenia's (56) 'something does not seem logical to me' seems to suggest that she seeks understanding (S2). Chen considers this idea (Chen 92, 114, 117, 153, 156, 178…), even after Yevgenia decides in an aggressive and patronizing way 'to change her mind completely' and to reject the idea (Yevgenia 88, 93, 96, 98…). A conflict between the two girls thus develops in a dialectical argumentation that quietly transforms into agreement after the teacher introduces the table model (Figure 1.2), which serves as a notation (S7) at a time when the students need such a tool to organize the sample-space events.

This example represents the kind of mathematics learning that serves as context for the constructing of knowledge and abstraction that we investigate.

While our standards have been exemplified here using one activity in elementary probability, they have been implemented widely and researched using the AiC framework in many topics, including rate of change, exponential growth, algebraic justification, probability and others.

Theory and research on the study of mathematical abstraction in classrooms

Our aforementioned standards bring to the fore the scrutiny of learning in groups and individuals through practices such as collaboration, argumentation and reflection, and of course the adoption of analytic tools that fit a socio-constructivist approach. Several leading researchers in mathematics education have already studied learning processes in classrooms in which we could recognize standards similar to ours. In *The Emergence of mathematical meaning: interaction in classroom culture* (edited by Cobb & Bauersfeld 1995), researchers including Bauersfeld, Cobb, Krummheuer, Voigt and Yackel develop tools to observe classroom learning in elementary schools according to a socio-constructivist approach. Among the constructs developed, one may cite *evolution of practices*, *taken-as-shared understandings*, *socio-mathematical norms* (Voigt 1995), *collective argumentation* and *collective reflection* (Yackel & Cobb 1996). Such constructs enable the tracing of processes at the level of the collective. These methods have been synthesized by Cobb, Stephan, McClain and Gravemeijer (2001) in what they called the *analytic method*. The methodological tools developed particularly enable tracing the emergence of understandings in classroom discussions, sometimes through the observation of shifts in practice and changes in norms during successive activities. In the examples that Cobb and colleagues analyse to exemplify their methodology, the role of the teacher is central as she generally moderates discussions with a small group of students or with the whole class. In such discussions, the teacher can instil socio-mathematical norms and instigate the enactment of productive collective practices. If the teacher is attentive to students' actions (arguments, explanations, questions, etc.) and skilful enough to capitalize on them, these practices support the emergence of mathematically valuable shared understandings. However, students also participate in other important kinds of activities: individual problem solving, collaborative autonomous problem solving, and the writing of collective or individual reports on problem solving. Such activities are not stressed in the works cited above, possibly because the researchers focus on elementary schools. For us, as curriculum developers and researchers in secondary schools, the alternation of successive collective and individual activities is a crucial opportunity for students to learn. For example, we showed how four adolescents engaged in collaborative problem solving mediated by technological tools collaboratively wrote a report on this activity and capitalized on their understandings in a reflective teacher-led class discussion about functions (Hershkowitz & Schwarz 1999). In another study, we showed how, after collaborative problem solving in small groups and collective reporting, a teacher redistributed reports to individual students in homework assignments to reflect on their own arguments in light of conflicting ones (Schwarz & Hershkowitz 2001). In both studies, reporting seemed extremely productive for further activities (collective and individual). Such findings add to theoretical considerations concerning the appropriateness of the succession of activities in different social contexts for identity formation (Packer & Goicoechea 2001) and to empirical findings (obtained according to a cognitive perspective) according to which the alternation of collective and individual activities is highly beneficial for learning processes (e.g. Fuchs & Fuchs 1998).

Our approach to designing successive activities while alternating different social contexts is rooted in empirical studies, theory and didactical experience. The question is then how could we trace learning during such varied successions of activities? Our motivation is to design successive activities that provide effective learning opportunities for groups and individuals. Our research tools should then be adaptable to a range of activities including small group or individual problem solving. This adaptability is crucial for studying learning in a span of time that encompasses more than one isolated activity. Our approach was to choose gradually different social contexts that seemed to open learning opportunities and to observe processes of abstraction in the context of the mathematics classroom.

As stated above, we began by largely identifying abstraction in context with vertical mathematization. Abstraction is then a reorganization achieved by means of actions on mental or material objects through which constructs are put together, structured and transformed; verticality refers to the depth of the connections established and the integration of knowledge (Treffers 1987). Abstraction is not an objective, universal process but strongly depends on the personal history of the participants in the activity of abstraction and on artefacts available to the participants. These artefacts are often themselves historical residues of previous activities. They include material objects and tools, such as computerized ones, as well as immaterial ones including language and procedures; in particular, they can be ideas or other outcomes of previous activities. The term 'artefact' will be mainly embodied in epistemic actions already operated that have the potential to mediate students' learning. In the next subsection, we present a short review of research on mathematical abstraction, showing that so far, contributions are mainly theoretical.

A short summary of research on mathematical abstraction

Abstraction has been central in Greek philosophy, principally due to Plato's *theory of forms* according to which each material form has an essence to be discovered by the philosopher. Plato's theory of knowledge and of knowing is based on levels of perception. Plato's *Meno* presents these levels through an example, now classical, in geometry. This very ancient idea has deeply influenced Western thought. In developmental psychology, it influenced Piaget in his theory of *genetic epistemology* on which his general theory of psychological development is based.

Piaget's theory presents reflecting abstraction as a general process that demands a high level of developmental maturity: reflecting abstraction requires that the subject attain the highest level of development, the level of abstraction. According to Piaget, reflecting abstraction is constructive, leading beyond simple generalization, typical for abstraction of physical qualities. Reflecting abstraction is constructive insofar as it is linked to the elaboration of a new action on a higher level than the action from which the characteristic under consideration was constructed. For Piaget, reflecting abstraction involves differentiation. It leads to a generalization that is a novel composition, because it involves a new scheme elaborated by means of elements borrowed from prior schemes by differentiation. According to Piaget, 'these new schemes are more mobile and more reversible, and consequently more equilibrated' (Campbell 2000, p. 11). Thus, reflecting abstraction is a developmental mechanism that may be facilitated by contextual factors (peers, instruments, etc.) but originates from the individual. This developmental mechanism is linked to the highest levels of development, and consequently cannot be fully operated in childhood and even in adolescence.

This orthodox view of reflecting abstraction could not be used as is by mathematics educators who experienced successes *and* failures in designing situations that intuitively foster mathematical abstraction, thus indicating the situated nature of mathematical abstraction. Mathematics educators' first attempts to define mathematical abstraction adopted the Platonic idea of level and hierarchy. Partly building on the earlier work of Dienes (1961), Skemp (1971/1986) viewed *abstracting* as an activity involving the recognition of similarities and their further classification leading to a new mental object/product: the *abstraction* that is used to recognize new experiences. Mitchelmore and White (2007) call this view of abstraction *empirical abstraction*, thus echoing Piaget's idea of empirical abstraction (Piaget 1977). Skemp viewed abstracting as a constructive process based on generalizing purposefully identified similarities. Constructivists refined this view of empirical abstraction by adding the notion of levels of abstraction. Hiebert and Lefevre (1986) distinguish a *primary level of abstraction* at which what is abstracted is at the same level of abstractness as the information itself is represented. At a higher level, which Hiebert and Lefevre call *the reflective level of abstraction*, relationships are less tied to specific contexts. They are created by recognizing similar core features in pieces of information that are superficially different, pulling them out and tying them together, thus transcending the level at which the knowledge is currently represented. This feature of decontextualization is contrary to the view that cognition is situated. For example, van Oers (1998) criticized the idea of decontextualization as the basis for abstraction, arguing that context is always relative to an individual so that decontextualization suggests removal of the individual and that removing context must impoverish a concept rather than enrich it. The theoretical ideas brought forward so far suggest that abstraction should be studied as an activity and that it should involve actions that distinguish between similarity and depth. Depth should be tied to recontextualization rather than decontextualization (as proposed by van Oers).

Noss and Hoyles (1996) also argued against the hierarchical and decontextualization views of abstraction. They characterized 'abstraction as a process of connection rather than ascension' (p. 48) and sought to break the word abstract 'free from its dehumanizing connotations' (p. 49). They introduced the idea of webbing as 'the presence of a structure that learners can draw up and reconstruct for support – in ways that they can choose as appropriate for their struggle to construct meaning for some mathematics' (p. 108), and introduced the term *situated abstraction* to describe 'how learners construct mathematical ideas by drawing on the webbing of a particular setting which, in turn, shapes the way the ideas are expressed' (p. 122). Whereas the critiques raised by Noss and Hoyles are well taken, the idea of webbing unnecessarily blurs the distinction between actions that refer to similarity and depth; webbing is thus too general to be productive for the study of abstraction.

Another stream of thought led by Soviet psychologists has dodged the pitfalls of decontextualization and proposed an articulated theory of abstraction which is not empirical and can be labelled *theoretical abstraction*. Vygotsky (1978) made a corresponding distinction between *everyday* and *scientific* concepts. Everyday concepts are formed by empirical abstraction, but the formation of scientific concepts has three features: the establishment of a system of relations among concepts; an awareness of one's own mental activity; and penetration to the object's essence.

Davydov (1990) continues Vygotsky's general ideas to note that scientific knowledge is not a simple expansion of people's everyday experience. It requires the cultivation of particular ways to abstract, a particular analysis, and generalization, which permits the internal connections of things, their essence, and particular ways of idealizing the objects of cognition to be established. For Davydov, these ways are argumentative and dialectical in nature (Ozmantar & Monaghan 2007). Abstraction starts from an initial, simple, undeveloped and vague first form, which often lacks consistency. The development of abstraction proceeds from analysis, at the initial stage of the abstraction, to synthesis. It ends with a more consistent and elaborated form. It does not proceed from concrete to abstract but from an undeveloped to a developed form. An important phase of theoretical abstraction is the identification of the *essence* of an idea (already alluded to by Vygotsky) as enrichment rather than an impoverishment of reality. According to Davydov the essence of something is the basis for all its changes in interaction with other things. Theoretical abstraction leads to the mental replacement of an object by its model. A way to contrast empirical from theoretical abstraction would be to contrast learning through examining several worked-out exercises (empirical abstraction) from learning through the deep analysis of one problem in order to identify its essential variables and relationships (theoretical abstraction).

The developing character of mathematical knowledge as a *theoretical* knowledge has been treated by Steinbring (2005). Steinbring contrasts between the social processes involved in teaching and learning mathematical knowledge and the historical development of mathematics, which was bound into social and cultural contexts, to understand the interactive generation of mathematical knowledge within the frame of the teaching culture. His *social epistemology of mathematical knowledge* (Steinbring 2005) stresses the particularity of the social existence of mathematical knowledge as an essential component of its theoretical approach. Mathematical knowledge is not understood as a finished product, but interpreted according to the epistemological conditions of its interactive development. Mathematical concepts are constructed in interaction processes as symbolic relational structures and are coded by means of *signs and symbols* that can be combined logically in mathematical operations. Epistemological characteristics of this knowledge are explicitly used in the analysis process: mathematical knowledge is characterized in a consistent way as a structure of relations between (new) symbols and reference contexts. The intended construction of meaning for unfamiliar mathematical signs, by building reasonable relations between signs and possible contexts of reference and interpretation, is a fundamental feature of an epistemological perspective on mathematical classroom interaction. This process of constructing meaning for mathematical signs is an essential element of every mathematical activity whether it is performed by a research mathematician or by a young child trying to understand elementary arithmetic symbols.

This review (by no way exhaustive) includes ideas from the best minds in the psychology of development and in research in mathematics education. However, empirical work on abstraction is limited. How should we then choose theoretical ideas from our predecessors? In fact, 'Piagetian' and 'Vygotskian' theoretical approaches do not conflict that much: the former focus on structures, levels and states and the latter on different kinds of ongoing activities. Both provide insightful ideas that have strongly influenced our model of abstraction in context. The standards we set and the design principles we formulated both have strong constructivist and social bases. In the activities designed to observe abstraction, individuals act alone or within groups, their actions being possibly mediated by a teacher or by tools. Giest (2005) showed that *activity theory* (Leont'ev 1981) provides a most suitable framework allowing the researcher to take a socio-constructivist approach in a cultural-historical frame: in activity theory, individual *actions* occur in a socio-historical context and are inseparable from overall *motives* that are perceivable only within the *activity* in which they take place. In addition, outcomes of previous activities naturally turn to artefacts in further activities, a feature that is crucial to trace the genesis and the development of abstraction through a succession of activities. The kinds of actions that are relevant to abstraction are *epistemic actions* – actions that pertain to the knowing of the participants and that are observable by participants and researchers. While the term epistemic actions has been used by other communities with different meanings (e.g. Neth & Payne 2002), Pontecorvo and Girardet (1993) have used it to describe how children developed their knowledge on a historical issue during a discussion. The observability is crucial since other participants (teacher or peers) may challenge, share or construct on what is made public. The epistemic actions we use to trace abstraction naturally convey the ideas of similarity and depth and may be nested in each other: nesting conveys the idea that some actions are performed to sustain constructions – actions whose object is new to the learner. The interplay between what is currently perceived and a new understanding suggests actions inside others.

The RBC+C model and its power for analyzing processes of AiC

In this section, we use the story of Chen, Yevgenia and Shany in order to elaborate a model for describing processes of mathematical abstraction in context (AiC) within the theoretical landscape drawn in the previous section, and we then demonstrate the analytic power of this model for explaining such processes. The aim of this section is to exhibit the added value that the model brings to mathematics education: We aim to show that the model provides micro-analytic tools for putting in evidence and interpreting processes of constructing and consolidating knowledge, interpretations that are frequently simply postulated or assumed a priori. This in turn will be shown to lead to insights into intricate issues such as how partially correct constructs explain students' inconsistent answers, and how justifications are produced and conceived of as such by learners.

In the first subsection, we select three epistemic actions as basic elements of the model. The second subsection deals with more elaborate aspects and forms of these epistemic actions as well as with relationships between them, which enable us to show that the model can explain the constructing of mathematically valuable constructs such as justifications. In the following subsection, we add another explanatory dimension to the model, namely that it can serve as a tool for identifying ways in which students' constructs may be only partially correct. In the next subsection, we expand on the brief discussion of consolidation given above, exhibit characteristics of the process and results of consolidation, and elaborate several mechanisms by which consolidation may occur. In the penultimate subsection, we do justice to the fact that all along we have named the object of our attention abstraction *in context*, discuss in detail some aspects of context and show how the model incorporates them. We conclude the section with an overview of research based on the model done by other research teams.

The RBC+C model

We recall that we define abstraction as an activity of vertically reorganizing previous mathematical constructs within mathematics and by mathematical means so as to lead to a construct that is new to the learner. We have also pointed out that the genesis of an abstraction passes through a three-stage process, which includes the need for a new construct, the emergence of the new construct and its consolidation. The need may arise from the design, from the student's interest in the topic or problem under consideration, or from combinations of both; without such a need, no process of abstraction will be initiated.

For the reasons pointed out above, we have chosen to use epistemic actions in order to model the central second stage of this process at the micro-analytic level. The three epistemic actions we have found relevant and useful for our purposes are Recognizing (R), Building-with (B) and Constructing (C). Above, these epistemic actions have been illustrated by means of the story. More generally, *recognizing* takes place when the learner recognizes that a specific previous knowledge construct is relevant to the problem he or she is dealing with. *Building-with* is an action comprising the combination of recognized constructs, in order to achieve a localized goal, such as the actualization of a strategy, a justification or the solution of a problem. The model suggests *constructing* as the central epistemic action of mathematical abstraction. Constructing consists of assembling and integrating previous constructs by vertical mathematization to produce a new construct. It refers to the learner's epistemic R-, B-, and possibly lower-level C-actions leading up to *the first time* the new construct is used or expressed by the learner, either through verbalization or through action. This definition of constructing does not imply that the learner has acquired the new construct once and for all; at this stage, the learner may, but need not, be fully aware of the new construct, and the learner's construct is often fragile and context-dependent. Constructing does not refer to the construct becoming freely and flexibly available to the learner. Becoming freely and flexibly available pertains to consolidation (see below).

C-actions, such as the one of the order principle in the story, depend on R- and B-actions; the R- and B-actions are the building blocks of the C-action; at the same time, the C-action is more than the collection of all R- and B-actions that make up the C-action, in the same sense as the whole is more than the sum of its parts. The C-action draws its power from the mathematical connections that link these building blocks and make them into a single whole unit. It is in this sense that we say that R- and B-actions are constitutive of and *nested* in the C-action. Similarly, R-actions are nested within B-actions since building-with a previous construct necessitates recognizing this construct, at least implicitly. Moreover, a lower-level C-action may be nested in a more global one, if the former is made for the sake of the latter. An example will be given in the next subsection. Given these characteristics, we named the model the *nested epistemic actions model of abstraction in context*, more simply the *RBC model*, or *RBC+C model* using the second C in order to point at the important role of consolidation.

The model emerged and was first described by means of illustrative examples in contexts differing by their mathematical content, social setting and research setting. We proposed and elaborated it on the basis of two case studies in which students were observed in laboratory settings: an interview with a single student (Hershkowitz, Schwarz & Dreyfus 2001, denoted henceforth as HSD), and the observation of dyads working in collaboration (Dreyfus, Hershkowitz & Schwarz 2001, denoted henceforth as DHS). It has since been used and validated in many further contexts; the following sections give an overview of this variety.

The model constitutes a methodological tool that we use for realizing the ideas of abstraction in context by means of a micro-level analysis of the learners' epistemic actions. In this sense, it has a somewhat technical nature. On the other hand, with a view to the discussion in the preceding section, it also has deep theoretical connotations. We refer the reader to the chapter by Hershkowitz in this volume for an in-depth discussion of the theoretical aspects versus the tool aspects of the model, and the relationships between these aspects.

Patterns of epistemic actions

In this subsection, we present two studies, in which the epistemic actions appear in patterns of differing complexity, and for good reasons. The first study (DHS 2001) will serve to further clarify what we mean by the need for a new construct, what C-actions are, and how they may be nested. Two knowledge elements were underlying the task design for the study: the use of algebra as a tool for justification and the extended distributive law, $(a + b)(c + d) = ac + ad + bc + bd$. The task was based on number arrays of the form shown in the diagram below (which we called seals).

X	X + 2
X + 6	X + 8

The study was carried out with pairs of introductory-algebra students, who had never used algebra as a tool for justifying a general statement, nor the expanded distributive law. The students were presented with two numerical seals and asked to suggest as many properties of such seals as they could find. They were allowed to use a spreadsheet. Most succeeded in finding many properties based on earlier similar experiences. The task then focused on the diagonal product property (DPP), namely that the difference of the products of the diagonals of a seal equals 12. The students were asked whether the DPP always hold, and to justify their claim. Most but not all student pairs succeeded in generating components of the equation $(x + 2)(x + 6) - x(x + 8) = 12$, which in turn led them to the need for the extended distributive law (although the law was not expressed in formal terms). The design of the task thus afforded the need for the intended new constructs. In what follows, we will denote these constructs by C_1 (algebra as a tool for justification) and C_2 (extended distributive law).

The epistemic actions of two student pairs working on this activity have been analysed at the micro-level in DHS (2001). We illustrate this analysis here by the following excerpt, in which two students start from the simple distributive law, which they had met before:

H 113: So, like, see, X plus 8, X, like, then it is 8X plus XX.
N 114: Again?
H 115: Wait. X times X plus 8, right? This is XX, like, X, X twice, so it's XX.
N 116: Yes.
H 117: And X times 8 is 8X.
...
H 121: So, like, one does the distributive law.
N 122: Ah, yes.
...
H 133: And this [pointing to (x+2)(x+6)]...
N 134: It's impossible to do the distributive law here. Wait, one can do...
H 135: This is 6X.
N 136: This is 6X times X and 6X times 2.
H 137: Wait, first, no...
N 138: Yes.
H 139: No because this is X plus 6, this is not 6X, it's different. Wait. First one does ... X; then it's XX plus 2X, and here 6X plus 24. Then...
...
H 152: Ah, it's XX plus 8X, but I don't know, like, how this [pointing to (x+2)(x+6)] will also be XX plus 8X. Like, it HAS to be.

In 113–121, the students recognize elementary algebraic elements (e.g. 115), and build-with them (e.g. 117) the simple distributive law in the present context (113, 121). In 133/134, they find themselves in front of an unknown situation, but, due to the design, firmly aware of the simple distributive law. They recognize (135) and build-with (136) algebraic elements, including the simple distributive law (137, 139), to reach a new (to them) result (139). Since this is the first time they have used and verbalized the extended distributive law, the process leading up to 139, including 133–139, is considered a C-action, namely C_2. This illustrates our definition of constructing, as well as of nesting: the C-action is constituted by and draws its power from the R- and B-actions in this excerpt and the connections between them.

The entire activity of the two students, is motivated by the encompassing C-action C_1, which begins when the students are asked whether the DPP always holds, and lasts until they are satisfied that they have established its general validity. C_2 occurs as part of C_1 and would presumably never have happened had it not been driven by C_1. This is most clearly expressed in 152. Thus C_1 subsumes, nested within it, not only a large number of R- and B-actions but also C_2. This relationship of C_2 being nested in and forming part of C_1 is symbolically represented in Figure 1.3, in which the time axis runs from top to bottom.

DHS (2001) is one of several papers in which we show in some detail that and how the RBC model provides tools for interpreting processes of constructing knowledge.

Nesting is one possible relationship between constructing actions. More elaborate processes of abstraction of more advanced mathematical contents are likely to lead to more complex relationships between constructing actions. We illustrate this by means of the construction of a justification for the second bifurcation point in a logistic dynamical system by a solitary learner L (Dreyfus & Kidron 2006). L is an experienced mathematician and her motivation for finding a justification drives her entire learning process. The researchers inferred her epistemic actions from her detailed notes during the learning experience and from her interaction with the computer. They found an overarching constructing action, within which four secondary constructing actions were nested. These secondary constructing actions relate to different modes of thinking: numerical (C_1), algebraic (C_2), analytic (C_3), and visual (C_4). They are not linearly ordered but took place in parallel and interacted (Figure 1.4; the time axis again runs from top to bottom; the numbers denote episodes). Interactions included branching of a new constructing action from an ongoing one (such as C_1 branching from C_2 at the beginning of episode 7), combining or recombining of constructing actions (such as C_1 and C_4 combining at the end of episode 10), and interruption and resumption of constructing actions.

The branching of C_1 from the ongoing C_2 can be explained by means of a refinement of the classification of building-with (B) epistemic actions. Specifically, a class of B-actions was discovered whose purpose it is to organize the problem space so as to make its further investigation possible. Such actions can lead to the requirement of additional constructions and thus branching. Interruptions and resumptions of constructions have been similarly explained by

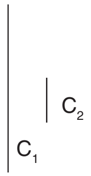

Figure 1.3 Nested C-actions.

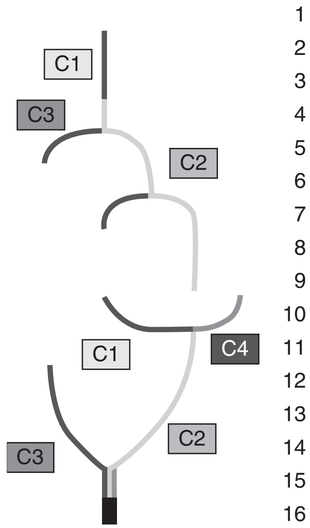

Figure 1.4 Interacting parallel constructions.

means of refined and/or modified R- and B-epistemic actions. Here, we relate more closely to the combining of constructions.

L aimed to justify results obtained empirically from her interaction with a computer. Her aim was not to convince herself or others, nor was she looking for conviction in the logical sense of the term; rather, she wanted to gain more insight into the phenomena causing the second bifurcation point. The term enlightenment, introduced by Rota (1997), seems appropriate to express her interpretation of the word justification (Kidron & Dreyfus 2007). Rota also pointed out that contrary to mathematical proof, enlightenment is a phenomenon that admits degrees. In L's learning experience, combining C-actions indicate steps in the justification process that lead to enlightenment.

In L's learning experience, we observe three successive degrees of enlightenment. They occur at the three points in time when C-actions combine, and each combining point was characterized by the integration of different C-actions and different modes of thinking. For example, the combining of C_1 and C_4 at the end of episode 10 expresses the connection in L's thinking of the numerical mode and a graphical mode, first a static graphical mode, and then a dynamic graphical mode. It led to L's enlightenment with respect to the nature of bifurcations in dynamic processes.

The relationship between combining constructions and justification has been confirmed in other contexts with students of different age groups dealing with different mathematical topics (Kidron & Dreyfus in press). Combining of constructions leads to enlightenment, not in the sense of a formal proof of the statement the learner wants to justify but as an insight into the understanding of the statement. This observation gives an analytic dimension to the RBC model and to its parallel constructions aspect: it allows researchers to use RBC analysis in order to identify a learner's enlightening justification.

Moreover, the analysis of the relationships between justification and parallel constructions led to the realization that often a weak and a strong branch are involved in the combining constructions (in the example, C_4 is the weak branch and C_1 the strong one), and that reinforcement of the weak branch played a crucial role in the construction of a justification. The realization that a weak and a strong branch combine considerably strengthens the theoretical root of the RBC model in Davydov's ideas as exposed above. Indeed, reinforcing the weak branch towards combination of constructions closely matches the description of the genesis of abstraction as expressed by Davydov's (1990) method of ascent, according to which abstraction starts from an initial, simple, undeveloped first form, which need not be internally and externally consistent, and ends with a consistent and elaborate final form.

Both examples in this subsection demonstrate vertical mathematization representing processes of constructing new mathematical knowledge within mathematics and by mathematical means. These processes often include a kind of insight or 'AAHA'. This expresses that the reorganization processes of the already constructed pieces of knowledge into a new construct are driven and strengthened by the genuine and creative mathematical thinking of the student.

Partially correct knowledge constructs

Another line of research on abstraction in context that also exhibits the analytic power of the RBC model deals with *partially correct constructs* (PaCCs). This term expresses a tension between the nature of actions undertaken purposely by students and the activities whose design invites normative construction. Ron, Dreyfus and Hershkowitz (2006, 2007) have used this term for students' constructs that only partially fit the mathematical principles underlying the learning context; they have shown how the RBC model may be used to identify PaCCs, and how PaCCs can be used as tools for interpreting situations in which some answers or actions of a student are inconsistent with others. These may be cases where the students' incorrect answers overshadow meaningful knowledge they have constructed, or cases in which students' correct answers hide knowledge gaps. For example, one of the analyses concerned students who gave inconsistent answers to problem 1 of activity 3 (Figure 1.1). In this problem, it is expected that many students predict Ruti to win in question 1b, basing their analysis on a 21-event sample space, just as Yevgenia did. However, it turns out that almost as many students predict Ruti to win in spite of answering in 1c that the sample space has 36 events. The answers 'Ruti wins' and 'the sample space has 36 events' appear to be inconsistent.

Micro-analysis of students' knowledge-construction processes by means of the RBC model has been used to explain such inconsistent answers. Indeed, using the epistemic actions of the RBC model as tracers, some of these students' constructs were identified as partially correct. In one specific case, for example, the 36-event answer for the size of the sample space was simply based on the product principle: the multiplication (6 times 6) of the sizes of the one-die sample spaces, without the student ever becoming aware of the implications of this multiplication for the order principle.

It thus turns out that PaCCs are useful as explanatory tools for correct answers based on (partially) faulty knowledge and for wrong answers based on largely correct knowledge, that abstraction in context is a suitable framework for defining the notion of PaCC and that the RBC model is an efficient tool for identifying PaCCs and their nature. Moreover, the elaboration of PaCCs avoids an important theoretical misunderstanding concerning the RBC model (Wagner 2005). In contrast to Wagner's approach, the RBC model does not tackle the problem of transfer. We do not use terms such as 'internal representations' or 'mental model' to refer to epistemic actions, and when we claim that a certain principle has been constructed, it always means that an understanding of the principle has been attained *hic et nunc*. What is recognized or built-with are often PaCCs or consolidations that resemble, in their flexibility, the readout strategies Wagner describes.

Consolidation

Neuro-psychologists define consolidation as the creation of long-term memory, typically for perceptual or motor skills (Robertson & Cohen 2006). Consolidation in the RBC model means something else; it refers to processes that involve a high level of consciousness. It does refer to a long span of time, but concerns the conscious reuse of a new construct. Recognizing a previous construct and building-with it may become immediate, self-evident and done with confidence, hence indicating high-quality consolidation. This development may be unlimited in time. We may thus be able to say that consolidation has started but not that it has been completed. Consolidation constitutes the third and last phase of the three-phase genesis of abstraction. Consolidation processes are extremely varied. However, in mathematics classrooms in which there is a didactic hierarchy, the design is expected to progressively ease recognition and use of new constructs by means of building-with actions. In other words, problem situations that are designed to generate a need for using a construct are crucial opportunities for consolidating it. Discussing the construct, examining it from different points of view and reflecting on it are other opportunities for consolidation. Further opportunities for consolidation potentially arise during later abstraction processes that make use of the new construct.

The activity described at the beginning of this chapter is part of a sequence of successive activities designed to lead to the construction of probability knowledge elements such as the order principle. This sequence helps studying abstraction as an ongoing process, consolidation being central because of the design decisions we took: we designed a 10-lesson learning unit with three stages: (1) calculating probabilities in one-dimensional sample space (1d SS); (2) calculating probabilities in two-dimensional sample space (2d SS) for cases where the possible simple events in each dimension are equi-probable – in such cases, the 2d simple events can be counted and organized in a table; (3) calculating probabilities in 2d SS for cases where there are two simple events in each dimension which need not be equi-probable. Although 'activity 3' (Figure 1.1) was the students' first opportunity to deal with a stage (2) situation, i.e. with events in a 2d SS, by the end of the second lesson, they confidently and immediately made use of the *all-events* element – identifying all possible simple events in a given 2d situation – to build-with it the answers to problems 1a and 1b. Judging from their written and oral (380, 397) answers, the action of recognizing all events in the 2d SS for this problem appears to be self-evident to them now. Adopting the criteria for consolidation proposed by Dreyfus and Tsamir (2004), immediacy, self-evidence, confidence, flexibility and awareness, we can conclude that the students have consolidated the *all-events* element within the particular context of using the event table as a tool for a sample space for two dice. Evidence from later activities and from the post-tests of the unit shows that all three students confidently and flexibly use the event table and recognize and build-with the all-events principle in other contexts also. We thus conclude that they continued consolidating this principle beyond the activity presented here.

A more detailed analysis of these episodes has been presented by Dreyfus, Hadas, Hershkowitz and Schwarz (2006). Several mechanisms of consolidation were identified. The first one consists of consolidating in the course of a new C-action: The students have been consolidating their construct of the all-events principle during the process of constructing the order principle. More generally, an earlier construct has been consolidated during the process of constructing a new one, with the earlier one serving as a resource in constructing the new one. An independent instance of the same mechanism of consolidation is a student's consolidation of her construct of derivative as limit during the process of constructing Euler's numerical method of solving differential equations (Kidron 2008).

Two other mechanisms identified by Dreyfus et al. (2006) have been extensively described by Dreyfus and Tsamir (2004), and confirmed by Monaghan and Ozmantar (2006). We therefore limit their discussion here to brief remarks. The second mechanism, consolidating a recent construct during building-with this construct, is the most frequent and most easily observed one. In the story, evidence for it can, for example, be found in students' answering further questions, which are similar but set in different contexts, progressively more quickly, more flexibly and with more self-confidence. Such flexibility expressed itself not only in students adapting to changing contexts but also in their becoming independent in generating their own, sometimes idiosyncratic tables, like our three girls in the post-test.

The third mechanism, consolidating a recent construct when recognizing it as an object of reflection, often stems from opportunities for reflection provided to students (e.g. requests for written reports). The data presented in the present chapter show reflective activity mainly with respect to the order principle. And in accordance with this, we can observe Chen's progressively more elaborate language for dealing with the *order* principle, from '1 and 2, and 2 and 1' (95) via '(1,2) and (2,1), are they different results?' (153), to '(1,2) and (2,1), that's not the same thing; it's a different event' (393). This more elaborate language expresses a more acute and fine-tuned awareness of the order principle; and as mentioned above, awareness is an important characteristic of consolidation.

The various mechanisms of consolidation point to learning processes involving abstraction: By fleshing out actions learners performed, successive consolidations subsequent to a construction stress important characteristics of learning processes linked to abstraction (e.g. comparisons between confidence, flexibility and awareness in the course of the successive consolidations). Research in these directions requires long-term data and is still at its beginning.

Context

Many contextual factors influence how a process of abstraction may be realized in a specific case. The components of context are varied and include: a mathematical curricular context, often including a sequence of activities designed with specific learning goals in mind, such as in our story; a historical context, including the students' previous learning experiences; a learning context, possibly including computerized environments with technological tools that may be at the students' disposal; and a social context – the norms enacted, with different social arrangements (small-group work, individual work and whole-class work). Abstraction is embedded in these contexts, and the term *abstraction in context* conveys the inseparability of abstraction from the context.

Research up to until now has considered many but not all possible contextual aspects; a sufficient number of such aspects have been taken into account to show how they are an inclusive part of the process of abstraction. For example, the mathematical context relates to what has been discussed in an earlier section of this chapter. When considering the mathematical context, the specific a-priori analysis of the activity design in terms of mathematical knowledge elements as well as the specific formulation of tasks has to be taken into account. When considering students' personal histories, not only the topics they have been dealing with previously are important factors but also the socio-mathematical norms in their present and previous classes and the typical organization of work in the classroom. Technological tools – a component of the learning context – have played a role in several research studies on abstraction in context, such as the computer simulation

> … in the second lesson of the story in the opening section or in the seals activity described above; the role of a technological tool has been analyzed in depth in the specific case of L's construction of a justification for bifurcations in dynamic processes mentioned above.
>
> (Kidron & Dreyfus 2007)

However, the influence of technological context in processes of abstraction needs to be investigated further. A rich semiotic system (e.g., several representations) may be very propitious for processes of abstraction.

Because of length limitations, we treat only one aspect of context in some detail here, namely social context. The social aspect of the RBC model of abstraction is inherent and ubiquitous. Classrooms provide different social settings in which individuals, small groups or the whole class are involved, and considering epistemic actions in such different settings is a challenge. The relationships between the construction of knowledge by individuals and the 'shared knowledge' of the class community or a group in the class, in which individuals contribute to the same activity, is a fascinating issue both from cognitive and socio-cultural points of views. Understanding the relations between the construction of knowledge by individuals and what we will call 'the constructing of the group's shared knowledge' is crucial in research about learning processes in the classroom, and evolves from the cognitive as well as the social domain.

As is apparent from the story in the opening section, knowledge may be constructed during group work, individual work or whole-class discussions, and social interaction may play a crucial role in processes of abstraction. In the story, we focus on processes of constructing knowledge in a group of three interacting girls, where personal diversity, the unique nature of each individual, is observed and analysed. We emphasize the flow of knowledge from one student to the others, until they have a common basis of knowledge. If their common basis of knowledge allows the students in the group to continue constructing knowledge collaboratively and actualizing it in further activities, we identify this as *shared knowledge* – a common basis of knowledge which allows the students in the group to continue together the construction of further knowledge in the same topic (Hershkowitz, Hadas, Dreyfus & Schwarz 2007).

Our research on shared knowledge stands on the shoulders of research of Voigt (1995), Cobb et al. (2001), and many others, but it goes beyond theirs in several perspectives:

- The *micro*-perspective: Our micro-analysis provides detailed evidence of the constructing process of individuals, of the group's shared base of knowledge, the constructions of individuals, of the manner in which this shared knowledge emerges from the individuals' knowledge-constructing processes, and the way in which it constitutes a shared basis that allows the students to continue constructing further knowledge together.
- The *continuity* perspective of the micro-analyses: We tie the data and their analysis together along a time span of several lessons in order to observe and analyse students constructing new mathematical knowledge in one activity, and consolidating it in further activities.
- The *theoretical* perspective: We consider how students go through an abstracting process in interaction with other students who go through parallel processes of abstraction.
- The *methodological* perspective: We use the RBC+C model as a tool for these analyses.

Our research on shared knowledge is based on, and continues the study of social interaction in knowledge construction undertaken with the seals activity (DHS 2001). In that study, we independently undertook a cognitive and a social analysis of the interview protocols, with the aim of comparing them. The cognitive analysis used the RBC epistemic actions, and enabled the generation of diagrams showing episodes of the constructing processes. The social interaction analysis used common categories such as explanation, query and agreement, as well as diagrammatic reference of each utterance to previous utterances. It enabled the generation of diagrams showing blocks of interaction. A main result of the research was that the sequences of epistemic actions and interaction patterns were congruent.

Hershkowitz et al. (2007) used the above story and similar ones to extend this analysis in two essential ways: The data stem from classrooms rather than from a laboratory; and the activity took place over an extended timescale, allowing us to focus on consolidating processes in addition to constructing processes.

The three girls from the story shared awareness of the issue of the order construct early on, but they did not arrive at an agreement until much later, in a dialectic process. There was a set of three interaction cycles in this story:

1. In the first cycle, the three girls engaged in an argumentative process to elaborate hypotheses concerning the importance of the order in a given pair. A shared agreement about the need to discuss this problem is evidenced.
2. In the second cycle, Yevgenia changed her mind about this principle and aggressively tried to force her opinion on the other two (93, 98, and 118). Shany followed her, but Chen opposed her whenever an opportunity arose (95, 153). The denial of the principle was shared by Yevgenia and Shany only.
3. In the third cycle, after the table was presented by the teacher and adopted by the girls, it became clear that Chen was right. Yevgenia became quite silent and worked alone, leaving Chen to lead the work of the group. A homework assignment and the post-test a few weeks later suggest that the order principle had by then been constructed and even consolidated by all three of them. The group reached shared knowledge.

In conclusion, our empirical work has strengthened the evidence for the high diversity in individual students' ways of participating in a group's construction of knowledge, as claimed by Cobb et al. (2001) when defining taken-as-shared activities of students in the same group or class. In addition, it exemplifies a number of paths by which individuals achieve C-actions. We also showed, as did Voigt (1995), that interaction – even between students in a small group – has many faces.

Research on the role of teacher–student interactions in abstraction through the lens of the RBC model has been quite limited in spite of the centrality of the teacher in the construction of knowledge in classrooms. The main reason for the lack of research in this domain may be the methodological problem that must be overcome to study the role of the teacher in construction of knowledge: in teacher-led discussions, most of the students remain silent most of the time; also, the degree of autonomy of the students is often quite limited. We nevertheless initiated a new path of research according to which we first identified the kind of dialogue the teacher initiates (grounding, prospective, critical, reflective, etc.) and within each of these dialogues types we identified teaching actions (Schwarz, Dreyfus, Hadas & Hershkowitz 2004). Although all dialogues may be important for knowledge construction, we showed the centrality of *critical* dialogue in which the teacher *elicited* or *mediated* argumentation and incessantly fed collective argumentation with *grounding* actions (linking problem-solving moves to students' informal knowledge). Such actions fit what Cobb and colleagues (this volume) call conceptual agency in contrast to disciplinary agency that generally does not help in conceptual shift. We showed that epistemic actions could then be enabled, and could lead to abstraction of probability knowledge elements (Schwarz, Hershkowitz & Azmon 2006; see also our summary below of Ozmantar & Monaghan 2007 on tutoring).

Trends in RBC-model-based research by independent groups

In this subsection we describe some trends which can be discerned in research by independent groups who make use of the RBC model, thus validating the AiC framework, or even expanding the model and discovering new meanings in it. Because of the dynamicity of the situation and due to space limitations, we can only partially exemplify these trends. These trends have a lot in common but the main focus of each, and how it relates to abstraction in context, is different.

A first trend, exemplified by Wood, Williams and McNeal (2006), is 'to describe, analyze and interpret the relationships between children's verbalized thinking and specific interaction patterns' (p. 228), in classrooms from different cultures. The RBC model served as 'conceptual framework employed to examine the quality of students' expressed thinking' (p. 225), while the interaction patterns were examined by a different framework. Here, the model is applied as a framework for both theory and methodology: it seems that on one hand the authors believe theoretically that the RBC model may express quite accurately children's level of mathematical thinking. On the other hand, they indeed use the model for analysing the protocols of different children and identifying each child's level of thinking. Based on these data, they gain the possibility for quantitative analyses of the children's levels of thinking. This research provides evidence for the maturation of the model as a theoretical framework and as methodology (Hershkowitz, this volume).

A second trend, a focus on the model itself and its features, is exemplified by the extensive work of Ozmantar and Monaghan (2007; Monaghan & Ozmantar 2006). It provides the most thorough independent validation of the nested epistemic actions model for abstraction in context to date. Their analysis of interviews with 20 high school students constructing absolute value transformations of functions with the scaffolding of an interviewer-tutor have not only confirmed the appropriateness and usefulness of the model for analysing processes of abstraction in context but also led to the significant theoretical developments mentioned earlier. This is not the place to give a full account of their work, and we therefore only mention a few of their conclusions:

- They present mediation by means of artefacts and by means of tutor intervention during students' processes of abstraction as two sides of the same coin; their data show clearly that scaffolding (or mediating) by the tutor, while being relatively minor during periods when students limit themselves to R- and B-actions, was crucial in the course of C-actions. In particular, they identified three functions of tutor intervention that were important for supporting constructing: reducing uncertainty, directing student attention and setting sub-goals.
- They also clearly identify Davydov's dialectical view of abstraction as proceeding from analysis to synthesis in the processes of abstraction they exhibit in their data. This includes the ultimate ascension to the concrete, where particular instances become more sensible after construction.
- Finally, they ask the question, what structures are being constructed? Throughout the years, this question was frequently discussed in our group.

> Although we have earlier used the term 'structure' ourselves, we have carefully avoided it in the past few years, referring instead to constructs.

A third trend relates to the expansion of the social contexts considered. For example, Dooley's (2007) classroom research investigates a whole-class interaction. Her aim is to show how the class community as one entity reaches 'sophisticated constructions' of knowledge. For this purpose, she carried out a micro-analysis of one lesson of the class community (students and teacher). In analogy to the 'collective argumentation' of Krummheuer (1995), we might call this a 'collective abstraction' process, where different students contribute different building blocks to the process of constructing a new knowledge construct (abstracting process). This 'collective abstracting' raises many questions: What can we learn from the above kind of research about abstracting in a classroom? What can we say about the classroom community, not only as one entity but as a community which consists of all the individual students who belong to this community? Do we have a methodological tool which enables us to give some answers to such questions?

The work of Stehlíková (2003a, b) similarly extends (or rather restricts) the social context, in that she examined the applicability of AiC and the validity of the RBC model in two different situations, both of which she had previously analysed using other means, such as the theory of proceptual thinking. Her previous analyses concentrated mainly on the mathematical achievements and understandings of the subjects. In one case (Stehlíková 2003a), the subject was the researcher herself, an experienced mathematician, and the method was introspection based on extensive and detailed notes taken during the learning experience. A similar case was investigated later by Dreyfus and Kidron (2006). In the other case (Stehlíková 2003b), the subjects were four mathematics teachers, each of whom worked separately; however, the RBC analysis was carried out on a model that integrated the work of all four. Stehlíková found that in both cases, AiC was not only applicable to her data but provided additional insights by prompting questions concerning the hierarchy of constructions, the means by which new constructs emerge and the kind of use that is later made of them. She thus helped raise, at an early stage of development of the AiC framework, a number of questions that have since been answered, for example by Ron et al. (2007) using the notions of knowledge element and PaCC (see above). Not unexpectedly, we thus observe that the research of both Dooley and Stehlíková have raised important issues and questions leading to further research and hence further generality and maturity of the AiC theoretical framework.

Finally, several researchers have successfully combined AiC with other theories. A partial review of such endeavours may be found in the chapter by Bikner-Ahsbahs and Williams in this volume (see also Kidron 2008).

New research trends in abstraction: issues on argumentation, motivation, design-based research and learning

In this chapter, we elaborated a model for studying abstraction in context. While the tools provided by the model enable tracing abstraction processes in various contexts, many of the basic issues involved in abstraction are still unexplored. For example, we mentioned that the genesis of abstraction stems from a need for a new construct. If the construction and its consolidation extend over a long period, we should question what drives students to engage in such a difficult and long process. The term 'need for abstraction', which we adopted, does not explain the origins of this need. The origins in classroom activities may be varied. Several researchers from outside the mathematics education community have indicated that argumentation may provide a fertile ground for abstracting (e.g. Baker 2003). In the social context of classrooms, students invited to account for their solutions to problems they solve in small groups naturally try to convince each other or to defend themselves. Also, teachers committed to co-construction of meaning in their classes, like some of the teachers involved in the experiment in probability, led whole-class discussions dominated by rich argumentation processes (Schwarz et al. 2004). After all, the relation between abstraction and argumentation is not surprising, especially when abstraction concerns, as in our case, conceptual knowledge. Baker, as well as Asterhan and Schwarz, show in this volume the role of argumentation in learning: argumentation, when it is dialogical and also dialectical, leads discussants to explain to others (or to themselves) their arguments and to open and/or consider critically new perspectives. These two processes, explaining and considering new perspectives, are crucial in general for conceptual learning and change. But still, we replaced a difficulty by another one, as it is known (Asterhan & Schwarz, this volume) that inviting students to participate in productive collective argumentation does not necessarily lead to what the educator expects, and students may remain stuck with their own arguments, without engaging in further discussion. Research on *achievement goals* may provide new, interesting directions on the untapped relation between motivation and abstraction.

Studying relations between argumentation, motivation and abstraction is a big challenge. Another route concerns design and learning: the design of sequences of activities according to the guidelines we listed constitutes an essential framework for the emergence and development of abstraction. This hierarchical design opened opportunities for learning processes involving abstraction. The combination of invitation to operate enquiry-based strategies (e.g. hypothesizing) and argumentation provided propitious grounds for abstraction to develop. In our efforts we initiated a programme of *design-based research* focusing on abstraction: the choice of a topic and of design guidelines, the development of a sequence of activities, followed by trial implementation in classrooms, systematic research about students' processes of abstraction supported by the design, and finally reflection and improvement of the design based on this research. An increasing number of such studies might progressively delineate topic-independent and specific design guidelines.

Seeking various constructions and consolidations is the empirical counterpart of the design effort to afford abstraction. The construction and the consolidation stages refer to different possible types of learning processes: construction may remain local and may become ephemeral if not sustained by appropriate design. At the present stage, our understanding of consolidation processes is still limited: we identified useful types of consolidation, or actions that may serve as evidence for consolidation, but we did not yet trace consolidation through an extended series of activities. We did not compare successive consolidations in immediacy, self-evidence, confidence, flexibility or awareness. A particularly interesting issue is to trace awareness in successive activities: in some cases, learning processes involved in abstraction may uncover progressively impoverished consolidations during which learners act more and more efficiently but more and more automatically. In other cases, consolidations may deepen meaning that emerged in initial constructions. Both cases may be desirable, depending on the circumstances. Opening opportunities for their occurrence and their actual coming out is at the heart of design, and of *design-based research*.

Observing learning processes involved in abstraction through series of consolidations is thus still a budding affair. In a pioneering research aimed at tracing learning in successive activities in which dyads solved a problem involving proportional reasoning, Schwarz, Perret-Clermont, Trognon and Marro (2008) discerned four processes: *unguided emergent construction in interaction*; *guided emergent construction in interaction*; *continuing construction from interaction*; and *retrieved construction from interaction*. This study does not focus on specific epistemic actions and the term 'construction' is used in a slightly different meaning from that used in AiC. This study may suggest ways to link between abstraction and learning processes by characterizing *chunks of epistemic actions*: We saw how, in their unguided interaction, three girls constructed a principle in probability. The variety of types of consolidation identified locally echoes the distinction between the more global processes of *continuing construction from interaction* and *retrieved construction from interaction*: The consolidation through constructing a new principle that we identified above fits very well the *continuing construction from interaction*, while the consolidation type we identified in operating building-with actions in an activity subsequent to the construction of a principle fits well *retrieved construction from interaction*.

Such considerations suggest the continuation of longitudinal studies of abstraction in context. Our research programme developed from interviewing one student during a single interview in an isolated problem situation in laboratory conditions (HSD 2001), continued by interviewing dyads who were working on a rather long activity in laboratory conditions (DHS 2001), to research on a single student over three meetings in laboratory conditions (Dreyfus & Tsamir 2004), dyads of students in a working classroom over three activities, with a few months' intervals between the activities (Tabach, Hershkowitz & Schwarz 2006), and groups of three students in their classroom during two lessons (Hershkowitz et al. 2007). This direction should be expanded to investigate abstraction and consolidation during longer sequences of activities.

Literature on research in mathematical abstraction has traditionally avoided treating the role of the teacher in guiding processes of abstraction. To some extent, this chapter does not make an exception to this rule. The tradition is understandable since researchers in this field have generally belonged to a constructivist approach according to which high-level processes involve autonomy. However, research is now more mature and theorists in socio-cultural psychology have pointed at the compatibility between guided participation and autonomy (e.g. Rogoff 2003). Schwarz et al. (2004) began in this direction and take into account the teacher's role in different social settings (individuals, groups and the whole class).

Clearly these future directions of research are not independent but can and should be interwoven. We should weave more threads between design processes, constructions, consolidations and further consolidations along considerable time intervals, as well as between constructions of individuals and groups of students with and without teacher support to better understand mathematical abstraction as it develops in classrooms. The nested epistemic actions model for abstraction in context gives us tools for this enterprise.

Acknowledgement

The analysis reported in this chapter was supported by the Israel Science Foundation under grant No. 1166/05.

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Chapter 2

The construction of physics knowledge in the classroom from different perspectives

The classroom as a community and the students as individuals

Andrée Tiberghien and Layal Malkoun

Introduction

This chapter contributes to research studies on physics classrooms from a methodological perspective. It is focused on the reconstructions of the knowledge that is involved in the classroom during teaching sequences devoted to a specific topic of physics at secondary school level. These reconstructions, mainly done from video recordings of the classroom during teaching sequences, can be carried out from different perspectives, that of one participant of the class, the teacher or a student, or that of the class as a group. Our methodological approach is based on several theoretical positions concerning knowledge, teaching and learning.

Theoretical position on knowledge

Our theoretical position on knowledge is based on the work of Chevallard (1991). First let us note that the term 'knowledge' has a broad meaning; it does not denote just content, but also includes the procedural components, the embedded epistemology of knowledge and the way its meaning is constructed. Chevallard addressed the subject of knowledge using the metaphor of life. Knowledge 'lives' within groups of people (a class in our case). Therefore knowledge depends on the group of people in whom it lives. So the knowledge that lives in a given classroom is specific to this classroom. We call this knowledge the 'taught knowledge' that is then specific to a classroom. The knowledge that the official curriculum requires to be taught is called the 'knowledge to be taught'; it mainly consists of the official curriculum and textbooks. Usually, several groups of people (scientists, inspectors, teachers) are in charge of producing it; they write texts to disseminate it. This knowledge to be taught, in physics for example, is different from scientific physics knowledge. The scientific knowledge lives in the scientific community.

In contrast to the knowledge to be taught, the taught knowledge in a classroom is largely enacted through oral and gestural productions, making it mainly interactional, and ephemeral. A major consequence of this is that the taught knowledge is not directly available to researchers through textual documents and must be reconstructed from emergent data. Until recently, relevant data was very difficult to collect; nowadays, the use of video and tools to manipulate digital files allows researchers to obtain relevant data to reconstruct the taught knowledge. Reconstructing it depends on an examination of what is being investigated in the classroom. This is why we are presenting our theoretical position on teaching and learning.

Theoretical position on teaching and learning

Following Vygotsky, we claim that learning physics will not happen without formal instruction and without the mediation of a teacher, or a more informed peer, and mediational signs.

Position on teaching and learning

According to Sensevy (2007), a classroom is an 'entity' viewed as a community of practice where two joint actions take place: *teaching and learning within a communicative process*. These actions are different but based on *communication* oriented by the goal given by society: the students' acquisition of the knowledge to be taught. This is due to the institutional role of the classroom, which is a response to the social demand to educate and instruct young people. These considerations lead us to investigate these actions of teaching and learning through the 'life of knowledge' in the classroom; that is, the taught knowledge.

In the following, we present concepts that characterize classrooms and come from a long French tradition of theorizing the didactic action. Then we relate them to other concepts that come from different research traditions. The following French concepts of chronogenesis, topogenesis, mesogenesis and didactic contract account for the life of knowledge, its condition, and the actors' relation to it in the classroom. *Chronogenesis* accounts for the development of knowledge during teaching and involves a relationship between knowledge and time. *Topogenesis*, still in relation to the metaphor of life of knowledge in a classroom, means the places of knowledge in the classroom; that is, which actors take responsibility for introducing/using elements of knowledge, and to what extent their responsibility is recognized by the class. *Mesogenesis* is related to the 'milieu', that is the environment, including the material components with which knowledge and meaning are constructed. The *didactic contract*, introduced by Brousseau (1998), constitutes a system of norms; some norms are generic and endure, and others are specific to certain elements of knowledge and should be redefined with the introduction of new elements. The concepts of chrono-, topo-, mesogenesis and didactic contract *characterize class-level phenomena* and not the level of the learner or the teacher as individuals. These concepts can be related to other concepts allowing the researchers to differentiate *collective and individual processes*. For example, Engle (2006) proposed to

> determine whether the generalizations or multiple examples became part of the *common ground for the collective*, and then consider this as content available to be potentially appropriated by individuals, with the recognition that content is often transformed during that process.
>
> (p. 455; emphasis added)

This idea of considering the common ground as content available to the students is also present in the notion of the 'learning trajectory of the classroom community' proposed by Cobb and McClain (2003). For them, this trajectory is part of 'the immediate social context of all the individual students' learning' (p. 5) that stresses the differences between individual learning and the learning trajectory. Like our concept of chronogenesis, the 'learning trajectory of the classroom community' addresses class-level phenomena and not an individual in a community.

In another research tradition oriented on individual students' learning in the classroom, the differentiation between an individual student's development of knowledge and the taught knowledge during a teaching sequence has also been emphasized (Niedderer, Budde, Givry, Psillos & Tiberghien 2007). An essential claim is that students' developing knowledge can be such that, at any given point, their knowledge can be closer to the taught knowledge than their knowledge at the beginning of the teaching sequence but still incorrect from the knowledge taught point of view. Moreover, students' development of knowledge does not happen in a rational decomposition of disciplinary knowledge; students can acquire new knowledge that is different from the taught knowledge.

Studying the articulation between the collective and individual processes that happen in the classroom is still an open question. In the following, we propose similar methodological approaches to analyse students' and classrooms' productions that could contribute to investigate this articulation. We now present how the concepts introduced above, and in particular chronogenesis and students' development, can be used to reconstruct the life of knowledge in classrooms during a teaching sequence.

Methodology

The aim of this section is to make the above concepts operational in order to characterize classroom practices.

Collected data

Our data was collected in the context of a design-based research project of teaching sequences in mechanics at Grades 10 and 11 (SESAMES 2007). At Grade 10, two classes were videotaped during the part of mechanics focused on dynamics; in one class, the teacher followed the SESAMES teaching sequence and the teacher of the other class used his own teaching sequence. The teachers of these two classes are experienced. The two schools are situated in middle-class areas in France. In each class, two cameras were used, one focused on the teacher and a part of the class and the other one focused on two students (the same students during the whole teaching sequence) and a part of the class. At Grade 11, one class was videotaped during the teaching part of dynamics, and the camera was focused on two students during the teaching sequence.

Two choices: references and scales

Our main source of data is video recordings; they account for a rich density of information. Dealing with such data to investigate complex phenomena characterizing a classroom necessitates several important choices; in particular, the 'reference points' and the 'scale' used in the analyses.

Reference points

Our aim is to reconstruct first the taught knowledge without limiting it to a label that, like a title, denotes the content, and, second, the knowledge development of a student during a teaching sequence.

Going from oral and gestural productions, given in a classroom by the different actors (teacher and students), to knowledge supposes that the researcher reconstructs a meaning of these productions. However, different meanings can be constructed from the same situation. For example, when the teacher says: 'the moving object has an acceleration', for the teacher (and the physicist) it means that the velocity of the moving object changes, either increases or decreases, and for the students it typically means that the velocity increases like in everyday meaning. More generally, following Bange (1992) in his interpretation of Grice, we have distinguished conventional and situational meanings. In the classroom, the meaning given to an utterance made by the teacher or by a student can be constructed from the perspective of knowledge to be taught (knowledge that the society requires to teach). The meaning of this utterance can also be constructed from a student's perspective. Then the taught knowledge is constructed from the classroom productions with the conventional meaning. The student's knowledge is constructed from his/her productions, in a given situation, from his/her perspective; in this case the referent is internal, and this reconstruction necessitates taking into account the student's 'history'.

Scales of analysis

The complexity of the classroom has led us to use several scales or levels of analysis. We follow Lemke (2001) on the idea that a very detailed analysis at a micro-level does not allow researchers to structure analysis at a higher level:

> Activities at higher levels of organization are emergent, their functions cannot be defined at lower scales, but only in relation to still higher ones.... Going 'up' we know the units, but we know *neither the patterns of organization nor the properties of the emergent higher-level phenomena*.
>
> (p. 25)

To reconstruct the taught knowledge we take three scales – macro-, meso- and microscopic – which include both time and granularity of knowledge. Studying the life of knowledge in a group necessitates several scales to grasp its development, like Hakkarainen and Paavola (this volume), who approach the creation of knowledge from the learner's perspective, and state that the process of creation is discontinuous in nature.

Macroscopic scale: This scale concerns the whole teaching sequence. The macro-analysis gives the conceptual structuring of the sequence in a chronological order but without duration. It also gives the main invariant elements of the didactic contract, in particular the norms established in a classroom. We do not develop it here (Malkoun 2007).

Mesoscopic scale: At the meso-scale, due to our approach, we have chosen a thematic analysis in order to keep the meaning of the taught knowledge involved in the classroom according to the chosen reference point, the conventional meaning. Structuring in themes is based on a thematic coherence *and* on a discourse analysis; most of the time there are discourse markers of introduction and conclusion. The theme is the mesoscopic unit of analysis; this unit can have different durations. However, it is at a mesoscopic scale of the physical time and the knowledge, in the sense that its duration ranges from a few minutes to half an hour; and the granularity of knowledge is lower than that of the knowledge included in the whole sequence and bigger than an element of knowledge given in a single utterance. Its delimitation depends on *knowledge and communication*. This unit is particularly relevant to investigate the students' and teacher's responsibility for knowledge development and display (topogenesis). The didactic contract, taking into account the situation, can also be analysed deeply in each unit. We do not develop this aspect in this chapter (Tiberghien & Malkoun 2007; Malkoun 2007).

Microscopic scale: At the micro-scale we have chosen two types of analysis: facets and epistemic tasks (we only present facets in this chapter). Facets correspond to small elements of knowledge. Our way of using facets comes from Minstrell (1992) and Galili and Hazan (2000), but our use differs in two respects: (1) we consider that facets are referents in discursive production analysis (either conventional or situational) – let us note that facets themselves are constructed with two reference points; (2) the second aspect is more methodological. When the analysis of data in terms of facets is done, we carry out our treatment of facets *before* interpreting them in terms of knowledge or conceptions.

Analyses from the students' perspective: students' developing knowledge

This study is a part of Küçüközer's PhD (2005) carried out at Grade 11 for a teaching sequence on mechanics. During this teaching sequence, a series of tasks (called activities in the classroom's language) has to be carried out by students working in pairs. The analysis of the students' developing knowledge is mainly based on one pair of students when the class is organized to work in small groups. This analysis is situated in the life of the classroom community. In particular, the unfolding of classroom events has been divided into three levels: activities (tasks), episodes, and steps. The activities (tasks) are under the teacher's responsibility to the extent that s/he assigns them to all the students, so these activities are part of the taught knowledge. The episodes often correspond to the different questions of an activity (task). However, they are, at least partly, under the students' responsibility to the extent that the students can change their order. The steps are under the students' responsibility; they depend on the way the pair of students construct their answer to the question. These three levels illustrate the relationship between the taught knowledge which is attached to the classroom community, and which is part of the social context of students' developing knowledge, and the students' knowledge. An activity (task), decided by the teacher, allows the student to develop his/her knowledge, and the student's way (or a pair of students' way) of carrying out the activity, in particular his/her steps, is an expression of his/her developing knowledge. Each step is analysed in terms of facets. For this, we have constructed a set of facets with an iterative process involving an a-priori list of facets deduced from students' conceptions, and the students'/teacher's productions. When the researcher considers that an element of a student's production has the same meaning as a facet, this facet is coded.

The example comes from the first activity of the part of a teaching sequence devoted to dynamics (just after the part on kinematics). The activity statement and the first question are given in Figure 2.1.

We present the transcript of the first step of the students' work in pairs; the students' work on the first question constitutes an episode. L and N are the students.

L 1: Look at and note the moments when you exerted [L reads] did you see the first question?
N 2: What? Which part?
L 3: When you exert an action on the medicine ball to begin you throw it using an upwards force
N 4: Yeah after
L 5: To catch it we exert a downwards force
N 6: Um no an upwards force when we catch it it's always an upwards force
L 7: [L experiments] Upwards like that [L experiments again]
N 8: Yeah but when we catch it you exert a force upwards as well
L 9: But you absorb [L experiments]
N 10: Yeah well [N takes the medicine ball] you make [N experiments] I am sorry I do not move
L 11: When you make that you move [L experiments]
N 12: Yeah but yeah

In this extract N and L agree on the direction of the action (or force) when they throw the medicine ball upwards but disagree when they catch the medicine ball; for L it is downwards and for N it is upwards.

This oral production corresponds to three facets:

- The way in which A exerts an action on B gives the indication of the direction of the force exerted by A on B.
- When an object is in contact with others then it exerts a force on these objects.
- Knowing the direction (orientation) of an action on an object, one realizes the direction of his/her own action on this object.

The first facet corresponds to explicit verbal productions (force upwards and downwards, even if it is incorrect for L). L and N claim that there is a force when

Throw the medicine-ball [a 'heavy' ball] up vertically and catch it.

1. Identify and note the moment(s) when you exert an action on the medicine-ball, state precisely each time and in what direction you exert this action on the medicine-ball.

2. ...

Figure 2.1 Activity statement given at Grade 10 and Grade 11 in 'sequence SESAMES'.

the medicine ball is thrown (turn 3 for L and 8 for N, who agrees) and is caught (L: turn 5; N: 6, 8). The second facet is inferred because we assume that, for the students, there is contact between the ball and the hands when the ball is thrown. (L did not maintain the direction of the action downwards; on his written note, the force is drawn upwards). The third facet is rather similar to the first but involves the learner's perception; it corresponds to the actions of the student, who throws and catches the medicine ball to convince him/herself and his/her partner (L: 7, 9, 11; N: 10).

The results of this type of analysis consist, for example, in the number of the most frequent facets used by a student during a teaching sequence or the evolution of the facets used (see Küçüközer 2005). More importantly, for each student, we have reconstructed the chronological list of facets during the teaching sequence in relation to the main component of Newton's Second Law introduced in the taught knowledge at this level, that is the relation between force and variation in velocity (acceleration is not introduced at this grade in France). For example, for student L, we obtained the following series of facets:

1. If the forces exerted on an object compensate each other, the object has a uniform rectilinear motion.
2. There is no link between the sum of the forces exerted and the variation (or the value) of the velocity.
3. The relation of dynamics is 'if the velocity of the inertial centre of a system varies then the sum of forces which is exerted on this system is not nil'.
4. The relation of dynamics is 'the vector sum of forces and the vector variation of velocity are collinear and have the same direction'.
5. The vector variation of velocity between two points is obtained by subtracting the velocity of the two vectors at these points.
6. For a vector subtraction, the vector 'minus' (vector with opposite direction) has to be constructed.

This chronological list shows three main points. First, L knows the principle of inertia (he uses it correctly during the steps 3 and 4 of activity 1, facet 1). Second, at the beginning of activity 3, L states a proposal in contradiction with Newton's Second Law (facet 2) and uses this relation correctly at the end (facets 3, 4). This evolution appears during this activity and we consider that it is because student L has acquired several elements of knowledge before this activity. This comment introduces the third point: the importance of the velocity in this construction of meaning of the Second Law (facets 3, 4, 5, 6); velocity is involved at different moments and contexts: vector calculus, relation between the results of a vector calculus and the variation of the velocity. The analysis leads us to consider that L constructs Newton's Second Law on the bases of the principle of inertia and of the vector construction of velocity and force. During task 1, L characterizes the type of motion and relates it to force; he also constructs relationships between the types of specific motion and variation of velocity. This result emphasizes the role of vector constructions of velocity and forces; these symbolic representations involve students' actions, they are used as references in the students' discussion and the tasks lead the students to interpret them at the conceptual level and also in relation to the material situation.

The decomposition into facets shows that small elements of knowledge which can seem uninteresting, like 'the vector's "instantaneous velocity" has a direction and a value' or 'one finds the vector "variation of velocity" by subtracting two vectors [velocity at two close points]', have to be learned and play a determining role in students' communication and in conceptual understanding. These results have consequences in the teaching sequence, its content, the duration and type of activities and the way of discussing and correcting them in order not to neglect the important role of these elements of knowledge. They also show the students' developing knowledge over several sessions, which can be compared to the development of the taught knowledge (presented in the following section). However, other approaches of students' developing knowledge should complement ours to analyse the understanding processes like abstraction (Schwarz, Dreyfus & Hershkowitz, this volume).

Analyses from the conventional perspective: the taught knowledge

These analyses are carried out on video data of two physics classes (mechanics, Grade 10) at several scales; they can be used to reconstruct the taught knowledge. That is, the classroom's productions are analysed from the taught perspective (conventional meaning). We present some of the reconstructions on both a meso- and microscopic scale.

Reconstruction on the meso-scale: themes

As we have introduced in the methodology, at the meso-level, our analysis is thematic. Figure 2.2 presents the series of themes in two classes to show that this analysis allows the researcher to compare the succession of themes (Tiberghien & Buty 2007; Tiberghien & Malkoun 2007). This figure shows that class 1 and 2 start with different concepts; class 1 with action and class 2 with the effects of force. It also appears that class 1 has several themes corresponding to a single one in class 2 (Figure 2.2 'modelling actions by the forces', in particular representation of forces in class 2). Figure 2.2 is a representation of the chronogenesis of the two classes.

Session	Time (min.)	Themes in class 1
		...
S I	1:25	*Introduction of the general theme of the notion of force*
	18:44	1. Determination of phases of motion of an object, direction of action on this object, variation of velocity
	10:41	2. Analysis of interactions for different phases of motion of an object (case of a medicine ball)
	4:41	3. Introduction of the force and its vector representation and of the principle of reciprocal actions
	9:23	4. Using (exercising force and its vector representation from interactions (use of the full model of interatctions)
S II	5:14	5. Interactions: relstions between a symbolic representation and one or several material situations
	10:10	6. Representation of force (with direction) modelling an interaction (not the length of the vectors)
	30:31	7. Representation of force modelling a moving object

Themes in class 2	Time (min.)	Session
1. Effects of force on the motion of the object	18	S I
2. *Interactions*		
2a. Interactions = A acts on B then B acts on A	14:33	
2b. Interactions at distance and contact interactions	4:39	
3. Revision of interactions	1:31	S II
4. Modelling actions by the forces (representation and measurement of forces)	34	
...		

Figure 2.2 Comparison of the development of themes in two classes (Grade 10) from the introduction of the notion of force to the introduction of the inertia principle (first sessions). The bold line between the cells means a new session (source: from Tiberghien et al. 2007a).

Reconstruction on the micro-scale: continuity and density

The second type of analysis is carried out in terms of facets. The main difference with the previous analysis with facets is that the reference is the physics knowledge to be taught (and classroom history) and not the students' comprehension. The method to go from verbal/gestural productions to facets is similar; we therefore do not present it. We just emphasize that the set of facets that we have created for this reconstruction is based on an analysis of the knowledge to be taught (curriculum, textbooks) and on the classroom's productions in an iterative approach. This set of facets is different from a set of facets constructed to analyse students' developing knowledge. The overlap of these two sets can allow for comparisons of the developments of the taught knowledge and of the students' knowledge. We focus our examples on the type of treatment done with facets involving two notions: density and continuity.

In our coding, we distinguish between a 'new facet' that corresponds to an element of knowledge introduced for the first time in the class, and a 'reused facet' that corresponds to an element of knowledge already introduced. This way

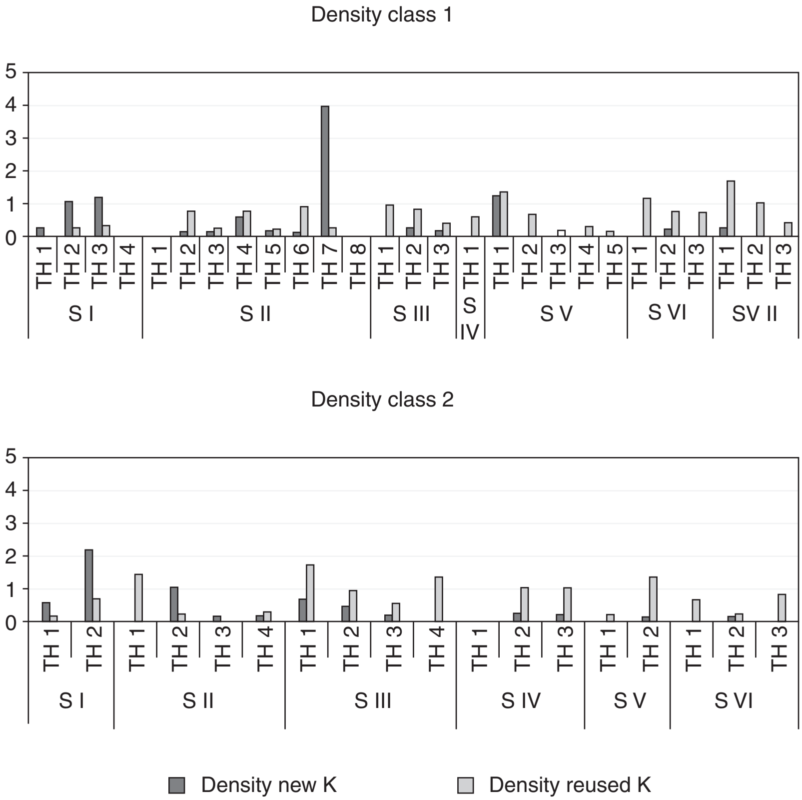

Figure 2.3 Density of new and reused knowledge by theme (Th) and session (S) for the two classes.

Table 2.1 Number of times that some of the most frequent facets are reused in the two classes (Malkoun 2007) (WC = whole class)

Groups of conceptual facets and representation	*Facets*	*Class 1 (WC)*	*Class 2 (WC)*
Action – Interaction	When object A is in contact with object B it acts on it	20	2
Action – Interaction	Earth always acts on (attracts) objects	15	1
Force – Interaction	When object A is in contact with other objects, it exerts a force on these objects	2	12
Force – Interaction	Earth always exerts a force on other objects	2	13
Representing	Force	11	6

of coding is related to our learning hypothesis that the reuse of an element of knowledge in similar or different contexts promotes learning.

We have introduced the notion of 'density' of knowledge that informs the dynamics of the taught knowledge. The density is the number of facets of one or several types in relation to the duration of a theme, a series of themes or a sequence (given in minutes) (Tiberghien & Malkoun 2007). Figure 2.3 shows that reused knowledge is denser at the end of the sequence and that almost no new knowledge is introduced during the last sessions in the two classes. It also shows the differences in the regularity of introducing new knowledge. For example, in class 1, theme 7 (session II) is very dense in new knowledge; in this theme the teacher presents the model of force (force which models action and force as a vector).

Another way of representing the taught knowledge is to select the facets which are the most reused. It allows us to know which aspects of knowledge are given emphasis in a given class; this is what we call the continuity of knowledge.

Table 2.1 confirms the difference between the two classes, in that action plays

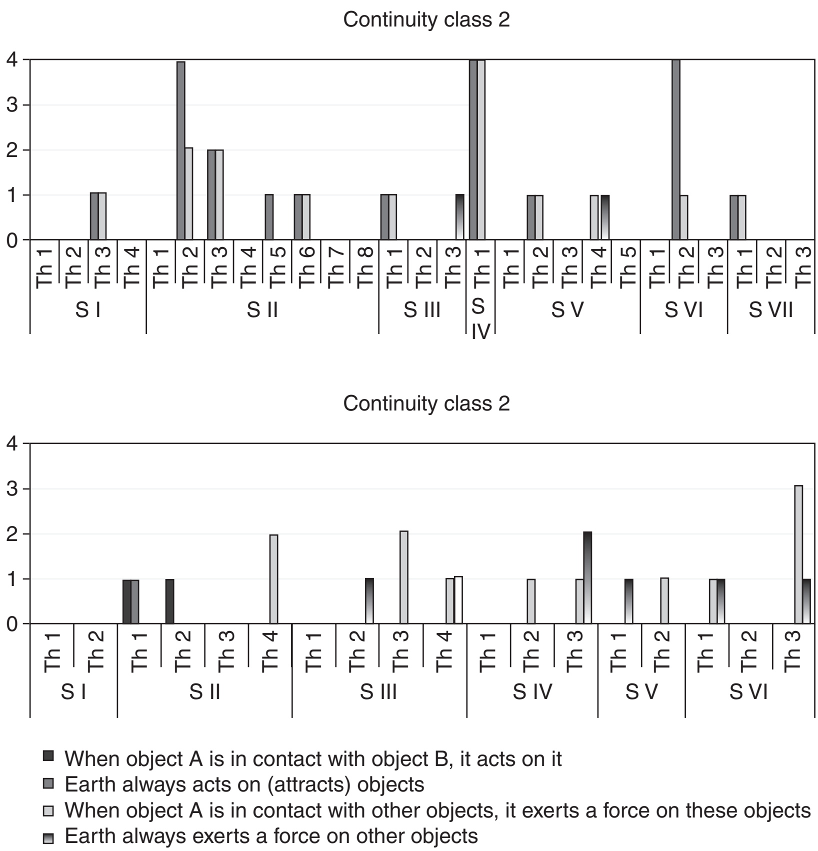

Figure 2.4 Distribution of the most frequent facets over the duration of the teaching sequence, presented in themes and sessions.

an important role in class 1, whereas in class 2, only force is involved. It also shows the importance of representations in class 1, in particular for vector force (last line Table 2.1). It is also useful to represent when these most frequent elements of knowledge are involved during the teaching sequence (Figure 2.4).

More generally, Figures 2.1 and 2.2 relate events on the micro-scale (an utterance or a verbal interaction corresponding to a facet) to the macro-level of the entire sequence. We do not present the relations with the meso-scale which are, however, essential, since they relate the sharing of the responsibility of knowledge in the classroom and the didactic contract with the facets (Tiberghien & Malkoun 2007). We just indicate again that the meso-scale corresponds to a timescale that is very relevant to describe and interpret classroom phenomena.

This way of reconstructing the taught knowledge in a classroom can seem too detailed, but shows the complexity of the taught knowledge as it is illustrated by its different representations according to the scales (Table 2.1, Figures 2.2, 2.3 and 2.4). It has allowed *close links to be established* between the taught knowledge in the two observed classrooms and the students' acquisition evaluated by a questionnaire given before and after the teaching sequence (see Malkoun 2007).

Conclusion

We have presented a methodological approach to analyse classroom data during a teaching sequence based on theoretical positions. First, from an institutional perspective, classrooms should allow the teacher to teach the students what society demands via the official curriculum and other instructions, the 'knowledge to be taught', and allow the students to learn what is taught. Taking into account its social position, the classroom is considered a community where two joint actions take place: teaching and learning, and where communication is key in sharing knowledge. Second, knowledge is studied through the metaphor of life. Knowledge lives in a group, a classroom in our case, and consequently depends on both teacher and students, and is specific to each classroom.

From these theoretical positions, we have proposed analysing a classroom's productions (oral, gestural, written) from two reference points: conventional and situational. From the first point, the meaning is reconstructed by the researcher from the knowledge taught perspective; we have called this knowledge the 'taught knowledge' of a classroom. This reference point is based on a collective perspective. From the situational reference point, the meaning of the productions is reconstructed from the point of view of the actors – the teacher and/or students – and the situation; it allows us to take an individual perspective and then reconstruct a student's developing knowledge.

Our methods of analysing the classroom productions are similar for each of these reference points. These methods involve three scales of time and granularity of knowledge to tackle the complexity of classroom situations. The classroom or students' productions are therefore analysed in terms of facets at a micro-level, and in terms of themes or episodes at a meso-level in relation to the time development of the teaching sequence. This meso-level is particularly relevant in studying the didactic contract, from the classroom's and from a student's point of view and the responsibility vis-à-vis knowledge of the teacher and the students (topogenesis). Each perspective, collective or individual, leads to different results, and comparing these can be profitable.

From the individual perspective, the method is used to construct a student's developing knowledge at a conceptual level and in particular, the various elements of knowledge involved in this development, which are not usually introduced in studies of students' conceptions. The possibility of identifying each facet with the situation it originates from, like the type of interaction, the role of the experimental device, the instruction, and the order of this situation in the teaching sequence, makes the micro-level of analysis relevant. Within this perspective, it is possible to compare different students' developing knowledge throughout teaching sequences.

From the collective perspective, the method is used to show the detailed characteristics of the taught knowledge, in particular, the density of introduction of new elements of knowledge according to the teaching session or theme, as well as the continuity of knowledge characterizing the number of times an element of knowledge is involved during the teaching sessions. Again, the links between micro- and meso-levels are essential to the extent that they allow elements of knowledge to be situated in relation to others as well as, more importantly, in relation to the way the teacher or student introduces them and in relation to the type of situation in which they are involved. Within this perspective, the characteristics of the teaching content, the way it 'lives' in different classrooms using the same curriculum and, in particular, the didactic contract that frames the life of knowledge in a classroom, can be compared; such a characterization opens a way to relate classroom practices to students' acquisition of knowledge.

The comparison of collective and individual analyses can be productive to the extent that they can be carried out in fine detail; that is, at a micro-level, while being constantly situated at upper levels. In particular, comparing the sets of facets obtained in the cases of a student's perspective and a conventional perspective can allow us to analyse the gap between the development of the taught knowledge during a teaching sequence and the students' developing knowledge in terms of content and of the rhythm of their respective developments (chronogenesis). This comparison can also be focused on the role of the taught knowledge viewed as learning conditions on a student's developing knowledge. Along the same lines, studies can be carried out on the ways in which different points of view emerge in a classroom and evolve in relation to the conventional meaning; this provides an operational way of analysing the positions of the different actors in a classroom regarding knowledge (topogenesis) and the classroom's didactic contract. The taught knowledge, which includes several components on different scales, can also be related to an assessment of students' acquisitions on the basis of the micro-scale analyses. It is possible to analyse questions asked in the assessments in terms of facets that should be hypothetically involved in the answer from the conventional perspective. These facets are then compared to the taught knowledge: this comparison is not only made in terms of the facets, their density and continuity, but also by analysing the way the facets are involved in the classroom teaching, at the meso-level in particular. However, if our methodology has the potential to articulate collective and individual analyses of the classroom and also to relate classroom analysis and students' acquisition, it is still at the case studies stage, and further work should be done.

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Chapter 3

Technology-based algebra learning

Epistemological discontinuities and curricular implications

Michal Yerushalmy

Introduction

Technology interacts in important ways with the desire of mathematics educators to promote long-term learning. Technology can upset the hierarchy of prerequisite skills that often seem to dictate the practice of mathematics in schools. And technology can assist teachers in making powerful mathematical ideas accessible to learners in a different sequence and at a different rate than has been traditionally deemed feasible. It is therefore important to view technology as an aspect of long-term learning. In a recent paper, Yerushalmy and Chazan (2008) used Tall's (2002) construct of discontinuities to identify and analyse curricular discontinuities (as distinct from the cognitive difficulties of learners). Their study of discontinuities is based on the assumption that it is impossible to design a long-term sequence that is smooth and free from abrupt transitions. The objective of this chapter is to illustrate the use of the notion of epistemological discontinuities as a means of analysing long-term technology-based learning of algebra. I argue that in designing a new curriculum based on use of new (technological) tools attention must be paid to identifying points of discontinuity that may be different in order or quality from those revealed by previous research. Several new technology-based algebra curricula have been developed and studied worldwide in the last two decades. The algebra curriculum is therefore a good example for demonstrating the issue of epistemological discontinuity and its curricular implications.

Although some learners enjoy solving equations in school algebra merely because the task has a clear goal and one knows when one is done, in general school algebra has been described as overly focused on meaningless manipulations. All too often, students learn to simplify expressions and solve equations with rules that are meaningless to them, and they usually survive by memorizing rules and ideas for a short time. Various proposals have been put forth and curricula have been designed to teach algebra in ways that promote conceptual symbolic understanding. Visualization and especially visual representations of functions, expressions and equations was found to be an important component of meaningful symbolization. But traditional views of symbol manipulation and solutions of equations in school algebra are disconnected from the study of functions. Technology, such as graphing calculators or function graphing software, provides students with opportunities to investigate algebraic ideas by linking the symbolic representation of functions and symbolic manipulations with their numeric and graphic representations. Central to such a function-based approach to algebra is the earlier introduction of a particular way of viewing equations – a definition of an equation as a type of comparison of two functions – and the use of this definition as a primary resource in constructing student understanding of school algebra. The two compared functions are graphed in a space with a dimension that is greater by one than the number of independent variables. Intersection points indicate a subset of the domain of the independent variables that make up the solution set. The equal sign is symmetrical when it indicates a comparison between an expression of a computational process on one side and an expression of another computational process on the other side ($f(x)=g(x)$), and is asymmetrical when it indicates the assignment of a name to an expression that represents a particular computational process ($f(x) = \ldots$). In this way of organizing the curriculum, the view of an equation as a symbolic string that requires defining a solution set is now second to the view of the equation as a statement comparing two processes.

In what follows I exemplify how a decision to design a technology-supported algebra curriculum (Visual Math 1995) based on the view of equation as a comparison of two single-variable functions offers a means to overcome the known discontinuity between algebraic manipulations and the study of functions, and inevitably leads to another discontinuity caused by the difficulty of thinking about multiple-variable equations as comparisons of functions.

Overcoming discontinuities

In general, an important goal of the Visual Math curriculum is to help students develop strong algebra skills and to learn to perform a variety of standard algebraic manipulations with understanding rather than by rote memorization. The early parts of the Visual Math curriculum focus on one-variable functions and equations conceptualized as the comparison of two one-variable functions. Thinking in this way about equations or inequalities, students acquire, in addition to algebraic procedures, alternative methods of solving equations. Before using symbolic manipulations, and in the process of doing so, students are encouraged to use systematic guessing and intuitive numeric and graphic analysis strategies, and to conjecture about the visual effects of symbol manipulations. For example, function graphing tools support the students' attempts to explore questions of equivalence as they learn to manipulate equations algebraically. Conjecturing, demonstrating and reasoning whether an operation on an equation or on an inequality would result in an equivalent equation (or an equivalent inequality) is a central activity in this curriculum. Legitimate manipulations are those involving the same operations on both sides of the equation or inequality. Such manipulation, as shown in Figure 3.1, changes each of the compared functions, each of the graphs, as well as the points of intersection of the two graphs, but preserves the solution set, so that changes should not affect the x values of the intersection points. Understanding equivalence equips learners with the tools they need to discuss questions such as: Why can't one always multiply each side of an equation by x? What happens when an inequality is multiplied by a negative number that

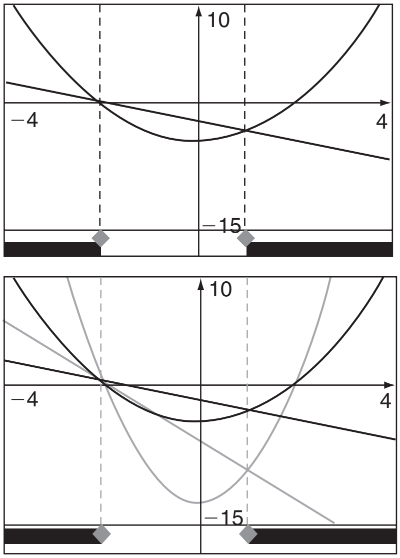

Figure 3.1 Graphs of two equivalent inequalities $x^2 - 4 > -2 - x$ and $3(x^2 - 4) > 3(-2 - x)$.

causes the inequality sign to change direction? Chazan and Yerushalmy (2003) described these questions as those that students seldom have opportunities to ask.

Another central activity in the Visual Math curriculum that takes advantage of early learning of functions is solving problems in context. Figure 3.2 describes a common algebra problem and a solution that appears commonly in the work of Visual Math algebra beginners (seventh and eighth graders). The solution identifies two processes that need to be compared in the story-problem. It consists of a sketch describing the structure of the situation in the problem (two intersecting graphs, each describing the change in position over time of one vehicle relative to point A), two algebraic expressions that match the functions in the graph ($g(x)=56x$ and $f(x)= 476–80x$), visual and numeric scripts of the rate of change on each graph (an annotated 'step' indicating its size and direction), and an equation ($476–80x=56x$) that represents the comparison of the two functions. The numeric solution is the x value of the intersection point.

In general, solutions included a graph and algebraic expressions of two functions that matched the graph. Graphs did not replace the algebra but rather served as a visual aid to formulate the algebraic equation. Viewing equations as models analogous to graphs and situations; viewing an equation as a comparison of two functions, most often graphically; and viewing algebraic letters as variables of functions served as powerful resources, leading to an exceptional success rate in comparative studies (Gilead & Yerushalmy 2006). Using these resources, students were able to solve problems for which they had not yet studied an algorithmic solution method, and although they were still algebra beginners they exhibited profound understanding of advanced calculus ideas related to rate of change of non-linear processes (Shternberg & Yerushalmy 2003). They demon-

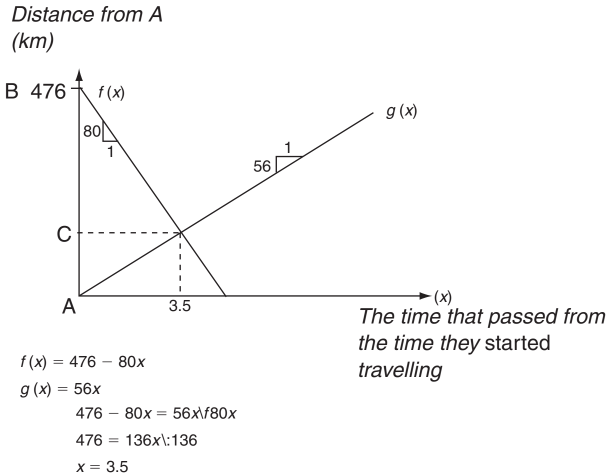

Figure 3.2 Common algebra problem and common solution of Visual Math students.

strated non-disruptive growth of symbolic algebra sense, and an understanding of ideas and representations of functions.

Approaching a discontinuity

Awni Bathish, teacher of a ninth-grade Visual Math algebra class, wondered whether his students could solve a task involving a system of equations in two variables without being taught a method for solving such systems. Taking into account the strength his students exhibited in the past in solving new problems and new types of single-variable equations by consideration of functions, he wanted to find out whether the students understood the new task. Could his students generalize their graphic and tabular representations to cope with the challenge? Did they understand the purposes, capabilities and structure of these representations sufficiently to modify and fit them to the new circumstances? With this in mind, he presented his students with a system of two linear equations in two variables ($x + y = 2x - y$, $2x + 1 = 3x + 3y$) and asked them to think about ways of describing a solution. The students were astounded by the appearance of this type of equation, which they had not seen before. For three consecutive lessons the work focused only on the first equation. I will describe the nature of the complexity the students faced by analyzing the attempts of two groups of students (Philip with Nidal and Saher, and Morad with Amin) to communicate to the class the work done in the first lesson by each group.

PHILIP: [at the board, attempting to graph the equation $x + y = 2x - y$] We saw that for the functions $x + y$ and $2x - y$, when x equals 1 or 0, y can be an infinite number or numbers; that means all over the y-axis.... It [y] is the same for any x; any positive or negative number.

Class: Draw!

Philip sketched a family of horizontal parallel lines. Each represents a specific y value and infinitely many x values.

SAHER: If y is fixed and x is not fixed, what do you do?

Philip started to draw the functions $x + y$ and $2x - y$, but in reality he graphed candidate values for x and y. He realized that each function should describe a rule operating on an infinite number of x values for any fixed y, and he considered only integers. Thus he drew lines representing infinite x values for each integer y. But he got stuck there, not considering representations of the statement $x + y$. Morad and Amin commented on what they found to be missing in Philip's description:

MORAD: Just a moment, Philip. There is an operation between x and y: addition. Where is the result of this operation?
PHILIP: $x + y$.
AMIN: $1 + 1 = 2$.
PHILIP: It may be all the numbers.
MORAD: It is possible to have lots of numbers, and it is also possible to have lots of results. Where are the results? Where is the result of the addition?

Philip seemed to think that his problem of infinitely many x,y values should be solved first, and he did not understand how to represent the operation. So Morad attempted to further clarify his question, and he and Amin outlined the analogy they found between the known and the new meaning of function representation.

MORAD: I want to explain to you how we concluded these things. We always take one variable. Add to it or subtract from it the other number, and we get a result.
AMIN: For example in $f(x) = x + 2$ we change the x, but the other number remains constant. When $x = 1$, we add 2 and we get 3...

We used to look at the point as two distances: one from the x-axis and one from the y-axis.

Now we have another variable. Some of you got confused because we used to take y as a result, and now it is a variable.
AMIN: Replace the y with $f(x)$.
MORAD: O.K., y is $f(x)$. We used to see the point in two dimensions. Because we now have another variable we need to see it in three dimensions: the distance from the x-axis, the distance from the y-axis, and a height.
AMIN: The result.

To overcome the difficulty they had visualizing their proposal, they first used their fingers, then they tried to hold their pens at a 90-degree angle to a sheet of paper, improved this by constructing a model of wooden sticks fetched from a nearby storage place and assembled with rubber bands, and finally used a sheet of paper to construct the $x + y$ plane that represents the function in two variables (Figure 3.3) in a three-dimensional model.

Figure 3.3 Constructing a representation of x + y: using fingers (top-left), pens (top-right), and sticks (bottom-right) with paper in three-dimensional space to describe the x + y plane (bottom-left).

Clearly, the conceptualization of an equation as a comparison of two functions introduced complications for the equation with two unknowns. First, there is an obvious difficulty in devising a graphic representation of a two-variable function; the task requires a conceptual change in understanding the role of the x-y plane and the meaning of a point on this plane. In describing a single-variable function, an x-y point describes arbitrary x values and y values which are constrained by the function's rule. A point in the x-y plane for a two-variable function is an arbitrary choice of both x and y and does not represent any constraint. The points constrained by the $f(x,y)$ rule are in three-dimensional space. Understanding that a point in the plane is not an appropriate graphic element any more to describe the function's expression was a major breakthrough. The constant value lines that Philip graphed as the generalization of a point make a sensible generalization, but it did not lead to a productive analysis. The 3D representation produced by Morad and Amin (and other models described in Yerushalmy and Chazan 2002) led to the correct representation of the function's expression but not to the visualization of the equation nor to an analogical representation of a solution involving a system of equations. Students were able to represent equations in two variables only later, after Mr. Bathish provided a 3D graphing software that enabled them to graph the functions and view the intersecting line of their projections on the x-y plane.

We may be puzzled by the students' choice to represent the original equation, $x + y = 2x - y$, as given rather than simplify it to $y = x/2$ and then graph it as a function of a single variable in the 2D plane. Obviously, generalizing familiar procedures and representations made more sense than manipulating them, not because they lacked manipulation skills but because they had not yet reached the more mature flexibility of viewing a manipulated equation in two variables as a function of a single variable. Thus, the task challenged students to reflect upon and consolidate their knowledge of solutions of an equation, because while attempting to generalize known representations of equations they had to rethink the meaning of letters as independent and dependent variables, as well as the meaning of representations of rules and operations. The discrepancy between the visual complexity that was an important part of this consolidation and the relatively simple visual representation of the manipulated equation created an opportunity for the teacher to make students learn about and appreciate the power of algebra as a system of symbols that supports various interpretations and views.

Discontinuities in the study of long-term learning with technology

In crafting and teaching a curriculum for long-term learning, designers and teachers often opt for a smooth sequence that enables them to unpack the mathematics to their students evenly by adopting a specific view of notations, representations or mathematical concepts. According to this view, students should benefit from the development of the cumulative knowledge they construct. I showed above the strength of a relatively long learning sequence in which the graphic qualities of single-variable functions were central to the construction of knowledge. I also demonstrated what I assume to be an unavoidable difficulty for students when they attempt to understand representations of equations in two variables. I argue that at that precise moment they reached a point of critical transition, which I call an epistemological discontinuity.

What is the nature of such points of discontinuity? How are they useful to mathematics educators? In what way are they different from the expected course of knowledge development consisting of changes in the functionality and form of central concepts? To explain the differences between the construction of knowledge within the algebra of single-variable equations and the abrupt transition from single to multiple-variable equations, it is helpful to map the situation in Awni's class along Brousseau's analysis of epistemological obstacles that occur in long-term learning.

Necessary conditions for obstacles (Brousseau 1997)	**Equation in two variables in function-based algebra: analysis of a discontinuity**
(a) *An obstacle is a piece of knowledge rather than a difficulty or a lack of knowledge.*	The central pieces of knowledge providing both the strength and the difficulty were (1) the view of an equation as a comparison of functions and (2) 2D graphs being the visual image of an equation in a single variable.
(b) *A piece of knowledge produces responses that are appropriate within a particular, context…*	These pieces of knowledge facilitated the effective learning of single-variable problems, including complicated ones.

(c) ... *but it generates false responses outside that context. A correct and universal response requires a markedly different point of view.*	The attempt to generalize graphic aspects from a single-variable to two-variable functions led to errors for some students and to difficulties for the majority of them. There are alternative views of equations, but it is difficult to imagine a view that would offer a universal, obstacle-free construction of knowledge of school algebra.
(d) *Possession of a better piece of knowledge is not sufficient to obviate the preceding one. It is therefore essential to identify it and to incorporate its rejection into the new piece of knowledge.*	As the difficulty cannot be made to disappear, there are two ways in which to construct a new piece of knowledge at this point: (1) teaching to generalize 2D methods to 3D methods, which can be accomplished relatively easily with technology, providing intuitive and natural views for learners in the digital age (e.g. Noss 2001); and (2) discussing the power of symbol manipulations to alter the meaning of the equal sign, and rewriting equations in two variables as functions of a single variable. In both cases, the old piece of knowledge is crucial and constructive.
(e) *After its inaccuracy has been recognized, the old piece of knowledge persists.*	A curricular choice needs to be made at this point. Once the obstacle has been recognized and treated as a curricular discontinuity, it facilitates the continual development of profound understanding of algebra and functions.

As co-designer of the Visual Math curriculum, I was aware of the complexity caused by emphasizing the view of an equation as a comparison of two functions. But we opted for this choice because it supported the learning of powerful ideas and of a long, smooth sequence of single-variable algebra, and because it instilled profound ideas for the upcoming calculus course. This incremental construction of knowledge by students was an important component in smoothing the traditional discontinuity between symbol manipulations and ideas related to functions. At the same time, the students' solid knowledge of single variable equations, while it helped them to make progress, created a new discontinuity from one- to two-variable equations. Thus, it served as an opportunity to consolidate existing knowledge and created the need for a change in perspective. Understanding the need for a fresh view, knowing where to look for a new vantage point, or even sensing that a generalization or analogy are difficult to make are at the heart of mathematics learning and should be thought of when designing a new curriculum. Learning and teaching opportunities of this type, although they are part of mathematical culture, are rare in the usual context of school algebra, which often emphasizes mastery of procedural knowledge.

I suggest that mathematics educators consider the analysis of discontinuities as a tool for curricular research. Yerushalmy and Chazan (2008) analysed the similarities and differences between two curricular sequences and showed how the analysis of discontinuities created an opportunity for a different type of curricular research, one that suggested ways to understand the nuanced differences between specific curricular choices. Another key question for educators is how use of a new curriculum, based on new epistemological assumptions, can change our ability to anticipate students' difficulties and strengths. Studies often report on the strength of student performance as a result of using new tools. In this chapter, we showed that in studying a new curriculum based on the use of new tools, attention must be paid to identifying points of discontinuity that may differ in the order or quality from the ones known from previous research. Focusing on the way students learn at these points can help educators anticipate, identify and handle their students' contribution.

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Chapter 4

Toward a trialogical approach to learning

Kai Hakkarainen and Sami Paavola

Introduction

The purpose of the present chapter is to examine a novel approach to learning, which we call *trialogical inquiry*. We will start this chapter by contrasting three metaphors of learning; the knowledge-acquisition metaphor, the participation metaphor, and the knowledge-creation metaphor (Paavola, Lipponen & Hakkarainen 2004; Paavola & Hakkarainen 2005; Hakkarainen, Palonen, Paavola & Lehtinen 2004). Anna Sfard (1998; see also Lave & Wenger 1991; Wenger 1998) has differentiated between two central metaphors of learning, the knowledge-acquisition metaphor and the participation metaphor. The division is very profound and considers there to be two fundamentally different approaches to learning. As we interpret them, the former emphasizes individual mental processes and the latter examines transmission of cultural knowledge and competence, from one generation to the next. We have proposed that in order to overcome the dichotomy between these approaches, a recognition of a third metaphor of learning is needed that addresses learning related to deliberate advancement of knowledge and transformation of social practices. Creating a theoretical framework that assists in conceptualizing, empirically studying, and facilitating knowledge-creating learning in education and workplaces is the focus of an integrated European Knowledge-Practices Laboratory (www.kp-lab.org) project.

The *knowledge-acquisition metaphor* examines knowledge as a property or characteristic of an individual mind. An individual is the basic unit of knowing, and learning is a process in which information is transferred to the individual agent. The acquisition metaphor may be based on the traditional assumption of the direct transmission of knowledge to the student, or, as Sfard (1998) herself emphasizes, the active and 'constructive' (but individual) process. This metaphor leads to an examination of learning from the perspective of a student's internal information processing and emphasizes the role of within-mind knowledge structures (e.g., schemata) in learning. Some versions of the knowledge-acquisition metaphor are based on the 'folk' psychological, metaphoric assumption that a person's mind is a container for knowledge, and learning is a process which fills this vessel, furnishing it with information (compare Bereiter 2002). This metaphor grew out of studying highly controlled tasks that often focused on memory, problem solving in toy domains, and simple aspects of language (e.g., word and sentence recall).

An alternative approach, according to Sfard (1998), is *the participation metaphor* for learning, which examines learning as a process of growing up and socializing to a community, and learning to function according to its socially negotiated norms (Lave 1988; Lave & Wenger 1991; Brown, Collins & Duguid 1989). Participation in various cultural practices and shared learning activities structures and shapes cognitive activity in many ways. Cognition is distributed across individuals and their environments (Salomon 1993; Norman 1993), and learning is 'located' in the evolving networking relations. From the participatory perspective, learning is the process of growing to become a full member of a community, in which there gradually occurs a shift from peripheral to full participation. From this perspective, knowledge is not a thing in the world itself or within the mind of an individual, it is simply an aspect of cultural practices (Brown et al. 1989; Lave & Wenger 1991). The focus of the participation view is on activities and 'knowing', rather than on outcomes or products (i.e., on 'knowledge' in the traditional sense).

Neither one of these metaphors appears, however, to examine in full processes of knowledge creation and advancement that are essential to an advanced knowledge society. The knowledge-acquisition metaphor concentrates on pre-given symbolic structures of knowledge that an individual student is directed to assimilate or construct in the process of learning and expertise development. Although this process may include creative elements and support the formation of new meaning connections, the creation of knowledge is not central to this metaphor; actually performance rather than learning was often emphasized, at least in older theories belonging to this tradition. The participatory metaphor, in turn, focuses its attention on the deepening knowledge of a community without intentional aspirations to bring about conceptual or social change. The focus is on prevailing cultural practices, and it does not pay particular attention to the creative change of these practices. Alternatively, it emphasizes discursive interaction but does not focus on how shared, concrete objects are developed collaboratively. The acquisition approach and the participation approach may both be developed so that they take innovative aspects into account, but it can be argued that this is not where these approaches are at their best (Paavola et al. 2004). There are, at least, theories and approaches focusing on how people collaboratively organize their activities for developing something new; theories that have an interventionist focus of not just explaining or describing existing practices but making research on the processes of change and how they are supported.

Based on these perspectives, we postulate that a third, *knowledge-creation metaphor* of learning is fruitful to overcome the limitations of the acquisition and participation approaches (Paavola et al. 2004; Paavola & Hakkarainen 2005). From the perspective of knowledge creation, learning is seen as analogous to innovative inquiry through which new ideas, tools and practices to support intelligent action are created, and the knowledge being developed is significantly enriched or changed during the process. The processes, practices and social structures promote focused creation of new knowledge and innovation rather than adjust to the culture or discourse at large or the assimilation of existing knowledge. One may contrast the metaphors as follows: The acquisition view may be seen to represent a *monological* view of human cognition in focusing on within-mind processes. The participation view, in turn, appears to represent a *dialogical* view because it emphasizes the interaction with the culture, the surrounding (material) environment or among people. The knowledge-creation view, in contrast, requires a *trialogical* approach to learning because it focuses on collaborative development of mediating objects or artifacts rather than monologues within mind or dialogues between minds (see Figure 4.1). In the middle of the knowledge-creation view are shared 'trialogical' objects because pursuit and development of these entities is the central aspect of the knowledge-creation approach. The objects may be conceptual (questions, theories, designs) or material (prototypes, concrete products) in nature or represent practices (industrial production procedures) to be collectively reflected on and transformed.

There are several models that examine learning and inquiry as a process of knowledge creation rather than just assimilating existing knowledge or adopting prevailing practices, such as Bereiter's (2002) theory of knowledge building, Nonaka and Takeuchi's (1995) theory of knowledge creation and Engeström's (1987) theory of expansive learning. These approximations to a trialogical approach guide the examination of learning as a process of innovative inquiry in which the aim is to progressively refine knowledge artifacts and engage in long-term processes of expanding a community's knowledge and competencies. According to these theories, persons are called upon to meet novel challenges and to engage in systematic, creative 'reinvention' of their epistemic practices so as to elicit knowledge processes characterized by novelty and conceptual innovation (Knorr-Cetina 1999). We have designated settings where knowledge creation occurs as *Innovative Knowledge Communities* (IKCs) rather than traditional communities of practice (Hakkarainen, Paavola & Lipponen 2004). Within these communities, social practices and knowledge practices are tailored to promote

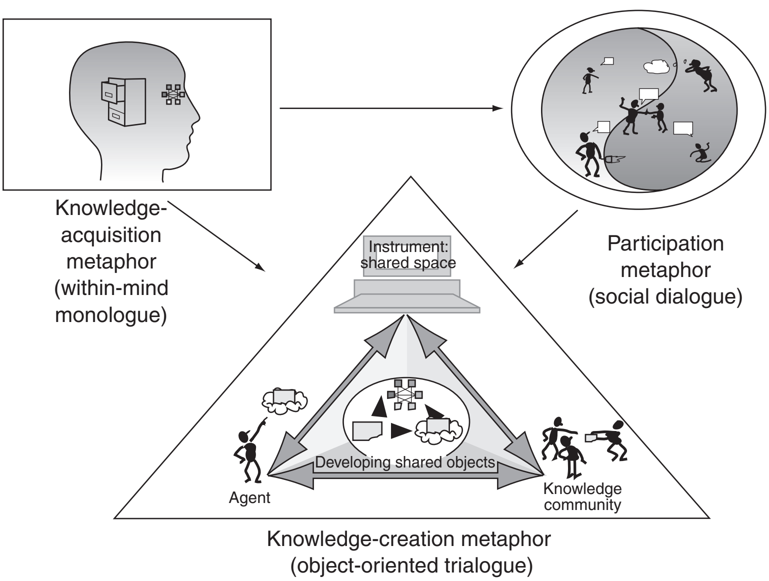

Figure 4.1 Three metaphors of learning.

continuous innovation and change. Table 4.1 presents an abstract description of some principal features of the three metaphors of learning.

Knowledge advancement and creativity can be understood as a form of *trialogical activity*, that is, one in which the persons working together are elaborating a shared *object*, whether it is a research problem, theory, plan, design, product or practice (to be reflected on and transformed) (Paavola & Hakkarainen 2004). The concept of 'object' has *philosophic* roots in Popper's (1972) and Peirce's studies (see Skagestad 1993), and *psychological* roots in activity theory as developed by Vygotsky (1978) and elaborated by Engeström (1987). The object arises within this perspective on human activity as essentially sign and tool mediated. A semiotic interpretation of Popper's (1972) notion of objective knowledge comes very close to the present approach (Skagestad 1993). According to the philosophic vision of three worlds:

> one day we will have to revolutionize psychology by looking at the human mind as an organ for interacting with the objects of the third world [World 3]; for understanding them, contributing to them, participating in them; and for bringing them to bear on the first world [World 1].
>
> (Popper 1972, p. 156)

Popper examined human cognitive evolution in terms of such objects: '*Human evolution* proceeds, largely, by developing new organs *outside of our bodies or persons*, "exosomatically", as biologists call it, or "extra-personally". These new organs are tools, or weapons, or machines, or houses' (Popper 1972, p. 238; original emphasis). The challenge is to develop methods of empirical, psychological inquiry into knowledge-laden objects of learning and working.

While trialogical inquiry focuses on generating concrete objects (such as a piece of text), in the background there are epistemic objects (Rheinberger 1997; Miettinen & Virkkunen 2005), i.e., deeper ideas prevailing at the epistemic horizon which the inquiry is focused on. The epistemic objects include in their scope or ambit characteristics that investigators do not yet know. Human beings pursue complex problems by crystallizing their emerging knowledge and understanding in a series of epistemic artifacts. While there might be critically important insightful moments, knowledge creation takes place through long-standing and extended efforts: weeks and months rather than minutes, hours or days (Gruber 1995; Schaffer 1994). New knowledge cannot be created from scratch; it emerges through elaborating prevailing knowledge and understanding across several – or a long series of – iterations.

The trialogical objects are concrete, epistemic artifacts that participants are creating, sharing and elaborating often by relying on information and communication technologies (ICTs). In this regard, the objects in question diverge from activity-theoretical objects embedded in complex dialectical theory of human activity (see Engeström 1987). A central characteristic of knowledge-intensive work appears to be that professionals are forced to create various knowledge artifacts in order to deal with the complexity and uncertainty of their work usually taking place in dynamically emerging and rapidly disappearing multi-professional projects at blurred interorganizational boundary zones. Epistemic artifacts refer

Table 4.1 Typical characteristics of the three metaphors of learning

	Knowledge acquisition	*Participation*	*Knowledge creation*
Main focus	An individual process of adopting or constructing subject-matter knowledge and mental representations 'Monological' perspective (within mind)	A process of participating in social communities, Enculturation, cognitive socialization Transforming norms, values, and identities 'Dialogical' perspective (between participants, or towards authentic situations)	Individuals and groups creating and developing new material and conceptual artefacts Conscious knowledge advancement, discovery, and innovation 'Trialogical' perspective (co-evolution of inquirers, communities and objects of inquiry)
Theoretical foundations	Theories of internal knowledge structures and schemata; individual knowledge construction Conceptual knowledge and individual skills Logically oriented	Situated cognition, various forms of social constructivism Practices and social interaction (communities of practice) Sociologically oriented epistemology Epistemology emphasizing dialogic interaction	Theories concerning mediated activity (Cultural-historical activity theory, knowledge-creating organizations, knowledge-building theory) Transformations between various forms of knowledge Epistemology of mediation; triangulation
Unit of analysis	Individuals	Groups, communities, networks and cultures	Individuals and groups creating mediating objects and artifacts within cultural settings

Source:
Paavola and Hakkarainen 2005.

to various knowledge-laden products of human activity (Sterelny 2004), such as ideas, concepts, theories and models. These entities may be visual-conceptual representations in nature, such as text, drawings, graphs or engineering designs. Epistemic artifacts also involve concrete products, tools, and instruments embodying and crystallizing human intelligence. All of these entities are used as tools for thinking. These objects are *hybrids* (Latour 1999) in being both epistemic entities and physically embodied as digital or other types of artifacts. The objects may be conceptual (questions, theories, designs) or material (prototypes, concrete products) in nature or represent practices to be collectively reflected on and transformed.

As mentioned above, the knowledge-creating processes of learning have become available for educational and professional communities to a much greater extent due to the revolution in collaborative learning technologies during the last two decades or so. These technologies furnish collaborative spaces for creating, sharing and developing objects – which we have designated *trialogical* – as well as offer provisions for collective memories capturing various aspects of the socially, spatially and temporally distributed inquiry processes in question. The KP-Lab project is focused on developing 'trialogical' technologies that are deliberately designed to scaffold and foster sustained processes of knowledge advancement, and refinement of the related objects. The emerging practices of technology-enhanced learning and instruction appear to make such epistemic mediation to some degree accessible even to elementary school students.

Six basic characteristics of trialogical learning

The trialogical approach is intended to elicit innovative practices of working with knowledge within educational and professional communities, and to foster the development of tools for understanding deliberate processes of advancing and creating knowledge typical of knowledge-intensive work in the present age. While professionals working in various domains have stronger personal, social and cultural resources for knowledge creation, deliberate knowledge creation appears to be attainable to students of education at least to some degree. Our talk about knowledge creation does not presume generation of historically novel bodies of knowledge, but rather systematic pursuit of solving societally significant problems important for students, teachers and their local communities. Rather than being a purely descriptive approach to learning, the trialogical framework guides investigators as well as teacher-practitioners in exploring to what extent it is possible to stretch educational practices toward trialogical ones. That is why it is closely connected to an interventionist view on research, and on the design-based research approach where the aim is to produce knowledge to help to develop novel practices and technology, not just to explain existing practices (see, e.g., Design-Based Research Collective 2003) A central characteristic of knowledge-creation practices is their artifact-mediated nature; the participants are engaged in externalizing and objectifying their evolving knowledge and understanding in the form of shared artifacts, conceptual or material in nature, which they can utilize in their inquiries. These artifacts are 'objects', according to the term already introduced. Within the knowledge-creation framework, further, we consider there to be a growing collective network of these artifacts of cognition or practice, which provides a basis for the participants' subsequent inquiry efforts.

Within the KP-Lab project, the following six interrelated, principal features are defined to characterize the trialogical learning approach.

1. Focus on *shared objects of activity* which are developed collaboratively, whether they are conceptual artifacts (e.g., ideas, plans, designs), concrete, material products (e.g., prototypes, design artifacts) or practices (e.g., standard procedures in a laboratory) taken as objects of inquiry. Knowledge creation takes place through collectively advancing shared knowledge objects that vary in terms of their abstractness, embodiment and trajectory of development.
2. *Sustained and long-standing pursuit of knowledge advancement.* Knowledge creation takes place across extended timescales and requires sustained, long-standing working for the advancement of the objects of inquiry. Because the process of knowledge creation is discontinuous and nonlinear in nature, it is full of sudden breakdowns, obstacles that appear insurmountable, accumulating and resolving tensions and contradictions, as well as occasional leaps of inquiry. Consequently, it is essential to address both longitudinal transformation of knowledge as well as critical, short-term processes, stages and moments of trialogical activity.
3. Knowledge-creation processes taking place in mediated *interaction between individual and collective activities.* While the knowledge-acquisition approach tends to reduce learning and cognition to individual mental processes, and some versions of the participation approach examine only social practices and structures, the trialogical framework addresses the reciprocal personal and collective transformation involved in knowledge advancement (Engeström 1999).
4. *Cross-fertilization of knowledge practices* between educational, professional and research communities. An essential aspect of the KP-Lab project is hybridization between schooling/studying and research cultures as promoted in various investigative learning practices. The background for this is that it is essential to bring cultures of schooling into closer contact with professional cultures and to engage students in expert-like knowledge practices from the very beginning of their studies.
5. *Technology mediation* designed to scaffold long-standing collaborative creation, building and sharing of knowledge. Trialogical activity cannot easily be pursued without appropriate technologies that help the participants to create and share as well as elaborate and transform knowledge artifacts. Novel collaborative technologies should provide affordances to trialogical learning processes.
6. *Development through transformation and reflection.* Models and theories belonging to the trialogical approach emphasize development through interaction between various forms of knowledge and between practices and conceptualizations, etc., that is, interaction and transformations between tacit knowledge, knowledge practices and conceptualizations are a driving force in processes of knowledge creation.

Three examples of application domains of a trialogical approach

The trialogical approach has been a kind of a meta-theory until now, emphasizing features pointed out in theories belonging to the knowledge-creation metaphor of learning. It needs to be further grounded on empirical research in general, and research and development of technology-enhanced learning in particular to be developed further. The problem of accessing and characterizing the 'object' necessitates using multiple methods of learning research, such as participant observation, structured interviews and validated self-report instruments. Trialogical processes may be captured by examining knowledge produced by collaborative technologies designed to provide affordances for such objects; the portfolios or folders offered by the software have allowed one to examine, at all stages of their elaboration, sketches, photos and plans posted to the common database. Several research projects carried out by us and our collaborators have been *design experiments* (Brown 1992; Collins 1999) in nature. Such investigations involve cycles of (a) developing technology-enhanced learning environments, (b) implementing these technologies in educational practices, and (c) collecting empirical data that guided further theoretical and technological development. Such investigations indicate that there are, indeed, empirical phenomena that are better understood in terms of trialogical rather than monological or dialogical processes. In what follows, we will briefly describe three such empirical cases. We will present two cases from primary education; the third, paradigmatic KP-Lab case, concerns universities and polytechnics.

Case 1: progressive inquiry learning

Hakkarainen's research group has been developing the so-called *Progressive Inquiry (PI) model* of learning for over 10 years. By 'progressive inquiry', we refer to the sustained processes of advancing and building of knowledge by pursuing the participants' own research questions and explanations (Hakkarainen 1998; Hakkarainen & Sintonen 2002). The PI model relied on Bereiter and Scardamalia's (1993) theory of knowledge building and was inspired by Hintikka's (1999) interrogative model of inquiry. Progressive inquiry may be seen as one type of a trialogical process because it engages the participants in systematic collaborative efforts in advancing shared knowledge artifacts, such as questions, working theories, results of scientific experiments, scientific information, and so on. The external representations, jointly created or developed, consist, for example, of text, diagrams and photos.

Hakkarainen (1998, 2003b, 2004) carried out detailed qualitative analyses of 10- to 11-year-old students' inquiry culture in a computer-supported classroom. The technological infrastructure of the students' inquiry was provided by an early version of Knowledge Forum, i.e., knowledge-building environment specifically designed to facilitate working with shared knowledge artifacts (Scardamalia & Bereiter 1994). The investigation indicated that knowledge produced by the school class in question was at a very high explanatory level both in biology and physics. In accordance with knowledge produced by the participants, practically all research questions posed by them were explanation seeking in nature. Moreover, the students pursued their research questions in depth, following the pattern of interrogative activity (Hakkarainen & Sintonen 2002). Accordingly, the students undertook their inquiry with initially very general and unspecific 'big' questions and tentative working theories, and tried to solve the initial questions by searching for answers to a series of subordinate questions (Hintikka 1999). This process was facilitated in the CSILE environment by provision of a special scaffold, the I-Need-to-Understand (INTU) question, for generation of subordinate questions. The analyses indicated, further, that the students made considerable conceptual progress. Because the students' progress was assessed by examining their written productions, the evidence of conceptual progress was not, however, conclusive, and there were some indications that the physical study projects (gravity, cosmology) were too complex for students to understand in depth.

The above investigations of progressive inquiry represented several other characteristics of trialogical learning as well. These efforts took place in the context of larger study projects that engaged students in pursuing collaborative inquiry across relatively long periods of time (from one to several semesters). The practice of progressive inquiry relies on hybridization of knowledge practices between educational and research communities through involving students in research-like practices of pursuing their own inquiries, corresponding questions and explanations. Careful experiments indicate, however, that successful progressive-inquiry cultures cannot be created from scratch, but need to be cultivated interactively through sustained efforts involving expansive transformation of knowledge practices (Hakkarainen 2003a).

Case 2: Collaborative designing

Collaborative designing, we propose, is a trialogical process par excellence. Collaborative designing of products for everyday or business use focuses on creating a common design artifact (Seitamaa-Hakkarainen, Lahti & Hakkarainen 2005), and it, in a very concrete sense, emphasizes the development of shared objects. Designing has mental and material aspects: It is not only focused on developing the participants' ideas as they are taking part in knowledge-seeking inquiries, but at the same time on creating design prototypes and concrete, material products. The efforts of the participants are organized toward developing shared design ideas (conceptual artifacts), embodying and explicating those ideas in visual sketches (graphic artifacts or inscriptions), and giving the ideas a material form as prototypes or end results (e.g., mass-produced products). The process involves interaction with users whose needs and desires form constraints on the design process. Both conceptual and physically embodied design artifacts may be considered as trialogical objects around which the participants' efforts are organized. The design process appears from beginning to the end to be mediated by the trialogical objects being designed.

A computer-supported study project concerning artifacts was organized at Laajasalo Elementary School, Helsinki, Finland (Kangas, Seitamaa-Hakkarainen & Hakkarainen 2007). 'The Artifact Project – the Past, the Present, and the Future' was organized in close collaboration between the class teacher and the researchers. In the project, 31 students from Grades 4 and 5 participated. The aim of the project was to break boundaries of traditional schoolwork by supporting pupils' collaborative creation of knowledge with the help of various experts, such as museum staff, craftspeople and designers. The technical infrastructure of the project was provided by Knowledge Forum (Scardamalia & Bereiter 1999). The timescale of the project (18 months) represented a genuine trialogical process that takes place through sustained efforts across substantial periods of time (Bereiter 2002).

In the first phase of the project (the Past), pupils explored their own environment of artifacts, analyzed the design and usability of artifacts, inquired into the historical development of selected artifacts and built an exhibition of artifacts within a classroom. In the second phase (the Present), pupils investigated the physical phenomena of artifacts, such as mechanics (movement of a ball), light (electric circuits) and characteristics of metals. During the last phase (the Future) of the project, pupils explored existing lamps and designed new lamps with the help of a professional designer. They also outlined and visualized artifacts of the future, as well as analyzed the needs of future consumers and utilization of future artifacts. The investigation of the lamp design led the students toward the last activities of the project, focused on projecting what the design of their chosen artifacts would look like in the year 2020. An innovative aspect of the project was a novel way of integrating working with material and conceptual artifacts in the context of trialogical inquiry (Seitamaa-Hakkarainen, Engeström, Kangas, Bollström-Huttunen & Hakkarainen 2004). During the lamp-design process, and the whole project, the pupils were both 'hands on' and 'minds on'. In studying, investigating and designing material artifacts, things that can be touched, they also created conceptual artifacts, such as ideas, explanations and theories. Examining everyday artifacts from a design perspective assisted the pupils in going beyond mere appearances and digging to deeper levels of knowledge and understanding.

Case 3: Boundary-crossing KP-Lab course

A central idea of the KP-Lab project is to elicit cross-fertilization of knowledge practices between educational institutions and professional communities through organizing courses in which the participants solve complex problems for real customers, whether the latter are enterprises, public organizations or research communities. This approach is based on actual practices for giving university courses developed by Professor Göte Nyman and his colleagues at the University of Helsinki. Each course is designed to answer a challenge with which one or several enterprises are struggling. Rather than lecturing about the issue in question, the participants are guided to go to the field and collect data by interviewing experts and observing their practices. The students work in teams with team leaders (i.e., senior students who have already completed similar courses). The team leaders constitute a coordination team that takes care of negotiations with customers. While other teams focus on fieldwork, the research team assists the coordination team in real-time management of the course. The complex problem provided by the customer determines the trialogical objects with which the participants work. Activities of the course are mediated by a technology-based learning environment. Learning that matters does not take place just within the classroom, but involves fieldwork where students create contacts with external participants (users, professionals, researchers). The boundary-crossing nature of this kind of process separates the activity from pure progressive inquiry or collaborative designing.

Let's take a closer look at one example. The Virtual Distributed Work (VDW) course was organized in collaboration between the University of Helsinki, Helsinki University of Technology and Helsinki University of Business Administration (for details, see Muukkonen, Lakkala & Paavola in press). It was aimed at bringing the complexity of professional life to university education. Toward that end, 60 students took part in the course. In the VDW case, the shared trialogical object was a complex problem coming from telecommunication companies. The challenge was to investigate what kinds of digital services (provided by TV) people need today and will need in 2015. Pursuing the knowledge work required for answering such authentic and challenging problems is the very essence of the trialogical approach. Rather than merely attending lectures on virtual work (those were also provided), the participants engaged themselves in corresponding epistemic practices throughout the course. From the perspective of the telecommunication companies involved, the course provided access to academic research on practices of virtual and distributed work. Consequently, the course elicited parallel knowledge advancement (Scardamalia 2002) between academic and professional communities.

Trialogical inquiry *takes place across extended periods of time*. In the context of the VDW case, these artifacts started from each team's flyer explaining who they are and describing their competencies; team offers regarding what specific issues to investigate; project plans modified according to customer feedback; presentations to customers at the end of the project; and final reports that explained their recommendations and justified their choices. This kind of iterative work for developing shared epistemic artifacts is one of the central characteristics of trialogical inquiry. Moreover, the VDW course provided continuity across two academic years; senior students who had taken the corresponding course one year earlier functioned as team leaders. Moreover, the present generation of students was able to utilize knowledge artifacts (e.g., document templates) created during the preceding year.

Interaction between personal and collective levels was also involved. The participants worked in multi-disciplinary, virtual teams consisting of 5–6 students across all three universities. Although the teams had collective objectives, the students were individually searching for information as well as writing documents. Moreover, participants that had taken a corresponding course during the preceding year functioned as team leaders and constituted the coordination team that was responsible for negotiations with customers as well as decisions about how to divide the task between the teams. Consequently, a trajectory from less to more demanding cognitive challenges was built into the design of the VDW course.

Knowledge creation typically relies on the material agency provided by ICTs and collaborative technologies. Without technology enhancement, many trialogical inquiry processes would not be possible. Traditional objects of educational activity are narrower and more impoverished. Spatial and temporal expansion of objects of educational activity (Engeström, Puonti & Seppänen 2003), characteristic of trialogical inquiry, require technology designed to foster long-term pursuit of knowledge-creating inquiry. In the present case, the students worked in Optima, an environment that provided some support for creating, sharing and commenting on epistemic artifacts produced.

In order to promote *deliberate transformation of knowledge practices*, the participants were asked, at the end of the course, to reflect on their experiences. Having a personal experience of distributed virtual work provided them richer contextual understanding of the issues in question than reading any number of books would have yielded. It contributed to transformation of their knowledge practices from educational to professional ones. After going through the present epistemic experiences, it will be easier to take part in knowledge practices regarding virtual work in the future. While the development of agency and restructuring of agency take place over relatively long periods of time, there is reason to believe that boundary-crossing projects, such as VDW, significantly contribute to such processes. Taking part in a long series of such courses in which they are appropriating expert-like knowledge practices may allow the participants to gradually develop a corresponding *habitus* (Bourdieu 1977). Consolidation of such effects requires, of course, months rather than weeks. The challenge is to create methodological tools and approaches that will allow investigators to analyze, longitudinally, parallel transformation of knowledge practices and the participants' agency and identity.

While the present case shared many aspects of trialogical inquiry, the participants did not carry out deeper inquiry, for example, through further iterations, publishing their findings or initiating further inquiries. We maintain, however, that the participants engaged in deeper inquiry than usually takes place in university education. The Optima environment provided some support for the participants' inquiry, although it is not a tool supporting reflection or deliberate transformation of knowledge practices. Nevertheless, cases like VDW provide a productive direction for developing university education. By taking part in *expert-like knowledge practices* that involve breaking boundaries between educational and professional communities, students are able productively to take part in knowledge-creation.

The three types of design experiments reviewed above reveal how the trialogical approach can be implemented in various ways (and these three are not intended to give an exhaustive list of alternatives). There are different kinds of shared objects in these studies, and those knowledge practices that they aim at developing show great variety.

Discussion

The issues we have raised in our discussion of the knowledge-creation metaphor have similarities with those articulated in the debates about *constructivism* in its several forms. The main reason that we have not used this metaphor here is that constructivism exists in so many versions and interpretations (see e.g., Steffe &

Gale 1995; Phillips 1997) that the term, by itself, has become rather meaningless unless further qualified. As Paavola et al. (2004) argued, constructivism can, for instance, be interpreted to be an enhanced version of the *acquisition* metaphor of learning if the emphasis is on the basic Kantian idea that knowledge cannot be acquired directly but must be accessed through inborn schemes or the like. Yet it can also have many affinities with the *participation* metaphor of learning if the idea is that social and cultural practices are primarily constructed. The knowledge-creation metaphor may also be seen as a form of constructivism in the sense that it emphasizes aspects of creating something new collaboratively in the process of learning. The knowledge-creation metaphor has guided us to recognize features that have been seen as essential while organizing 'trialogical' processes, and to see connections between theories highlighting similar phenomena. In the KP-Lab project it has been used to frame design-based research in order to develop novel technology and pedagogical practices to support knowledge-creation processes in education and workplaces. The aim is to start from existing pedagogical practices having some elements of the trialogical approach and to support trialogical aspects with new technology and pedagogical ideas.

The present, *knowledge-practices* perspective on learning implies that participation aspects (social practices) and knowledge creation (deliberate advancement of epistemic artifacts) are not opposite, but closely related in a way that has crucial educational implications. Cognitive researchers have highlighted the importance of guiding students to take part in in-depth learning involving active processes of knowledge construction. The problem is that even if students and teachers were aware of the desirable characteristics of such learning, this does not in itself provide a basis of making corresponding changes. This is because learners have developed, in the course of their lives, an implicit and subconscious *habitus* (Bourdieu 1977; Roth 2002), i.e., predisposition to act and think in certain ways at school that is carved into their minds and bodies by social practices. Because educational activity is embedded with certain kinds of habitus, its transformation is difficult and may require efforts across long periods of time. Human beings do not have ready-made personal and collective psychological mechanisms for going through such expansive transformations. The challenge with theories belonging to the knowledge-creation metaphor of learning is that they require conscious and long-standing efforts to be implemented. Our cases above give different kinds of examples of the knowledge-creation approach we have been investigating. Some of these cases emphasize more 'hands-on' and others more 'minds-on' activities. It is not easy to combine various aspects highlighted in the knowledge-creation approach but these cases show different ways of aiming to do that. These three cases are not in opposition but rather may follow from different kinds of activities put before learners, or in pursuit of different types of investigations put forth by the social scientists. As a simple example, rote memorization of multiplication facts is probably best described using the monological framework.

Consequently, we would like to argue that in order to genuinely elicit educational transformations, it is necessary to put social practices into the middle rather than the periphery of discussion. Instead of mental dispositions, in-depth learning is about certain kinds of social practices of working with knowledge, as preliminarily explicated by the trialogical approach. Genuine elicitation of educational transformations should focus on creating shared knowledge practices that channel the participants' limited intellectual resources in a way that elicits meaningful engagement, advancement of collective knowledge as well as the development of the participants' agency. Pursuit of question-driven inquiry, collaborative design or boundary crossing are social practices related to creative working with knowledge. Such pedagogical approaches as knowledge building (Bereiter 2002) define and conceptualize a certain kind of innovative knowledge practice cultivated iteratively in collaboration between practitioners (teachers) and researchers in long-term processes. The initial phase of using ICTs in education was characterized by a propensity to apply new technologies within the existing institutional practices and computerization of traditional forms of teaching and school activities. Only after teachers and students have learned to use ICTs is it possible to apply new technologies broadly in schools as well as start to transform prevailing institutional practices according to the possibilities of the new technological infrastructure (compare Perez 2002). More than two decades of experience of studying computer-supported learning indicates that technology enhances meaningful learning and instruction only through transformed social practices (Hakkarainen, Muukkonen, Markkanen & the KP-Lab Research Community 2006). Social and technical aspects of technology-enhanced learning coevolve through novel technological instruments providing new affordances for educational activity and evolving practices affecting directions of subsequent technology use. On the basis of these kinds of considerations, we argue that various forms of trialogical inquiry are not only about pedagogical processes but define certain social practices as well. The technology as such does not automatically change educational practices; teachers' and students' deliberate efforts to cultivate new social practices are needed which facilitate trialogical inquiry.

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Chapter 5

Commentary on the chapters on the construction of knowledge

Ivy Kidron and John Monaghan

Our reflections on the four chapters in Part 1 took us in many directions but here we address just three themes: theoretical frameworks; knowledge and learning; individual and social construction of knowledge. Behind these themes is the question, can these chapters contribute to a consistent wider understanding and analysis of knowledge construction? For brevity we refer to the chapters as SDH, T&M, YER and H&P.

Theoretical frameworks

There are similarities and differences with regard to the theoretical frameworks of these chapters. Each chapter takes, explicitly or implicitly, a contextual view of knowledge construction, but there are different emphases in these contextual views. T&M present a hybrid theoretical framework which draws on 'concepts characterizing classrooms that come from a long French tradition of theorizing didactical action', for example, Chevallard, Brousseau and Sensevy, and also acknowledge 'a Vygotskian perspective … mediation of adult and of sign'. YER also draws on a concept from the French tradition, Brousseau's epistemological obstacles, but although she notes concepts that influence her work she is not explicit about the theoretical framework she employs, though it could be characterized as 'social constructivist' as it focuses on the construction of knowledge in social interaction. Both H&P and SDH draw on constructs from activity theory, though the theoretical framework of each is not simply activity theory: H&P's created and shared objects are concrete epistemic artefacts that 'diverge from activity-theoretical objects embedded in complex dialectical theory of human activity'; SDH's approach was inspired by the idea of vertical mathematization of the realistic mathematics education group. In the remainder of this section we consider aspects of consistency, first of T&M's hybrid framework and then H&P and SDH's activity-theoretic frameworks.

T&M's hybrid theoretical framework draws on the work of: Chevallard ('La transposition didactique' is referenced but T&M's position on knowledge is consistent with Chevallard's anthropological theory of didactics – ATD); Brousseau (his didactical contract is mentioned but T&M's position on teaching and learning is consistent with Brousseau's theory of didactical situations – TDS); Sensevy (the idea that the institutional roles of the classroom allows teaching and learning to share a common object); Vygotskian ideas on the mediation of knowledge.

There seems, to us, no problem of consistency in this mix. The 'French tradition' on which T&M draw is particularly noted for its focus on knowledge, especially forms of knowledge peculiar to classroom environments. This focus started with Brousseau, and Chevallard's ATD complements Brousseau's work, as has been noted by others; for example, Kidron, Lenfant, Bikner-Ahsbahs, Artigue and Dreyfus (2008), with particular regard to Sensevy, state:

> These combinations between the TDS and the ATD insert the analysis of social interactions proper to the TDS into a larger perspective … to relate the understanding of what happens locally in a classroom about a specific mathematical topic to characteristics and constraints situated at the more global levels of the educational systems.
>
> (p. 249)

We move on to consider H&P and SDH's so-called activity-theoretic frameworks. A comment following the original SDH presentation was 'How is that activity theory?', which we understood as 'What is the activity?'. Roth, in this volume, views activity in terms of activities 'that contribute to the survival of human societies'. This view is true to the Marxist base of activity theory as Marx was centrally concerned with the conditions (and activities) of human development on a grand scale. Can activity theory be applied to smaller-scale activity?

The scale of the activity has worried many educators: Daniels (2001) contrasts Wertsch's micro-focus on mediated action with Engeström's macro-focus on activity systems and notes a tension; Cole (1996, p. 334) sees value in both approaches and claims that 'Mediated action and its activity contexts are two moments of a single process'; Wells (1993, p. 3) goes to great pains to establish a 'framework of analysis, in which the classroom is seen as a site of human activity' by focusing on the operationalization of actions. While we agree with Roth that activity theory classically concerns large-scale social reproduction activities, we also accept Wells' argument that activity-theoretic approaches are valid in classroom studies at the level of operationalization of actions.

Notwithstanding the divergence from activity-theoretic objects noted above, we see substantial links between H&P's trialogic approach and Engeström's expansive learning which views schooling as in need of radical widening via the creation of networks of learning from below. For this, school students need to be empowered to embrace the contexts of criticism, of discovery and of application:

> expansive transition is itself a process of learning through self-organization from below. The self-organization manifests itself in the creation of networks of learning that transcend the institutional boundaries of the school and turn the school into a collective instrument.
>
> (Engeström 1991, p. 257)

We see this in the six basic characteristics of trialogical learning, for example, 'shared objects of activity … [and] development through interaction between various forms of knowledge and between practices and conceptualizations', and in the case studies, for example, 'involving students in research-like practices' and linking 'knowledge practices between educational and research communities'.

With regard to SDH, although the inspiration of abstraction in context was vertical mathematization, the theoretical basis is Davydov's (1990) ascent to the concrete.

> To know the essence means to find the universal as a base, as a single source for a variety of phenomena, and then to show how this universal determines the emergence and interconnection of phenomena – that is, the existence of concreteness.
>
> (p. 289)

This requires analysis and synthesis: analysis to establish initial abstractions, to link features of reality obtained through empirical thought, for example, similarities and differences; synthesis and the use of theoretical thought to establish essential relationships not directly available to the senses. The basis of this approach, to Davydov, is historically situated human activity, individuals drawing on features and potentialities of the objects (tools, concepts) and other people.

We use an activity-theoretic construct, the object of the activity, to end this section and distinguish between the approaches in the four chapters. SDH, T&M and YER focus on subject knowledge and the object of the activity is learning. H&P focus on more than subject-knowledge activity and the object is, really, schooling. Engeström (1991) uses this construct to address overcoming the encapsulation of school learning and contrasts Davydov's ascent to the concrete with his expansive learning. He concludes that although a Davydovian approach can create powerful intellectual tools, the 'social basis of school learning doesn't seem to be altered by this strategy' but expansive learning could 'break the encapsulation of school learning by a stepwise widening of the object and context of learning' (p. 257).

Knowledge and learning

Knowledge construction is at the heart of the four chapters. We consider three themes related to knowledge construction: epistemological considerations; pedagogy and teaching; and need and anticipation.

Epistemological considerations

Learning concerns both knowledge recreation and creation. H&P are primarily concerned with knowledge creation and the others are primarily concerned, even when their focus is students' knowledge construction, with knowledge recreation. H&P appear to share with Engeström (1991) a desire to break the encapsulation of school learning. It is, we feel, not a coincidence that H&P are also the odd chapter out in not having a subject focus (mathematics or physics in the other three chapters) to their considerations of learning. Mathematics and physics educators, in our experience, have a pride in both advances in mathematics and physics and in their histories; knowledge recreation is central to their identities as educators. This recreation–creation distinction, however, should be viewed dialectically: knowledge advances from and to a historical knowledge base. We return to this theme in the section on *need and anticipation* below.

We view all knowledge as social knowledge inasmuch as knowledge resides in social praxis. This view is consistent with the theoretical frameworks of the four chapters though this is only explicit in T&M, where 'knowledge "lives" within groups of people'. All four chapters are concerned with knowledge in educational settings. Knowledge recreation–creation in educational settings requires an embedded, though usually implicit, epistemology: a teacher literally cannot teach without an epistemology. It is, then, not surprising that epistemic issues are central to all four chapters, but the epistemic issues they address differ. YER is centrally concerned with epistemological discontinuities and tool use, in 'long-term technology-based learning'. This serves as a useful reminder that epistemic issues cannot be divorced from consideration of artefacts/tools. Tool use with regard to epistemology is also central to H&P but they are further concerned with the creation of epistemic artefacts, 'knowledge-laden products of human activity', through epistemic practices. By focusing on facets rather than epistemic tasks, T&M may appear to bypass epistemological considerations, but this is not so as their dissection of knowledge into 'taught knowledge', 'knowledge to be taught', 'development of the taught knowledge', 'evolution of knowledge during teaching', 'intermediary knowledge', etc. is, of course, an epistemological categorization. The embedded epistemology of the SDH chapter is that of Davydov's ascent to the concrete, and epistemological considerations in SDH are confined to the formation of (their version of) mathematical abstractions, not learning in general. We end this subsection with a consideration of what SDH's specialized knowledge is.

SDH define abstraction 'as an activity of vertically reorganizing previous mathematical constructs within mathematics and by mathematical means so as to lead to a construct that is new to the learner', where vertical reorganization 'points to a *process of constructing* by learners that typically consists of the reorganization of previous mathematical constructs within mathematics and by mathematical means'. So abstraction to SDH is one specific meaning of abstraction – it is the realistic mathematics education group's vertical reorganization of mathematical knowledge. Freudenthal (1991) gives examples of horizontal/vertical mathematizing, for instance:

> *Commutativity*. Replacing 2 + 9 by 9 + 2 may be due to horizontal mathematising if 2 and 9 are visually or mentally combined as linearly structured sets and their combination is read backwards. It may be vertically interpreted as soon as the law of commutativity is generally applied.
>
> (pp. 42–43)

So, abstraction to SDH not only takes on a specific meaning, it does not include the recreation–creation of all mathematical knowledge.

Pedagogy and teaching

Pedagogy and teaching are often equated but they are not identical. Pedagogy encompasses educational norms including forms of social interaction and acceptable forms of knowledge, whereas teaching is directed at enabling knowledge recreation–creation within a pedagogical framework. The four chapters do not say a great deal about pedagogy or teaching. This is, in our opinion, a little strange because pedagogy and teaching are central to the matters the chapters discuss. In this subsection we address pedagogy and teaching with regard to these chapters.

T&M state that 'learning physics will not happen without teaching' but say very little about teachers. In the 'French tradition', Brousseau's theory of didactical situations, a distinction is made between 'adidactic situations', where interaction between students and knowledge can function without teacher intervention, and 'didactic situations', where teachers play an essential role in this interaction. An 'adidactic milieu' is a system in which the student interacts in an 'adidatic game' with materials, symbolic artefacts and other students. The teacher has a responsibility to link the knowledge built by the student in adidactic situations with the intended institutional knowledge. YER too says little about teaching and makes repeated reference to (elements of) Brousseau's theory. Perhaps both T&M and YER assume that the devolution of adidactic situations will take place.

H&P's vision for knowledge creation involves 'breaking boundaries between educational and professional communities'. They state that 'forms of trialogical inquiry are not only about pedagogical processes but define certain social practices as well'. Seeing limits on what pedagogic practice offers knowledge creation is important but H&P sketch a plan without details.

SDH attend to pedagogy and teaching in several ways: through standards and design guidelines; through their exposition of the ideas of Davydov; through their literature review. It is clear, however, that most of the extant studies of AiC are focused on (small groups of) students and knowledge development within students' minds and that an account of the role of the teacher in relation to 'AiC in the classroom' is in its initial stage.

Considering all four chapters, there is a need for further consideration of pedagogy and teaching. Given that these chapters report on ongoing research it is understandable that there will be areas of work not fully developed. But given the importance of pedagogy and teachers to knowledge recreation–creation, this is an important area to which to attend.

Need and anticipation

In the chapters we observe connections between learners' *past* knowledge, *present* learners' needs and the framing of *future* knowledge construction. At a cognitive level, anticipation has an important effect on the analysis of construction of knowledge. In the theory of didactical situations, a-priori analysis takes into account contextual factors and their influence on epistemic processes. For example, a-priori analyses consider cognitive difficulties that accompany the specific mathematical domain. This may help to clearly define the knowledge that has to be constructed and will influence the design of appropriate activities. It allows the analyst to build situations that reflect in depth the mathematical epistemology of the given domain. The a-priori analysis also has a referential role with regard to analyses of the didactical phenomena: the hypotheses of the a-priori analysis are often not confirmed in reality and differences between what was expected and what is observed lead to a deeper understanding of the learning situation.

T&M extend consideration of anticipation by means of establishing links with Engle's construct of 'framing time' which includes 'framing current learning episodes as building upon past ones' and 'framing current learning episodes as relevant to the future', which are situated chronological constructs related to T&M's hypothesis on the reuse of elements of knowledge. This relation between past knowledge and future construction of knowledge is also present in SDH in the process of vertical reorganization of previous mathematical constructs. This process interweaves previous constructs and leads to a new construct. Even though AiC analyses are not usually preceded by an explicit a-priori analysis, in each AiC study the researchers were aware of the (mathematical) constructs to be constructed, and this awareness is shown in their standards and design principles.

YER also relates past knowledge and future construction of knowledge but by means of curricular choices. Her chapter supposedly adopts a macro-level perspective (the influence on the curriculum) but in fact looks at moments that she calls 'epistemological discontinuities' in which cognitive reconstruction is needed. Her focus on need arises from the epistemological nature of the mathematics domain. She emphasizes that 'the old piece of knowledge is crucial and constructive' and that old knowledge should not be rejected or neglected under any circumstances. She highlights the importance of understanding the need to change a view and that 'striving to achieve a new perspective is at the heart of mathematics learning'. H&P refer to the importance of past knowledge in the claim that new knowledge cannot be created from scratch and that it emerges through elaborating prevailing knowledge and understanding across several iterations.

The authors of three of the four chapters agree on the importance of linking past knowledge and the construction of (new) knowledge but there are differences in why this is important: H&P do not address this issue; T&M view it in terms of knowledge to be taught and their hypothesis that the reuse of an element of knowledge favours learning; YER focuses on the curriculum; SDH focus on the learners' needs in the construction of new knowledge. In this subsection we focus on SDH to highlight differences between 'external needs' – arising from the designed didactic situation or from curricular issues – and 'internal needs' – arising from the learner, though usually within a designed didactic situation.

In SDH the learners' need for new knowledge is inherent to the task design but this need is an important stage of the process of abstraction and must precede the constructing process, the vertical reorganization of prior existing constructs. This need for a new construct permits the link between the past knowledge and the future construction. Without their Davydovian analysis, this need, which must precede the constructing process, could be viewed naively and mechanically, but with Davydov's dialectic analysis the abstraction proceeds from an initial unrefined first form to a final coherent construct in a two-way relationship between the concrete and the abstract – the learner needs the knowledge to make sense of a situation. At the moment when a learner realizes the need for a new construct, the learner already has an initial vague form of the future construct as a result of prior knowledge. Realizing the need for the new construct, the learner enters a second stage in which s/he is ready to build with her/his prior knowledge in order to develop the initial form to a consistent and elaborate higher form, the new construct, which provides a scientific explanation of the reality.

Individual and social construction of knowledge

This last section concerns the social and its influence on the analysis of the construction of knowledge by the individual within the group. It focuses on analyses of the construction of knowledge and the grain size of analyses.

The social is inherent in the frameworks represented in T&M, SDH and H&P inasmuch as they view individual actions as occurring in a social context. This view is not inconsistent with YER but this is not stated, in so many words, in that chapter. T&M, SDH and H&P are socio-cultural in the sense that cognition cannot be separated from context and mediation (through artefacts and people). Even so, the different frameworks offer different interpretations to the word 'context' which lead to different analyses with regard to the construction of knowledge of the individual within the group – different modes of learning have distinct cognitive and social strengths; as Engeström (1991) notes, there is a great difference between a school textbook being the object of activity and the instrument for activity. The distinct cognitive and social strengths depend on the way the frameworks view social interactions. Is it the learning itself or a part of the context? What do we exactly mean by context? We address these questions with regard to T&M and SDH's frameworks.

Our starting point is the knowledge addressed. T&M deal with 'taught knowledge' in a classroom and 'knowledge to be taught', the knowledge society requires the education system to teach. SDH deal with a specific form of knowledge, mathematical abstractions. Unlike experimental studies carried out in the 'French tradition' that generally concern classroom situations or some kind of institutional design, AiC studies consider a greater diversity of learning situations inside and outside the classroom. Comparing the two frameworks, a difference is the focus of each approach: in the TDS, for example, learning situations are central objects while in AiC the focus is on the learner or an interacting group of learners. In TDS, contextual factors are part of the *situation*, the system of relationships between teachers, students and mathematics. The process of setting up such situations, including their phases, conditions and false avenues to be avoided, constitutes the major part of the theory and the role of the teacher is paramount. Noting what has been said above of didactic and adidactic situations it is clear that social interactions, between students and between students and teacher, are a central focus in TDS. AiC notes the importance of students' interactions in collective activities but research on the role of teacher–student interactions in the construction of knowledge is in its infancy. To better understand how social interactions are viewed in AiC we consider SDH's meaning of context.

Context, in AiC, is an integral component of the abstraction but has two parts. The first consists of elements known in advance such as students' prior learning history. The second part of the context relates to knowledge in development that develops in interactions with peers, teachers and texts (including technology). To AiC researchers such as SDH the processes of constructing knowledge and patterns of social interaction strongly influence each other. The point of our considerations is that both theoretical approaches view social interactions as integral to the analysis of the construction of knowledge but the foci and categories of analysis are different: SDH focus on learners while T&M focus on the situation; SDH's basic elements are epistemic actions while T&M distinguish between functionalities of knowledge which serve to organize the development of students' conceptualizations through appropriate situations.

Vygotsky's general genetic law, which we view as being almost general, holds that every development function appears twice, first at an interpersonal level and then at an intrapersonal level. But this is not linear, it is dialectic. Tracing the learning of the individual within the group is complex. Where do we start, with the individual(s) or with the group? The answer will depend on the focus of the approach. T&M analyse the taught knowledge (the class) and the understanding of the individual (for this purpose it uses different grain sizes of analysis). AiC is characterized by its fine-grained size of analysis. The aim is to offer a micro-analysis of the group without losing the individual. The focus is on the learner(s) both in collective and individual activities.

SDH start by considering the processes of knowledge construction in a group of three learners without losing the personal diversity and the unique nature of each individual. In their review of AiC literature they outline distinct approaches to individual and group epistemic actions: group epistemic actions – the group as a unit (Dooley 2007); analysis of epistemic actions of individuals within the group and in conjunction analysis of the social interactions and identification of patterns of interaction likely to support abstraction (Dreyfus, Hershkowitz, & Schwarz 2001); emergence of the construction of the group's shared knowledge from the individual's knowledge-constructing processes (Hershkowitz, Hadas, Dreyfus, & Schwarz 2007); micro-analysis of the epistemic actions of a solitary learner (Dreyfus & Kidron 2006); and at a second stage the analysis of the role of the contextual influences (learner's interaction with the computer as a partner) on the different patterns of epistemic actions (Kidron & Dreyfus 2007).

T&M offer different types of analyses: at the meso-level the analysis of reconstruction of taught knowledge in the class and at the micro-level the physics knowledge to be taught and students' comprehension by means of analysis in terms of small elements of knowledge, the facets. In T&M's decomposition of knowledge in smaller elements we find an expression of TDS a-priori analysis which has a referential role and reveals didactical phenomena.

We end this section with a consideration of the complementary insights on cognitive aspects offered to each other by means of the different analyses. T&M

provide an analysis of students' different interactions with the milieu, and the observation that the interaction with the milieu was not as expected may reveal important cognitive aspects to researchers. The micro-analysis of AiC focuses on learners' epistemic actions and, in parallel, on social interactions. The same micro-perspective is used to analyse the construction processes for the individual and for the group. Unlike T&M, with their analysis of taught knowledge at the meso-level and their micro-analysis of the evolution of knowledge, SDH propose two fine-grained analyses which finally merge into one. SDH claim:

> We provide detailed evidence of the constructing process of the group's shared base of knowledge, the manner in which it emerges from the individuals' knowledge-constructing processes, and the way in which it constitutes a shared base that allows the students to continue constructing further knowledge together.

Sometimes, the analyses of the social influences and the analysis of the epistemic actions are in parallel, but links are soon established and the linked analyses offer a deeper view on the construction process of the group without losing the individual.

Conclusion

We return to our opening question, can these chapters contribute to a consistent wider understanding and analysis of knowledge construction? Analysing the similarities and differences between the chapters, we realize that they addressed similar issues in different ways. They deal with students' knowledge construction but there is a distinction between knowledge creation and recreation. With regard to epistemic issues there are connections between learners' *past* knowledge and the framing of *future* knowledge construction, but these are addressed in different ways. Side by side with these different ways of addressing similar issues are distinct approaches to the analyses of knowledge and the grain size of this analysis. These different approaches have the potential to be complementary; they allow us to observe the limitations of each framework but also the richness and additional values offered by each framework to the others. This is especially important in the complex matter of analysing the construction of knowledge in classrooms.

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Part 2

The role of the teacher in the transformation of knowledge in classroom interaction

Chapter 6

Expert support for group work in elementary science

The role of consensus

Christine Howe

Introduction

Evidence has been amassed over the past 20 years to indicate that group work among children can support knowledge and understanding in elementary science (see Howe, Tolmie & Mackenzie 1995; Sherman 1999). Group work has therefore been promoted in many countries as a key component of teaching. Its relevance to science education is, for instance, a recurring theme in contemporary guides for practitioners (e.g. Harlen & Qualter 2004; Sherman 1999). Group work is also emphasized throughout a recent issue of the professional journal *Primary Science Review* (Association for Science Education 2004). Within the United Kingdom, it has even been enshrined in national policy (Department of Education and Science 1989; Learning and Teaching Scotland 2000). Perhaps as a result of these initiatives, group work is already well-embedded within elementary science teaching. A recent survey of 111 British primary schools (Baines, Blatchford & Kutnick 2003) revealed that 28% of the teaching in science involves 'peer interaction', compared with 5% in mathematics and 12% in English.

Nevertheless, no matter what its significance, group work among children will never be sufficient to deliver the science curriculum. Children working with each other are not going to construct Newton's laws or Darwin's theory of evolution, nor, given the difficulties that adults are known to experience (Dunbar & Fuselgang 2005; Kuhn, Amsel & O'Loughlin 1988), are they going to master the full intricacies of hypothesis testing. Expert guidance will be required in addition, primarily from teachers but perhaps also from computers. However, although this point is taken for granted in most relevant discussions, little is said about how guidance should be given. This is troubling, for expert input to group work is theoretically challenging from a number of perspectives. For instance, within the Piagetian tradition (e.g. Doise & Mugny 1984; Piaget 1932), knowledge growth is thought to depend upon *coordinating* existing beliefs with alternatives, and the unequal power relations between children and experts are believed to undermine coordination. Within the Vygotskyan tradition (e.g. Vygotsky 1978; Wood 1986), expert guidance is critical, but it is depicted as requiring interventions that are carefully tailored to individual needs. It is hard to see how comparable interventions could be achieved with groups.

Studies highlighting the challenge posed by expert guidance have recently been reviewed in Webb (in press), and the challenge is amply illustrated in research that I have conducted with children in the 8- to 12-year age range. For instance, mimicking characteristic teaching strategies, Tolmie, Howe, Mackenzie and Greer (1993) gave children a quiz with feedback, after group work relating to object flotation. Conceptual growth in children who experienced the quiz was significantly *worse* than in otherwise equivalent children who did not have this experience. Howe, Tolmie, Thurston, Topping, Christie, Livingston, et al. (2007) report a classroom intervention, where teachers took children through extended programmes of instruction first relating to evaporation and condensation, and then relating to force and motion. Group work made a positive contribution to learning, but its impact was *weakened* when teachers intervened directly within group interaction. One clue to interpreting these results may lie with evidence that the benefits of group work are not always immediately apparent with the 8- to 12-year age group (Howe, Tolmie & Rodgers 1992; Tolmie et al. 1993). Children in this age range often require post-group opportunities to consolidate group experiences and resolve the contradictions that group work generates (Howe 2006; Howe, McWilliam & Cross 2005). Arguably one problem with the quizzes used by Tolmie et al. (1993) and the interventionist teachers observed by Howe et al. (2007) is that they pre-empted crucial post-group processes by forcing premature closure. The implication is that expert guidance requires subtle balancing acts.

Without doubt then, group work in elementary science poses a conundrum. On the one hand, group work will never be sufficient to deliver the curriculum; expert guidance is also essential. On the other hand, expert intervention can often undermine the value that group work is known to possess. One potential response to the conundrum is to conclude that group work is more trouble than it is worth, and focus upon the delivery of expert guidance in non-group settings. Despite the emphasis on group work in policy documents relating to science (as noted above), there are signs that government thinking in the United Kingdom is moving in this direction. Recent policy initiatives emphasize whole-class or ‘interactive’ whole-class teaching (Alexander 2006). However, as also noted above, group work is now firmly entrenched in science classrooms, and there may already be a momentum that is hard to shift. It may therefore be prudent to explore whether group work and/or expert guidance can be planned in a fashion that avoids the potential pitfalls, permitting productive combination rather than antagonism. This was the issue examined in research reported by Howe, Tolmie, Duchak-Tanner and Rattray (2000) and Howe and Tolmie (2003), and therefore it is aspects of this research that will provide the focus for this chapter. After summarizing the theoretical underpinnings, the chapter will outline the relevant procedures, and conclude by describing and interpreting the main results.

A possible role for consensus

The approach taken in the research reported by Howe et al. (2000) and Howe and Tolmie (2003) revolves around the achievement of *consensus* during group interaction. Any search of the worldwide web (e.g. via www.google.co.uk) will reveal that the term ‘consensus’ is associated with a multitude of definitions.

However, two themes recur. The first is that consensus is achieved when group members agree among themselves about some issue, even if for some members the agreement is not whole-hearted, but rather a decision that can be 'lived with'. The second theme is that agreement is reached through discussion about the issue, during which group members express opinions and if necessary debate these. Jointly the two themes highlight a subtle distinction between consensus and agreement, which implies a subordinate–superordinate relation. Consensus is, in other words, a specific instance of agreement. First, consensus seems to be restricted to issues that have been the focus of discussion (i.e. the topic of conversation), while agreement can also apply with side issues. Second, consensus seems to require that several group members are party to the agreement, while agreement can also be achieved by one member alone. For example, in the following extract, the children can be regarded as achieving consensus about where the water will be cooler, because this is the focus of their discussion, and because several (in fact, all) of them are involved in the concluding agreement. On the other hand, when Graham concurs with Nicola about the reasons why the water in the black cup will be cooler, agreement occurs, but not consensus. This is because reasons are a side issue in a discussion that focuses upon predictions, and no child apart from Graham is involved in the agreement. Nicola made the claim that is accepted, but even she does not contribute to the agreement per se:

GRAHAM: [*Reads from workbook*] Talk very carefully about whether the water will be cooler in the orange cup or the black cup. You're starting [*points to Joanna*].

JOANNA: The orange one. I think the orange cup could get more air in than the black cup.

NICOLA: I think the black cup can get more air in than the orange cup, so it will be cooler.

ROSEANNA: Same as Nicola – the black cup will be cooler.

GRAHAM: Well, I thought it was the black cup because the black cup has more space. So Nicola's right – the black cup would have more air. Do you all agree now that the black cup will be cooler [*all three girls nod*].

The above amounts to a working definition of consensus along the lines of 'agreement among several individuals about some topic, after the topic has been discussed and (where necessary) differences have been resolved'. Addressing consensus in this sense rather than general agreement, the research reported by Howe et al. (2000) and Howe and Tolmie (2003) examined the consequences of requiring groups of children to converge on consensual positions in the context of science, and making these positions the focus of the expert guidance that the groups were offered subsequently. Two, essentially theoretical, considerations suggested that the approach might prove helpful. First, it seemed possible that if experts address the consensual positions of whole groups, their interventions might, in principle at least, be seen as *serving* children's needs, rather than having authority over these. This might address Piagetian concerns about relative power. In addition, it also appeared feasible that if experts tailor their interventions to positions that group members have converged upon, these interventions might come close (or as close as possible) to the individualized support that Vygotskyans require. Nevertheless, regardless of any theoretical appeal, the use that Howe et al. (2000) and Howe and Tolmie (2003) made of consensus was not grounded in an extensive body of supportive research. Many scholars have used consensus as a strategic device for promoting interaction. For instance, this is the reason for the emphasis upon consensus in the *Thinking Together* programme developed by Mercer and colleagues (e.g. Dawes, Mercer & Wegerif 2003; Mercer & Littleton 2007): as Mercer and Littleton note 'the imperative to reach consensus in the context of *Thinking Together* is designed to motivate the children to engage, and keep engaging, with each other's ideas and suggestions' (p. 72). It is also the reason for task instructions like 'Talk about your different ideas until you all think the same way' throughout my early studies of group work in science (e.g. Howe, Rodgers & Tolmie 1990; Howe et al. 1992). However, in these studies, nothing was done with the consensual positions once they were achieved. Indeed, it may not have mattered whether consensus was in fact achieved; the crucial thing was that the *quest* for consensus stimulated interaction. By contrast, because consensual positions were the focus of expert guidance in the work to be discussed below, these positions were not simply crucial, but also the object of subsequent activity.

The subjection of consensual positions to further activity in the service of learning seems, on the face of it, to resonate with Vygotskyan ideas. Within the Vygotskyan tradition (see Vygotsky 1978), learning is believed to depend upon the joint construction of superior understanding ('inter-psychological function'), and the subsequent assimilation of these constructions to individual knowledge ('intra-psychological function'). Since the notion of inter-psychological functions implicates consensus (and inter-psychological functions underpin eventual intra-psychological functions), the emphasis is indeed upon the creation and subsequent usage of consensual positions. This said, the inter-psychological concept also implies that, to be useful, consensual positions will involve progress from what group members were capable of before they worked in groups. My previous research indicates that such progressive consensus is seldom achieved during group work in science, at least at the elementary level (see, e.g. Howe 2006; Howe et al. 1990, 1992). Moreover, individual learning after progressive consensus (on the rare occasions that such consensus is achieved) seldom surpasses what occurs without such consensus. Thus, when Howe et al. (2000) and Howe and Tolmie (2003) examined the role of consensual positions in the research to be summarized below, they did not expect that these positions would necessarily display the superior understanding emphasized within the Vygotskyan tradition. Equally, they did not anticipate that the success of their interventions would depend upon jointly achieved progress.

In sum, the research reported in Howe et al. (2000) and Howe and Tolmie (2003) explored consensus rather than the more general concept of agreement. Moreover, the role that was envisaged for consensus was subtly different from (although related to) roles that have been flagged elsewhere. The research examined the significance of consensus using tasks, which started with children being invited to reflect upon the factors relevant to shadow size (Howe et al. 2000 – hereafter referred to as the 'shadows study') or rate of cooling (Howe & Tolmie

2003 – hereafter referred to as the 'cooling study'). Subsequently, children were asked to design tests to explore whether the factors they had been considering were relevant in practice, and to draw conclusions from test results. The key issue was the value of requiring children to discuss and achieve consensus about relevant factors during the first stage, as a prelude to subjecting consensual positions to empirical test during the second stage and receiving expert assistance over test design. Howe et al. (2000) and Howe and Tolmie (2003) recognized that the type of task they were using is employed in British schools with the intention of fostering understanding of experimental procedures *and* using test results to promote conceptual growth. Therefore, the research covers the implications of consensus for both conceptual and procedural understanding. Here, due to space limitations, the focus will be upon the conceptual dimension alone, and readers are referred to the original publications for the largely parallel results in the procedural domain.

Design and methods

Of central interest in the context of this chapter is the comparison made in the shadows and cooling studies between two groups of children (called Types 1 and 2 in the original reports). The Type 1 children held group discussions about factors during the first of the two stages outlined above, and reached consensus about which factors were relevant. They then tested consensual positions, receiving expert guidance while they did this. The Type 2 children also held group discussions about factors, and received expert guidance while testing. However, they were not asked to achieve consensus during the first stage, and in the absence of consensus the factors that they tested were described as 'ideas that people have'. If consensus was playing its anticipated role, conceptual growth in the Type 1 children should be superior to that in the Type 2 children. Besides the Type 1 and 2 children, both the shadows and the cooling study included two further groups, referred to as Types 3 and 4. The characteristics of all four types are summarized in Table 6.1. The Type 3 children discussed and achieved consensus over factors, but did not receive expert guidance during testing. By comparing the Type 1 children with the Type 3 children, it was possible to establish whether expert guidance (at least when 'protected' by consensus) was actually adding something to what could be achieved from group work alone. This could not be inferred from simply showing that Type 1 was superior to Type 2. The Type 4 children considered factors individually without discussion (and therefore without consensus), but they received guidance when testing the 'ideas people have'. Comparison of the Type 1 children with the Type 4 children indicated

Table 6.1 Key features of group task types

	Type 1	*Type 2*	*Type 3*	*Type 4*
Discussion	Yes	Yes	Yes	No
Consensus	Yes	No	Yes	No
Guidance	Yes	Yes	No	Yes

whether group discussion (again when accompanied by consensus) added something to what could be achieved by guidance alone. Again, this cannot be guaranteed simply by comparing the Type 1 and Type 2 children.

Both the shadows and the cooling study began with pre-tests to 9- to 12-year-old children (*N* = 272 for shadows; *N* = 164 for cooling). To pre-test conceptual knowledge, arrangements of apparatus were shown to whole classes, i.e. for shadows, triangles of varying sizes and colours at varying distances between a screen and a lamp, with the lamp varying in brightness; for cooling, hot water in containers that varied in width, height, thickness, material and reflectivity. Using answer booklets, children were asked to write down (and, via short sentences, justify) predictions about whether shadows projected from the apparatus would be bigger, smaller or the same size as a previously projected 'standard shadow', or whether water would cool down faster, slower or at the same rate as a previously presented 'standard container'. Children were also invited to explain real-world instances of shadow formation or rates of cooling, for example, the size of a pantomime giant's shadow under bright and dim spotlights, and the fast or slow cooling of the water in a white, rubber hot-water bottle that is left overnight. For both shadows and cooling, pre-test responses were scored for their adequacy as: (a) explanations of predictions (EP), which tapped understanding of how factors manipulated via the apparatus affect outcome; (b) applications to real-world instances (AR), which tapped understanding of how the same factors operate in everyday contexts. With cooling only, scoring also covered the adequacy of the causal mechanisms (CM) that children used to explain the cooling process, i.e. their embryonic understanding of conduction, convection or radiation. Background literature (e.g. Howe 1998; Osborne, Black, Smith & Meadows 1990) indicated that causal mechanisms would seldom be used with shadows but would be frequently used with cooling, which proved to be the case. Scoring procedures meant that each child was given a total pre-test score from 0 to 9 on each of the EP, AR and CM dimensions.

Subsequently, pre-tested children worked in same-age (but otherwise randomly assigned) triads through a two-stage task (*N* = 72 triads for shadows; *N* = 36 triads for cooling). Both stages were videotaped throughout, and both employed the apparatus that had been used in the pre-tests. For the children assigned to the Type 1 task, the first stage started with them making individual predictions about outcomes by ticking on cards. Predictions were then disclosed to the rest of the group, and joint predictions were formulated by discussion. Joint predictions were tested, by switching on the lamp and observing the shadow's size (shadows study), or by using a thermometer to measure the water's temperature (cooling study). Finally, to promote consensus, triads were asked to discuss and agree what makes a difference to outcome and what does not matter, and record their joint conclusions in writing. The first stage for the Type 3 children was identical, but for the Type 2 children there was discussion but no requirement to reach consensus, and for the Type 4 children there was no opportunity for discussion, let alone consensus. The second stage involved inviting triads to use the apparatus to test the accuracy of their agreed ideas (Types 1 and 3), or 'some of the ideas that people have' (Types 2 and 4). For the Type 1, 2 and 4 children, testing was supported in accordance with Wood's (1986) principles of 'contingent prompting', which involved praise for success and feedback of increasing explicitness in the event of error. The Type 3 children received no support for testing. In the cooling study, a researcher provided the support, acting very much as a teacher might. In the shadows study, 50% of the children received support from a researcher, and 50% received support from a specially programmed computer. A few weeks after the group task, the children were post-tested, following procedures that: (a) corresponded with those used in the pre-test; (b) were scored in the same way; (c) allowed the same (0 to 9) range of scores. In both studies, two judges independently scored 25% of the pre- and post-test responses. Inter-judge agreement ranged between 81% and 96% across the three (EP, AR, CM) scoring dimensions.

Results

In general, analysis of the progress that the children made from pre-test to post-test (as assessed via post-test scores with pre-test scores subtracted) suggested that there was value in requiring groups to achieve consensus before they received guidance. Supportive results were obtained in both studies, and in the shadows study regardless of whether researchers or computers provided expert support. In fact, the researcher vs. computer manipulation had no effect on either the amount of progress that the children made or the process by which progress was achieved. Of particular interest was the difference between the Type 1 and Type 2 children over pre- to post-test progress, for the only difference between the group tasks here was whether or not consensus about how factors operate was required before researchers or computers assisted with testing. Therefore it is important that, as shown in Table 6.2, the progress that the Type 1 children made in conceptual understanding was superior to the progress that the Type 2 children made on all five comparisons. In three cases, the differences were statistically significant. Moreover, among the Type 1 children, there were significant positive correlations ($r = 0.33$ to 0.38, $p < 0.05$) between the number of children concurring with each

Table 6.2 Conceptual change from pre-test to post-test as a function of task type

	Type 1	*Type 2*	*Type 3*	*Type 4*	*Significance*
Shadows	$n = 52$	$n = 53$	$n = 49$	$n = 52$	
EP	$+2.19_a$	$+1.43_{ab}$	$+0.90_b$	-0.02_b	$F = 4.19$**
AR	$+1.08_a$	$+0.03_b$	$+0.39_b$	$+0.19_b$	$F = 4.00$**
Cooling	$n = 25$	$n = 25$	$n = 25$	$n = 27$	
EP	+1.04	+0.64	+0.96	+0.78	ns
AR	+0.20	+0.08	+0.24	+0.11	ns
CM	$+1.36_a$	-0.28_b	$+1.60_a$	$+0.70_b$	$F = 3.26$*

Notes
1 Data analysis was restricted to children who completed all stages of the procedure, i.e. pre-test, group task, and post-test – numbers (n) are shown in the table.
2 EP = Explanation of predictions, AR = Application to real-world instances, CM = Causal mechanisms.
3 * = $p < 0.05$, ** = $p < 0.01$.

written idea (as ascertained from the videotapes) and pre- to post-test change on three measures (EP and AR for shadows, and EP for cooling). This contrasted with the Type 2 children where the number of children agreeing with each idea that was proposed (whether or not agreement amounted to consensus) was unrelated to pre- to post-test change.

From Table 6.2, it is clear that in the shadows study, the Type 1 children did not merely outperform the Type 2 children as regards conceptual growth; they also surpassed the Type 3 and Type 4 children. This means that here consensus did not just 'protect' the children from antipathy between group work and expert guidance; it also helped them go beyond what they could achieve from either of these alone. In this context, it is revealing that for the Type 1 children, EP and AR change were both significantly correlated (r = 0.24 to 0.34, $p < 0.05$) with the number of valid tests conducted during the second stage of the group task (defined as tests that manipulated the focal factor while holding other factors constant, and counted from the videotapes). The corresponding correlations for the Type 2 children were non-significant, even though the two groups of children conducted similar numbers of valid tests in absolute terms (M for Type 1 = 3.39; M for Type 2 = 3.83). The implication is that consensus allowed the Type 1 children to treat the second-stage tests as providing *evidence* about factors, and therefore to benefit from the valid results that were obtained under guidance. In the absence of consensus, the Type 2 and 4 children probably treated testing as an isolated exercise. While the Type 3 children most likely achieved the same, evidential perspective as the Type 1 children (due to their common experience of discussion and consensus), the relatively low number of valid tests that these children conducted in the absence of guidance (M = 1.56) meant that they were unable to profit from test results.

In the cooling study, as in the shadows study, the Type 1 children outperformed the Type 4 children as well as the Type 2. However, this time, they were only equivalent to the Type 3 children, and not superior. This implies that even if consensus pre-empted the unproductive combination of group discussion and guidance that characterized Type 2, it did not allow the Type 1 children to obtain actual benefits from the expert guidance. One possible reason for the difference is signalled by the fact that valid tests, which were again more frequent in Types 1, 2 and 4 (Ms = 1.11 to 1.33) than Type 3 (M = 0.67), were less relevant to conceptual growth with cooling than they were with shadows. There was, in fact, no relation in the cooling study between pre- to post-test change and number of valid tests with any of the task types. This lack of relevance might stem from the fact that in the cooling study (and once more in contrast with the shadows study), the interpretation of test results was coloured by preconceptions about underlying causal mechanisms. Thus, in the following extract, preconceptions about the passage of heat dictate how variations as a function of reflectivity are interpreted:

JANE: [*Puzzled because rates of cooling differ between two containers that are identical apart from reflectivity, i.e. black or white*] They're both exactly the same, except they're different colours.

MARY: So they must let exactly the same amount of heat pass through. I just don't understand why the temperature's different. Maybe there's something wrong with the thermometer.

As noted earlier, it was known prior to the research that causal mechanisms, specifically ideas about conduction, convection and radiation, are central to children's thinking about heating and cooling. This was, after all, the main reason for introducing the CM measure in the first place. Nevertheless, the fact that test results would be interpreted via mechanisms was not anticipated, and it is potentially significant. It implies that the material that is crucial to conceptual growth is information about mechanisms, and not empirical data. Therefore, had mechanisms been the focus of expert guidance, there might, once more, have been differences between the Type 1 and Type 3 children.

Implications

Further research is needed to ascertain the implications of providing information about causal mechanisms in domains where, like heating and cooling, mechanisms appear to be pivotal. However, no matter what the outcome of such research, it is clear from present results alone that consensus creates receptivity to expert guidance, which, as can be inferred from the Type 2 data, would not otherwise be apparent. At the very least, this underlines the genuineness of the issue addressed in this chapter: the successful coordination of expert intervention with group work among children is most definitely challenging, is likely to misfire and is fully deserving of detailed research. In addition, it is to be hoped that the results also suggest a positive way forward as regards achieving coordination. In this context, it is worth reiterating that both the shadows study and the cooling study addressed the implications of consensus for procedural understanding, as well as conceptual. While space restrictions precluded discussion of the procedural data, the implications of the Type 1 task were positive here too (for details, see Howe et al. 2000; Howe & Tolmie 2003).

However, while the implications of consensus were positive, it is not clear that they were positive for the reasons that triggered the research in the first place. In particular, it is uncertain whether, by virtue of addressing consensual positions, expert guidance was seen as serving children's needs rather than having authority over these, and even if this did happen, whether this was relevant to outcome. Equally, it is unclear whether, through being tailored to consensual positions, expert intervention was regarded as providing individualized support, and whether this too was relevant to outcome. These uncertainties are inevitable, given that the research was designed to explore consensus and not the underlying theoretical considerations. Nevertheless, even though this must be accepted, there can be no doubt that the Type 1 (and Type 3) children saw what happened during the second stage of the group task as addressing positions that they themselves 'owned', and it was this sense of ownership that engaged them. This is consistent with the sense both of being served and of receiving individual tailoring, and upon reflection this consistency should not be surprising. The emphasis upon 'being served' stems from the anti-authoritarian approach taken within the Piagetian tradition, and arguably the concept of 'consensus' is itself non-authoritarian. Certainly, the working definition of consensus advanced earlier, i.e. agreement among several individuals about some topic, after discussion and (where necessary) resolution of differences, conveys something more collective and egalitarian than agreement per se. Likewise, the emphasis upon 'individual tailoring' derives from Vygotskyan thinking, and as noted already, the image of consensus that was developed in the shadows and cooling studies resonates, in part, with the Vygotskyan concept of inter-psychological functions (e.g. Vygotsky 1978). More generally, the Vygotskyan tradition places considerable emphasis upon social products as 'tools' for learning, and the present use of consensus is entirely consistent with this approach.

Nevertheless, while there are Piagetian and Vygotskyan elements within both the concept of consensus and its use in the present research, the results of the shadows and cooling studies are not fully compatible with either approach. Regardless of its egalitarian connotations, it remains unclear how an inherently *social* product like consensus should be incorporated within the Piagetian tradition, given the latter's emphasis upon coordination within *individual* cognition (Piaget 1985). It is also uncertain how Piagetians would deal with the finding that consensus allowed the children to *use* test results to develop conceptual understanding, in other words transformed them into *bona fide* hypothesis testers. The children were, after all, only 9 to 12 years of age, when hypothesis testing (particularly of the multi-variate kind addressed in the research) is regarded by Piagetians as formal operational (Inhelder & Piaget 1958) and therefore only accessible from adolescence. As regards Vygotsky, the results continue to challenge the emphasis, inherent within the notion of 'inter-psychological functions', upon socially constructed *progress*. The consensual positions achieved in the shadows and cooling studies were often inferior to the ideas expressed at pre-test, when they were compared against received science wisdoms. Furthermore (and consistent with data reported in Howe 2006; Howe et al. 1990, 1992), there was no relation between the scientific merit of consensual ideas and pre- to post-test change. Progress was as likely after poor ideas as it was after good.

In general then, the research points to a theoretical position that is a rapprochement of Piagetian and Vygotskyan thinking, but goes beyond this thinking in several respects. Social products are emphasized, but children are viewed as responding to these products actively rather than passively assimilating them. Whether the indicated position is theoretically significant depends of course on the extent to which it is endorsed across a broad range of contexts, and this remains to be seen. However, whatever the case on the theoretical level, the research has undoubted practical implications for science education itself. First, because the research suggests a means, namely consensus in the context of the Type 1 task, by which group work can be coordinated with expert guidance, it indicates that recent moves in the United Kingdom (see earlier) to dispense with group work (and its potential benefits) may be premature. Second, as emphasized already, the Type 1 task was grounded in an approach that is already widely used for purposes of teaching, at least in the United Kingdom. In particular, it is commonplace to invite children to reflect upon the factors relevant to outcomes, and to design tests to explore whether the factors under consideration are relevant in practice. The research endorses the continued use of this approach, at least in its

Type 1 format. Finally if, as suggested, the Type 1 task succeeds in inculcating a hypothesis-testing *perspective*, it will have helped relatively young children to emulate professional scientists. In the United Kingdom (and perhaps in other countries), emulation is a curriculum requirement (Department of Education and Science 1989; Learning and Teaching Scotland 2000).

Acknowledgement

The research described here was supported by ESRC award R000236714 to Christine Howe and Andy Tolmie.

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Chapter 7

Guided construction of knowledge in the classroom

The troika of talk, tasks and tools

Richard Sohmer, Sarah Michaels, M. C. O'Connor, and Lauren Resnick

In the process of guided knowledge construction, tasks, representational tools and talk are inextricably intertwined. This chapter explores the role of the task (and the participant structure entailed by the task), the representational tools that mediate and scaffold knowledge construction, and how tasks and tools are coordinated and animated through carefully structured talk. We first explore each of these facets in isolation, briefly reviewing the literature and our own position on these facets. We then consider these facets together, in an example of guided knowledge construction from the Investigators Club, an after-school program which takes a conceptual, 'physics first' approach to science. Any instance of knowledge construction is unique – instantiated in a particular time and place, and with a unique set of participants. Our example is of a five-minute interaction which took place on the sixth day of the Investigators Club, with a group of low-SES middle school students learning about air, air pressure and the atomic-molecular theory of matter. The challenge in making sense of an example is to examine particulars (particular content, particular students, particular age group and setting) while developing general claims about nature of tasks, tools and talk, and the way they interact.

Our work on guided knowledge construction grows out of a Vygotskian theoretical framework which emphasizes the social formation of mind, that is, the importance of social interaction in the development of individual mental processes. Sensemaking and scaffolded discussion, calling for and elicited by particular forms of talk, are seen as primary mechanisms for promoting deep understanding of complex concepts and robust reasoning. We will briefly review ideas on talk, classroom activities or tasks, and cultural tools, as an introduction to our example, where a carefully constructed cognitive tool is at the center of a productive task and talk amalgam, led by the first author as teacher in the Investigators Club after-school setting.

The role of talk: reasoned discussion in the classroom

Over the past two decades, a great deal of research has demonstrated the role of certain kinds of structured talk for learning with understanding. A synthesis of this work can be found in Cazden (2001) and in a recent handbook chapter on classroom discourse (Michaels, Sohmer & O'Connor 2004).

We can point to a number of 'success stories' in the literature on instructional change and school reform, where elements of academically productive talk are demonstrated (cf., among others, Ball & Bass 2000; Beck, McKeown, Worthy, Sandora & Kucan 1996; Chapin, O'Connor & Anderson 2003; Chapin & O'Connor 2004; Goldenberg 1992/3; Lampert & Ball 1998; Lee 2007; Minstrell 1989; Rosebery, Warren, Ballenger & Ogonowski 2005). The common thread in these cases is the use of discourse-intensive pedagogical practices combining rigorous tasks with carefully orchestrated, teacher-led discussion. Through talk, students are encouraged to draw on their Lifeworld experience and home-based genres of argument and explication, while being scaffolded into using effective (canonical) representational and discursive tools. These practices have been shown to result in robust academic achievements for students from a range of economic and linguistic backgrounds.

Over the past several years, key features of academically productive talk (by both teachers and students) have been cataloged, characterized and subsumed by the term 'Accountable Talk' (Michaels, O'Connor, Hall & Resnick 2002). 'Accountable Talk' is one of nine 'Principles of Learning' developed by the Institute for Learning (IFL – see www.instituteforlearning.org).

What is 'accountable talk'?

Accountable Talk is talk that seriously responds to and further develops what others in the class have said. It puts forth and demands knowledge that is accurate and relevant to the issue under discussion. Accountable Talk uses evidence appropriate to the discipline (e.g., proofs in mathematics, data from investigations in science, textual references in literature, documentary sources in history) and follows established norms of good reasoning. Accountable Talk sharpens students' thinking by reinforcing their ability to explicate, use and create knowledge (Michaels et al. 2002).

Accountable Talk has been defined operationally as talk that is:

- *Accountable to the learning community*: This is talk that attends seriously to and builds on the ideas of others; participants listen to and learn from each other, grappling with ideas together.
- *Accountable to knowledge*: This is talk that is explicitly based on a body of knowledge that is public or accessible to the group as a whole. Speakers make an effort to get their facts right and make explicit the evidence behind their claims or explanations.
- *Accountable to rigorous thinking*: This is talk that emphasizes logical connections and the drawing of reasonable conclusions. It is talk that involves explanation and self-correction.

These forms of *accountability* can be seen in what the students and the teacher say. They are supported by explicit teacher moves, by classroom norms, recurring activities and talk formats, as well as by carefully designed tasks (Michaels et al. 2002). The e-book on Accountable Talk (Michaels et al. 2002) suggests that, in order for Accountable Talk to happen reliably, two components need to be in place: (1) teachers need skill using talk in particular ways, understanding the power of particular talk moves and talk formats in orchestrating student talk and participation, and (2) teachers need rigorous tasks (or problem sites) to talk about. Talk moves taken out of context, not linked to academic purposes or academically rigorous tasks, will not, in and of themselves, produce learning gains.

The three dimensions of accountability are analytically separable. It is possible to have a discussion that is accountable to knowledge, with correct facts, that is *not* accountable to rigorous standards of reasoning because the argument rests on faulty premises. But in practice, it is often impossible to separate out these strands. As linguists and philosophers of language have long noted, there is no one-to-one mapping of form and function in language. The same utterance can be used to accomplish multiple functions; the same function can be served by numerous, often widely different forms of language. It is often the case that a particular utterance supports accountability to community, knowledge and reasoning, all at the same time. For this reason, we have found it helpful to focus on 'talk moves' rather than on the three facets of accountability.

What are talk moves?

A talk move is a turn at talk that (1) responds to what has gone before; (2) adds to the ongoing discourse; and (3) anticipates or 'sets up' what will come next. A talk move is inextricably tied to the context. It reaches beyond a single turn – both backwards and forwards – and both reflects and constructs the interactional context. It is thus helpful to think of a talk move, not as a particular form serving a single function, but as a move that positions the speaker and hearers and ideas in very particular ways, moment to moment.

On the basis of detailed studies of teachers who have been effective in using talk to promote learning, we have identified a set of moves that teachers can use to model and to elicit academically productive talk:

Talk Move Action Description	Prototypical forms
Revoicing	'So let me see if I've got your thinking right. You're saying XXX?' (with space for student to follow-up)
Asking students to restate someone else's reasoning	'Can you repeat what he just said in your own words?'
Asking students to apply their own reasoning to someone else's reasoning	'Do you agree or disagree and why?'
Prompting students for further participation	'Would someone like to add on?' 'Say more about that.'
Asking students to explicate their reasoning and provide evidence	'Why do you think that?' 'How did you arrive at that answer?' 'What's your evidence?'
Challenging or providing a counter-example	'Does it always work that way?' 'What about X…?'
Using wait time	'Take your time … we'll wait.'

These talk moves enact (or operationalize) accountability to community, knowledge and reasoning, sometimes in a single move. In addition, these moves shift or modify – moment by moment – the configuration of social and intellectual roles: roles and rights for thinking and speaking. In subtle and not-so-subtle ways, these moves position students as thinkers, theorizers and providers of explanations, rather than as 'parrots,' trying to echo the answer the teacher has in mind. This level of positioning work is critical if we want to understand how talk (a) guides knowledge construction and (b) socializes identity, granting students the sense that they have the right to make knowledge claims and critique and build on others' thinking.

In the anthropological and sociolinguistic research literature on classroom talk, one finds a variety of terms, such as participant structures, speech activities or speech events, each highlighting a different aspect of the organizations and arrangements within which talk takes place. As we discussed at greater length in an earlier paper (O'Connor & Michaels 1996), work by Goffman (1981) and Goodwin (1990) has become useful in our study of classroom discourse. In our earlier work (O'Connor & Michaels 1996) we argued that Goffman's ideas, particularly as elaborated by Goodwin, enable us to see what goes on *inside* extended speech activities. These ideas let us explicate how larger speech activities are actually built up through utterance sequences. This is especially helpful in the analysis of teachers' orchestration of group discussion. In this chapter we will not go into micro-level detail, but will simply provide examples of how utterance-level teacher moves position students into particular kinds of intellectual roles, which then may socialize them into the kinds of scientific modes of thinking we are concerned with. These ideas will be revisited briefly in our later analysis of student turn at talk in an after-school science program.

Consider, for example, the common tripartite Initiation–Response–Evaluation pattern (IRE). The teacher asks a question (Initiation), a student responds, and the teacher follows up with some form of feedback or evaluation. An IRE question is typically designed to elicit a single, correct answer – a 'fact' – and student responses are typically quite short. An example:

TEACHER: What's the capital of Michigan?
STUDENT: Um … I think, Detroit. No, wait, I know that's the biggest city, but I think the capital is um … [*someone whispers 'Lansing'*] oh yeah, Lansing, so I guess it's not always the biggest city in the state.
TEACHER: That's right. It's Lansing. Good.

Contrast this pattern with what we have called a 'revoicing' move – on the surface, a similar sequence of turns which we have identified in many successful teacher-guided whole-group discussions (O'Connor & Michaels 1993, 1996). In our formulation of the revoicing move, the teacher initiates with a question, to which the student responds —— but then in the follow-up, rather than evaluating the contribution as right or wrong, the teacher does something that is interestingly different. Consider the example above playing out a bit differently:

TEACHER: What's the capital of Michigan?
STUDENT: Um … I think, Detroit. No, wait, I know that's the biggest city, but I

think the capital is um … [*someone whispers 'Lansing'*] oh yeah, Lansing, so I guess it's not always the biggest city in the state.

TEACHER: So, are you saying that the capital of a state isn't necessarily the biggest or most well-known city?

STUDENT: Yeah, like in some cases … [*student continues*]

Note that in a revoicing move, after repeating or rephrasing some aspect of what the student said, the teacher always checks back with the student to see if her interpretation of the student's utterance is what the student intended.

The IRE pattern and the revoicing sequence in the first two turns above look similar, but the revoicing move in turn 3 provides the crucial difference. The use of the discourse marker 'so' (Schiffrin 1987) and the use of the question that follows ('Are you saying…?') automatically opens up a *new* slot for the student to chime in, to agree with or disagree with the formulation of the student's meaning that the teacher has put forward. Even if the student merely nods assent, there is a slot in the exchange that has been opened up for the student's voice. Whereas the IRE is a three-part move, ended by the teacher evaluation, the revoicing move is a four-part move, with the student having the ultimate interpretive clout, the right to agree or disagree with the teacher's formulation. The marker of a warranted inference and the verb of speaking ('so' plus 'you're saying') credits the originating student with the reformulation. It is the *student*'s idea that is being formulated and made public, not the authoritative knowledge of the teacher.

Though the linguistic differences between IRE and revoicing moves are small, the socializing potentials of the two sequences are poles apart. In the IRE pattern, students are positioned as 'reciters' or 'getters of the answer in the teacher's head,' while in the revoicing pattern, students are positioned as thinkers, discoverers and advocates of their own ideas, positions, theories or explanations. 'So, let me see if I have your theory [explanation, idea] right. Are you saying…?' The IRE sequence positions the teacher as the authority figure, with the right to evaluate the student's answer. In the revoicing move, teacher and student are positioned (for the moment) on an equal footing, building together a complex idea, which is then credited to the student.

What are talk formats?

Different talk formats carry, by their design, particular arrangements of roles and motivate or promote very particular talk moves, which, in turn, create differentially socializing participant frameworks. Consider the different talk formats of recitation, partner talk, student presentation and group critique, and position-driven discussion.

1. *Recitation*: In a recitation, the teacher is fully in control of the content and direction of the conversation, by virtue of her right to ask the questions and evaluate the answers. The dominant structure of the talk in a recitation is the Initiation–Response–Evaluation (IRE) sequence. Students are positioned as getting the answers the teacher is looking for, with a premium on getting the correct answer.

2. *Stop-and-talk (Partner talk)*: This hybrid talk format combines elements of whole-group discussion and small-group or partner talk, under the guidance of the teacher. It occurs when students, in the midst of some form of whole-group, teacher-fronted presentation, are asked a pointed question and told to discuss it with one or more partners. How does strategically placed partner talk engender academically productive dialogue? Students are expected both to contribute ideas and listen carefully to their partners' thinking (as any individual may be called upon to summarize their joint reflections). In partner talk, students are positioned as 'active reflectors,' on an equal footing with their partner or partners, poised to contribute something of note to the ideal rehearsal space as students formulate thoughts and try them out in a relatively private, non-threatening arena. Students get to hear their partners' ideas and use them as a base on which to build, and get plenty of 'air time' in trying out their own ideas before going public. The format allows the teacher to hear a range of ideas as he circulates among the students, and select among them key voices to be heard by the entire community. In this way, the teacher can strategically select students to speak who have opposing views or seed the territory with ideas that will further his academic agenda.
3. *Student presentation and group critique*: Here a student (or sometimes partners or small-group teams) presents his or her work – a solution to a math problem, an explanation, a theory – to the rest of the class. Typically, the 'presenters' are positioned centrally, either at the board, or standing up at their seats, which marks them more formally as having the floor. The presenters are allowed to talk for an extended time with follow-up questions from teachers and other students. The role of the students in the audience will differ from classroom to classroom. In some cases, students are expected to take on active roles as critics and questioners, offering challenges or suggestions for improving the presentation. In other cases, students merely watch the presentation, and the teacher takes on the role of primary responder, questioner or evaluator.

In this talk format, in contrast to partner talk, the students are expected to explicate their reasoning in a formal manner, that is, to 'go public.' In student presentations, the presenters are positioned as 'experts' on their own work, and critiqued and questioned by both teachers and other students. They must develop an explicit enough account of their work, reasoning, theory, or project so that it will be clear to others who have not been a part of their earlier group or partner conversations. At the same time, the other students are expected to attend and respond to this presentation as a critic or questioner.

4. *Whole-group 'position-driven' discussion*: In this talk format, a teacher leads a group of students to consider a single problem or question. A position-driven discussion (PDD) can be carried out in any subject area (O'Connor 2001; Michaels & Sohmer 2001; Sohmer & Michaels 2005). What is crucial is that the question posed has more than one answer for which reasonable arguments can be made. In the Investigators Club (a conceptual, 'Physics First' approach to science), for example, PDDs begin with a centrally placed exper-

imental setup (drawn from a carefully selected and sequenced set of 'demos' which bear upon aspects of the domain being investigated). The demonstrations at the heart of these position-driven discussions are not simply 'science demos' of the traditional sort – performed and explained by the teacher. The demos, by design, use commonplace materials (a jar with balloons, empty gasoline cans, candles, soda bottles, cups and water) which build on students' embodied experience and highly developed physical intuitions but pose intriguing and non-obvious questions (Liem 1987; Arons 1997). 'If I pump the yellow volleyball with 10 pumps of air (with a bicycle pump), will it weigh more, less, or stay the same?' (Figure 7.1).

How do position-driven discussions guide knowledge construction, promote academically productive talk and socialize students into valued intellectual practices? First, PDDs promote active participation before students are fully competent in a domain. In many cases, everyone is expected to commit to a position (and explain why) but it is perfectly fine to build on (or even copy) someone else's reasoning, provided the student states it in his or her own words. The teacher's role is to help students clarify and make explicit their position and the evidence for that position. Taken together, this kind of group discussion provides opportunity and support for students to listen to one another, build on one another's ideas, and take on new 'ways with words.' Position-driven discussions combine academic rigor with an enjoyable, game-like character. They offer each student the opportunity to *make something that is highly (and rigorously) valued*, i.e., the opportunity to develop and marshal evidence for a well-reasoned position about a matter of personal interest. At the same time, the game has a 'low entry threshold' (Gee 2007): each student is credited as a 'player,' a holder of a position or theory, whether or not his prediction or explanation is supported, in the end, by the outcome of the demo. Students – even those reluctant to participate in other kinds of group discussion – enjoy both 'playing the game' and gaining competence in valued ways of talking and thinking. This provides a productive bridge from students' home worlds and everyday experiences to the more academic forms of reasoning, socialized in school.

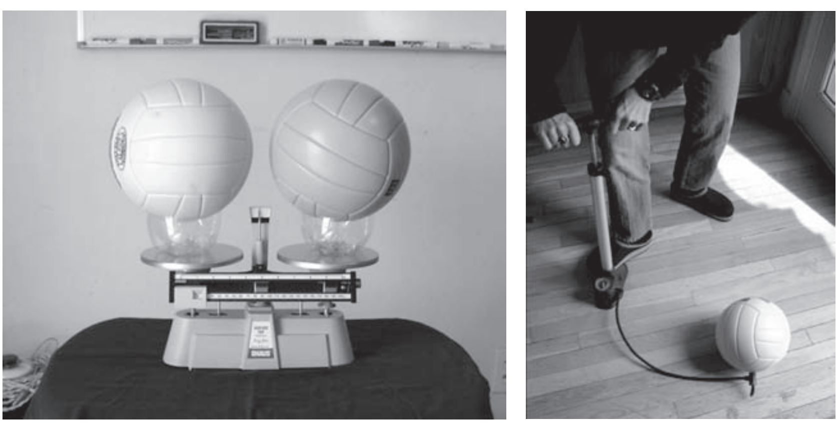


Figure 7.1 'Does air have weight?'.

The cognitive demand of tasks

The effectiveness of discourse-intensive instruction depends significantly on the quality of the tasks used in instruction. Tasks with high-level cognitive demands are characterized by multiple entry points, solution strategies and interpretive claims. They allow different students to approach the task in different ways, before being guided by the teacher into explicit formulations or arguments. High-level tasks can also feature multiple representations and opportunities to form connections between different ideas or representations (Hiebert et al. 1997). Tasks that are classified as having low levels of cognitive demand involve either memorization or the application of procedures with no connection to meaning or understanding (Stein, Smith, Henningsen & Silver 2000).

While the quality of tasks plays a significant role in learning, simply providing students with high-level tasks is insufficient for effective instruction. Research indicates that the level of cognitive demand of a task is often altered over the course of an instructional episode (Stein & Lane 1996). Teachers and students accustomed to traditional American styles of almost purely procedural teaching can be uncomfortable with the open discussion and intellectual struggle that often accompany high-level tasks. Stein and Lane (1996) found, however, that the greatest student learning gains occurred in classrooms where students were consistently exposed to high-level tasks and in which the high-level cognitive demands were sustained throughout the lesson. A set of factors that contribute to the decline of cognitive demands during classroom task implementation have been identified (Henningsen & Stein 1997). These factors are particularly strong in U.S. classrooms, according to the TIMSS 1999 Video Study, which showed less than 1% of classroom time in the United States is spent on high-level mathematical work (Hiebart et al. 2003).

There is evidence in a number of the studies cited that the demand of tasks often degrades during discussion. Teachers may try to 'help' students in ways that diminish the cognitive demand of the task (e.g., telling students which step to do next rather than helping them figure out what comes next and why). To keep the conversation moving and socially comfortable, discussions often devolve into teacher-led recitations, where teachers ask a question, a student answers and the teacher evaluates the answer, and then moves on to the next student, a sequence often referred to as the Initiation–Response–Evaluation, or IRE, sequence (Cazden 2001).

While the work examining the cognitive demand of tasks in classrooms is quite extensive, less has been done on the role of activity structures in guiding knowledge construction. In our work on Accountable Talk, we have used the term 'talk format' to focus on the general goals and characteristics of the speech activity.

The role of carefully-designed cognitive tools

While utterance-level talk, talk formats and task level have been explored in the literature on classroom learning, there has been little systematic work linking representational tools to these three. In the Vygotskian framework, tools – including physical, semiotic and psychological tools – are inextricably part of human cognition, action and identity (Cole 1998; Gee 1992; Wertsch 1991). The unit of intelligent agency is not the individual *in vacuo*, but the person (the participant) who is always already a social being, acting upon the world, him/herself, and others with historically situated and socially provided tools. One way to conceptualize the guided construction of knowledge is to focus on the transformation of thinking through mediational means.

The realization of human dependence on tools has led many (notably Vygotsky and Luria) to take the position that the primary concern for learners should be to acquire competence in the most efficacious tools: namely, 'scientific tools' (Vygotsky 1978) or, in Wertsch's technical term, 'decontextualized mediational means.' Few 21st-century parents (certainly not the authors of this chapter) would disagree.

The educational plan, then, should be straightforward: Identify the learners. Identify the best tools. Deliver the best tools to the learners.

In the United States, it hasn't worked (TIMSS 1996). To the question, 'What is the net effect – the "take-home lesson" – of all the science and math classes you've taken over the years?', most secondary school graduates respond with some version of:

> As a kid I loved it [science/math],
> but then I found out I'm no good at it.
> So now I know…
> That I don't like it, and I'm no good at it.

Within the socio-cultural paradigm, much is known about the problems plaguing science and math education. A short list – by no means exhaustive – includes:

- the problem of oppositional Discourses (Gee 1992);
- emphasis on algorithms instead of conceptual understanding;
- premature focus on the most-decontextualized tool versus useful representation(s) of a canonical tool;[1]
- failure to acknowledge and deal with naive conceptions: e.g., the mistaken notion of 'suction' (as in a vacuum cleaner 'sucks up dirt') has great explanatory currency in the everyday 'Lifeworld.' A more canonical tool must be both understandable and *more* useful in order to displace 'suction' as the default explanatory mechanism.

In addressing these problems, we have found that acquisition of a given 'scientific tool' can be facilitated by presenting students with a carefully designed cognitive tool (CDCT) which (1) is representative of the canonical 'scientific tool,' (2) is readily understood, and (3) quickly empowers students to understand, predict and manipulate physical events. A successful CDCT effects changes in students' thinking and behavior. When students use the CDCT iteratively to qualitatively transform their experiences towards a more canonical, more coherent and more controllable, physical world, they simultaneously transform their self-stories, their identities, in the direction of increased agency, higher status and legitimated participation in the Discourse of Science.

The notion of mediational means locates performance deficits in the 'lack of fit' between tools and tool users. What kinds of tools will fit the hands of our diverse students? How do we guide them to pick up those tools and practice using them in effective ways? Moreover, once they have picked up the tools, how do we guide them to signal to one another that they are using them (talking and reasoning with them) competently? This is where the norms for Accountable Talk and the importance of well-designed tasks and talk formats assume central importance; indeed, all must be in play and thus are inextricably linked.

Putting talk, task and tools together: how well-structured talk and well-designed tasks and tools build the mind

In what follows, we examine a particular episode of guided knowledge construction. It takes place in the after-school program known as the Investigators Club, which supports the teaching of physics to urban middle school students, most of whom are failing in school. The example focuses on the physics of air pressure and a narrative tool developed to help students understand and explain phenomena in this domain. Students are introduced to the tool through particular talk formats, and guided to use the tool in demonstrating their competence to their peers, through talk.

The Investigators Club ('I-Club') setting is an after-school program that meets three times a week for 15 weeks. Participants are middle school students (seventh graders) from a wide range of cultural and linguistic backgrounds, predominantly those who are struggling and/or failing in school.

The I-Club recruits students' everyday ways of speaking about the world – while gradually scaffolding them into the use of new discursive tools (new ways of giving scientific explanations and using representational tools). In this program, the activities ('tasks' or 'demos') are designed to promote active theorizing, prediction and argument about puzzling physical phenomena, often called 'discrepant events.' In the process, having a well-argued theory is the name of the game. If a student's prediction, or theory, or both are in the end disconfirmed by the evidence, that is OK: the job of the scientist is to make cogent predictions and theories so that they may be cogently disconfirmed. The goal is to make one's claim as explicit and persuasive as possible. Everyone benefits from seeing the (ultimately) best theory in the field of contesting, less effective theories, and everyone can appropriate the results for their own use in the next task.

I-Club students ('Investigators') explore a range of shared 'theorizable situations,' all of which have unexpected outcomes. These discrepant events are (by design) made from everyday objects – balloons, soda cans, drinking glasses, candles, water, fire, etc. – and while the I-Club students use their diverse, culturally derived, everyday ways of speaking about them, the facts of physics to which they refer are identical. In the process of explicating contesting predictions and theories, the students are guided by the teacher, using the norms and forms of Accountable Talk, toward the Discourse of physics – which is not anybody's primary discourse. In the process of *doing* science, the students take on a new identity (scientist) – which builds upon (and transforms) their current understandings and ways of speaking as the basis for new ways with words.

The role of the task/participant structure in guiding knowledge construction

Investigators Club tasks are embedded in a set of participant structures and expectations which model the way scientists actually talk, think and act. Students accomplish both identity work and cognitive work in the practice of these activities – and it is the students who are doing the science. Two primary and recurring talk formats in the I-Club are among those discussed above: 'position-driven discussion' and 'student presentation with group critique.' In both of these talk formats, the teacher's job is *not* to provide 'right answers.'

In position-driven discussions, one of the I-Club teacher's major concerns is to avoid shutting down the discussion by prematurely 'telegraphing' (indicating in any way) which theory is closest to being canonically correct. The teacher might say, 'OK, so let me see if I've got your theory right. You're saying that the volleyball will weigh less when I put more air into it because balloons are lighter when full of air?'. Having a good 'sayable' theory (conjecture, or position) is more important than having the right theory, until the final phase of the discussion, where, for example, the science demo is run and there is consensus on the outcome. (At that point, the teacher's role may shift toward a focus on correctness, getting the right solution, and actively explaining to the students how to think about the situation.)

In student presentations, the teacher is primarily a coach, whose job is *not* to talk the students *out* of their home-based knowledge and the theories implicit in that knowledge, but rather to help them to explicate, clarify and sharpen their theories and support others in the group to understand, critique and improve the presenter's explanation.

In the I-Club environment, heterogeneity of students' experience and cultural background is a valuable resource. When the group evaluates competing theories in their most persuasive forms, in the shared context of the demo at hand, cognitive growth in the form of movement toward more effective (and canonical) ways of seeing and talking is self-motivated and self-enhancing, grounded in individual and collective experience (rather than a concern for the 'right answer').

Background: the science and the narrative tool, the 'Air-Puppies' story

In the 'Air-Puppies' story, the 'puppies' referred to are mythical or fictional beings combining *some* of the properties of real, live puppies with the behavioral characteristics of air molecules. The air-puppies are the bumbling (mindless) agents in a modifiable story with a particular setting (always including two rooms separated by a moveable wall-on-wheels), participating in a series of events, always resulting in some kind of lawful effect – that is, the wall moves as it *must*, given the air-puppies' opposing impacts upon both sides.

We typically introduce the air-puppies story to students (in a 10- to 20-minute session) by telling them the basic story, followed – always – by several variations. As the story progresses, the situation and changes in it are illustrated with simple, freehand drawings (on whiteboard, paper or chalkboard). We begin by asking the kids to imagine a big room divided into two smaller rooms by a wall on frictionless wheels (like roller skates). In each of the rooms on either side of the wall-on-wheels there are *air-puppies* – initially, equal numbers and kinds of air-puppies – mindlessly bumbling around. (Figure 7.2 shows a top–down view of the situation.) The dividing wall-on-wheels moves[2] whenever a puppy bumps into it (not intentionally, just mindlessly moving around). As the puppies bumble around and mindlessly bump into things (all the walls and each other), 'What,' we ask the kids, 'will happen to the wall?' Even at this point, in this first session, one or more kids will confidently 'read' the situation to predict that 'the wall [on wheels] will stay in the same place.'

Once the scenario in Figure 7.2 is set in motion the wall-on-wheels (henceforth simply, as the kids say it, 'the wall') is pushed a little bit to one side or the other each time a puppy bumps into it. Because the wall gets, on average, the same number and kind of bumps from each side, the wall stays over time in approximately the same place, oscillating about the centerline (Figure 7.3).

A number of variations on this basic story are possible:

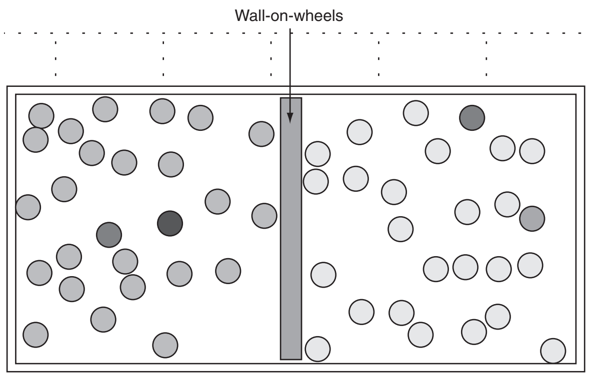


Figure 7.2 The view (from above) of the beginning of the 'Air-Puppies' story. In this version of the story there are equal numbers and kinds of air-puppies on each side of the wall-on-wheels.

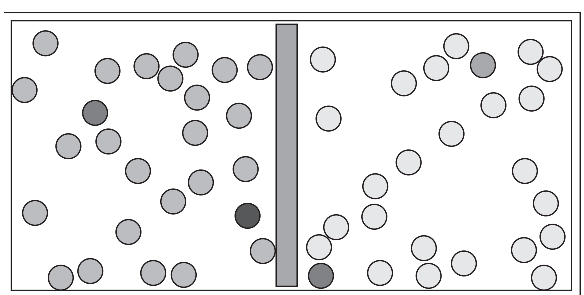


Figure 7.3 With equal (numbers and kinds of) air-puppies on each side, the wall-on-wheels is continually bumped from side to side. The net impact of the puppies on one side of 'the wall' (the wall-on-wheels) is, on average, equal to the net impact of the puppies on the other side, making the wall oscillate about the centerline.

Story variation #1

STORYTELLER: What if we start out with 20 air-puppies on *this* [*right-hand, e.g.*] side of the wall, and *more* air-puppies – say, 100 – on the other [*left-hand*] side? What do you think will happen to the wall-on-wheels?

KIDS WILL SAY (SOMETHING LIKE): The wall's gonna move towards the 20-puppies side [*i.e., the wall will move to the right*] because there's more puppy hits on the other [*100-puppy*] side.

Story variation #2

STORYTELLER: What if we start out with the same number of air-puppies on both sides of the wall, but we get the puppies on *one* side really excited – so that they bumble around *much* faster than the puppies on the other side … What do you think will happen to the wall-on-wheels?

KIDS WILL SAY (SOMETHING LIKE): The *fast* puppies are gonna bump into the wall faster and more times and harder so the wall is gonna be pushed *away*, towards the slow puppies.

Story variation #3

STORYTELLER: What if the air-puppies start out the same on both sides, but then a door opens on the right side of the wall. What will happen? [*Figure 7.4.*]

Most people see, use and accept 'suction' as a perfectly adequate explanation of ordinary actions like using a vacuum cleaner to clean a carpet or drinking a milkshake through a straw. An ordinary person who does not know much physics sees sucking (or, what sounds more scientific, a 'vacuum') at work when they see a person drinking a milkshake through a straw. A physicist, in contrast, sees pushing. The actual forces of pulling and pushing are both invisible, but practitioners of physics see atmospheric pressure *pushing* the milkshake up into the straw.

The initial invention and use of the Air-Puppies Model stems from Sohmer's observation that, in practice, novice physics learners simply do not retire or replace 'suction' as an explanatory tool after (repeated) reading or instruction in

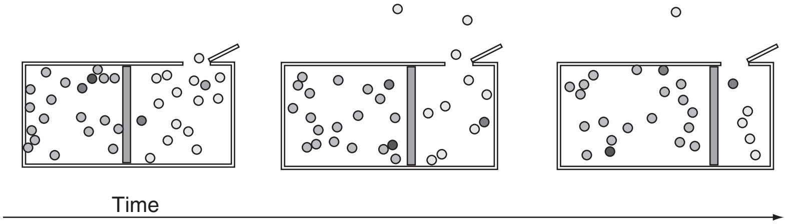


Figure 7.4 Views at times 1, 2 and 3. As air-puppies in the right room 'bumble' out the open door, there are fewer and fewer air-puppy impacts from the right upon the wall-on-wheels. Increasingly unopposed air-puppy impacts from the left push the wall away – to the right.

the details of a canonical account of air pressure. By contrast, these same novices *do* take on the Air-Puppies Model – which encodes the canonical explanation in a narrative – to successfully 're-see' the physics of air pressure.

The spud-gun

The following example shows the way the air-puppies tool is taken up by a student, with support from the task, the teacher and fellow investigators. The episode is taken from Day 6 of one I-Club (Fall, 2000). The talk norms for 'Circle Up time,' the group-discussion activity-supporting argument and peer critique, are still being developed. The main event of each day is 'Circle-Up time,' which is focused on discussing, theorizing and predicting the outcomes of intriguing air-pressure demos. Since Day 3 (when they were introduced to it), they have been exploring and practicing the use of the Air-Puppies Model.

During this particular 'Circle Up time,' the students are asked to give an explanation – using the Air-Puppies Model – of *any* of the many air-pressure demos they have seen thus far. In addition, the game requires that 'You've got to explain it *in words* such that even someone who couldn't see you – someone just listening to your explanation over the telephone – would understand it.' If the presenter's explanation is deemed acceptable by her peers, she gets to shoot the 'spud-gun' (itself an air-pressure demo) (Figure 7.5), trying to knock down as many film canisters, stacked in a pyramid, as she can in 30 seconds.

Although the game-like structure of this activity (and the students' excitement about shooting the spud-gun) disguise it, the Investigators Club teacher has several pedagogical purposes for getting the students to present, and for getting his/her peers to critique the presenter's performance.

Presenting to their peers prepares I-Club members for public presentations, like science fairs, or teaching younger students (both valued activities which students work toward in the I-Club). The process is an important part of their apprenticeship as scientists. The presenter and the audience member take on the identity (and the responsibility) of an expert *Investigator* – very different from

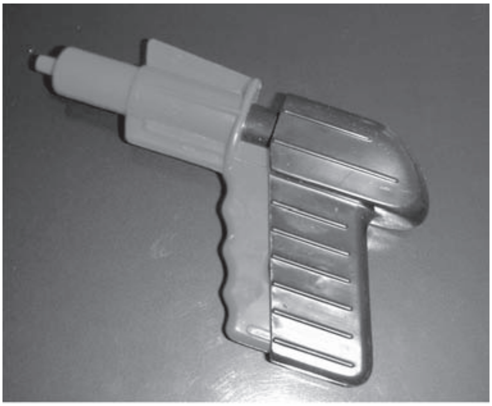


Figure 7.5 The spud-gun.

their experience in school, where many of these students are 'failing.' The Investigators Club presenter stands up *as an expert*, even if she has to try a couple of times to get it right. The presenter's peers are positioned as coaches – 'expert enough' to critique the presenter's performance. With their feedback (and the teacher's, when necessary), the presenter gets the guidance and practice she needs to improve as an explainer.

In the transcript excerpt below, Daheesha selects the spud-gun as the air-pressure demo she's going to explain. Daheesha explains the spud-gun and the teacher asks students for comments (Figure 7.6).

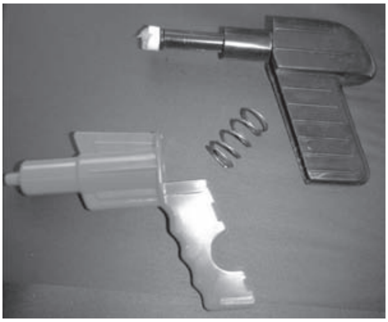
Daheesha's turn lasts 5 minutes. Announcing that she's going to explain how the spud-gun works, Daheesha gives a step-by-step and somewhat stilted procedural account (with some help from her peers in naming the parts of the spud-gun). She explains, in a sense, 'how *you* work *it*.'

The other I-Club members are not satisfied with her account. They tell her she's got to explain *why* the spud-gun works, and be more scientific by using the air-puppies. Daheesha demands a second chance, saying, 'this time I'll amaze you guys.' On her second try, she shifts from procedural to explanatory mode – saying, 'Now *how* it works…' She gives two explanations in succession. She begins her first why-explanation by saying that the spud-gun works 'because there's a little spring in it.' Laughter erupts and someone quickly points out that the teacher had pointedly taken out the spring at the outset of the activity (Figure 7.7).

Without missing a beat, Daheesha goes on to her second why-explanation saying that there's air inside the chamber and that's what makes the potato fall out.

Her peers are still not satisfied and again tell her she's got to use the air-puppies. 'Explain about the wall, how when you push [the piston] the wall comes up and squishes [the puppies] in so they run into the wall more and the potato'll go out.'

Figure 7.6 Daheesha explains how the spud-gun works.



At the outset, the teacher dis-assembled the spud-gun. He then re-assembled it – showing the students that he was leaving the spring out, while telling them that 'This spring doesn't have anything to do with propelling the potato pellet. We don't need it, so we're going to leave it out.'

Figure 7.7 The spud-gun, dis-assembled.

Daheesha presents once again, this time using the Air-Puppies Model as an explanatory tool. Interestingly, in this third account, as she moves into narrative mode with the puppies, she changes her discourse style dramatically. She shifts into African-American Vernacular English prosody in giving a fully performed narrative (Hymes 1981 calls this kind of shift a 'breakthrough into performance'), complete with dialogue and animated hand gestures.

DAHEESHA: Right now the air-puppies are havin' space // … now that/now I put it in [*looks down and pushes the red piece into the black piece*]/and they don't have no space so they – so they're like [*high pitch, hands moving wildly*] Oh let's get out of here // So they .. push outta this hole thing // and they all sss-[*flying motion*] … [*push out the potato*]

The students register the improvement in her account ('Better. Better.') and vote that her account is good enough to warrant a turn at shooting the spud-gun.

Below, we provide a turn-by-turn account of Deheesha's entire presentation, with interpolations (in italics) marking the changes in Daheesha's discourse. The question to keep in mind while reading the transcript is: 'What is doing the guiding here?' Is it the teacher's talk, the task/participant structure, or the tool?

Daheesha's first attempt

DAHEESHA: I'ma explain how this thing works //
STUDENT: What is this *thing*? //
TEACHER: A spud-gun //
DAHEESHA: Yeah // the spud-gun //
TEACHER: A potato gun .. spud-gun //
DAHEESHA: A potato gu:n // I 'on know //
TEACHER: Okay how's it work? //
DAHEESHA: Okay // [*loud presentation voice*] It works by/you/um inserting this black thing //
STUDENT: What's the black thing? //
STUDENTS: [*laugh*] What's the black thing?
TEACHER: The piston //

DAHEESHA: Yeah the piston // Insi:de // … the … the…
STUDENT: chamber
DAHEESHA: the chamber // … and as you .. do that/you: … put your fingers … where them things are /
FRANK: the grip //
DAHEESHA: the *grip* … and then you put … this whole thing/ … inside a potato/ and you scoop it up // … and you can shoot it wherever you want to //
FRANK: What's this whole thing? // What's this whole thing? //
DAHEESHA: The whole .. spud-gun // … Okay: // this little thing/that's stickin' out/ … I mean – I mean/ … I call it … the peanut/or/whatever // … Anyway/[*laughter in background*] … the thing sticks out of the spud-gun/ … and then you scoop it .. into the um … the peanut [*laughter*] there // [*louder laughter*]
STUDENT: potato //
STUDENTS: [*laughter and talking*]
DAHEESHA: Yeah [*laughs and covers mouth*]
STUDENT: potato //
DAHEESHA: And then it shoots out // and it shoots out //
TEACHER: Okay // is that your explanation? // Okay/so what do you think? //
STUDENTS: Ba:d // bad // [*lots of talking*]

Comments: Daheesha's first attempt is marked by numerous indicators of a procedural account ('How You Work It').

- *sequentiality – indicated by 'then' as a temporal sequence marker;*
- *recurring VP structure: It works by you doing X, you Y, you Z it;*
- *invocation of 'technical' terminology: the pistons, the chamber, I call it 'the peanut'.*

TEACHER: How's it work?
DAHEESHA: It works by *you* inserting the *piston* into the *chamber* a:nd as *you* do that, *you* W, and *then you* X it, and *then you* Y, and *you* Z. … And this little thing, … I call it the *peanut* [*stipulating a new technical term*] … *you* X it and *then* it shoots out.

As soon as Daheesha finishes, the students critique her performance on two grounds, (1) vagueness and (2) the lack of an explanatory mechanism. Arrows indicate these points in the transcript, with ➡ *indicating vagueness and* → *indicating lack of explanatory mechanism.*

TEACHER: What's wrong with it // as a – as a .. s-scientific explanation // as a … day-to-day explanation/if you were just tellin' somebody/ … you know/what to do/ … it'd be fine // if you were just tellin' somebody how to use the spud-gun // but we want to know what makes the spud-gun work // so: … um: … OK we've got three people here // so:/uh … let's see what Alysha has to say // What do you got to say Alysha? // … What's – what didn't you like about this as a scientific explanation? //
➡ ANDREA: She kept using *thi:ng* //

➡ STUDENT: and *that* //

➡ TEACHER: Okay/so it's a little – a little – a little – little vague to be a really good scientific explanation //

ANDREA: ==And she didn't mention anything about ==

DAHEESHA: If you want I can explain it over // [T: Okay]

→ ANDREA: == *how* it pushes the potato out //

TEACHER: Okay //

→ ANDREA: She didn't mention any/air-puppies or any molecules //

TEACHER: Okay // Do you have something to add to that? // I think that's right //

→ BILL: Yeah like um/ … she didn't tell us/*ho:w*/.. *how*/it pushes the potato out/like with the wall movin' over and the puppies goin' out //

DAHEESHA: I – I – I messed up on the first time so I'll do it over and I'll==

TEACHER: ==Okay/wait-wait-wait // let's see what these people have // You got something to add? //

➡ ANTHONY: I think she needs to use like more nouns and pronouns/instead of usin' thing and this and like //

TEACHER: Okay so you're picking up on what Andrea had to say? //

ANTHONY: Yeah // ==

DAHEESHA: ==Okay I'll do it over //

TEACHER: Wait one second/it's hard to be criticized but/it's … it's not about you really // it's just about how to do this // Go ahead //

→ DAVID: I think you – you like – you like .. explained all the things about .. what it does but you didn't explain like how it w–*works* // like – you – you – we know it pushes out all the tomatoes – I mean the .. potatoes but not – … but *how* does it – how does it push it out though? //

DAHEESHA: By there's like a little spring in here //

TEACHER: Oka:y? // What do you think? //

→ BILL: She needs to do more about like/ … scientific/..air-puppies and the wall //

DAHEESHA: This time I'll amaze you guys //

BILL: All right // do it again //

Comments: In response to Daniel's question 'But HOW does it/how does it push it [the potato] out though?', Daheesha responds: 'By there's like a little spring in here,' as she looks down at the spud-gun in her hand. The spud-gun comes with a spring inside it, which functions to move the handle (piston) back to its original position after the gun has been shot, recocking it as it were. But this has nothing to do with how it shoots potato pellets, and is not essential for the spud-gun to work. Indeed, the teacher had at the outset made a show of opening the spud-gun and removing the spring. The spud-gun that Daheesha was holding was known by most of the students to have no spring inside. Daheesha's comment is evidence that she has not understood the role of the Air-Puppies Model in explaining how the spud-gun works. But the fact that Daheesha offers the spring as an explanation suggest that she HAS understood that she needs to shift from a procedural to a causal account. This is signaled by her emphatic stress on 'Now ***how*** *it works' and is further confirmed in the course of her second attempt.*

Daheesha's second attempt

DAHEESHA: All right // … Okay // this is called a s– … a spud-gun // Okay // … now *ho:w* it works/ … is that .. there is a little spri:ng … in it /
STUDENTS: [*laughter*]
DAHEESHA: For real // I'm not jokin' //
STUDENT: There's a spring in it //
STUDENT: Stop laughin'
DAHEESHA: It's like a thing that's round //
BILL: There's *not* a spring in it // … He took it out //
STUDENT: If you open it up [Daheesha: Oooh] there's no spring //==

Comments: In suggesting that the spring is what makes the spud-gun work (again), Daheesha is indicating that although she recognizes the need for a causal explanation, she does not yet know what actually counts as one. Instead, she makes a rhetorical move common to school science, adducing a 'black-box' mechanism, that is, offering up a putatively causal mechanism without explaining how it works ('It's because of X'). To someone who doesn't know what's actually happening, the black-box explanation sounds scientific and smart. Daheesha has, no doubt, picked up this move from many years of school science. In typical school science, it's sufficient to name the black box, even if you don't know what it does. A big part of school science in the United States, in fact, is just memorizing the names of black boxes: density, volume, area, mass, weight, buoyancy, vacuum, pressure, inertia, force, acceleration, velocity. In this particular case, everyone else knows there is no spring, and more importantly, everyone knows that a genuine explanation is available in terms of the Air-Puppies Model. Thus it can be inferred that Daheesha has not yet come to grips with what the parameters of a good explanation are.

DAHEESHA: [*Noticing there's no spring*] Oooh //Yeah // Okay // and then what it does is that/when you push it/like this/ … there's a little/there's air in here/ … so like when you push it/you can feel like air comin' out of this/ [T: Uh huh] [*holds gun up to her face*] [*laughter*] and then/ … and that's what makes the um/.. when you scoop the potato/that's how–what .. makes it fall/ just like.. so when you push it/the air/goes tshooo/and it pushes [*laughter*]

*Comments: Daheesha is a quick study. Her initial attempt at a causal explanation has gone awry, immediately giving the lie to her claim 'I'll amaze you guys.' Nonetheless, she infers correctly from her peers' criticisms that while the specific mechanism she's adduced (the spring) cannot be right, the explanatory genre ('**how** it works') has not come in for criticism, and has thus (implicitly) passed muster. Rather than acknowledging failure and asking for another try, she frames what is essentially another turn as a simple continuation. This happens in the course of four words. She acknowledges her mistake ('Yeah'), shifts gears while holding the floor ('Okay'), uses a discourse marker of continuation ('and **then**') and marks the continuation of explanatory mode 'what it does is that…' She identifies air as a likely candidate for the role of causal mechanism. The notion of air as an invisible but powerful agent that is everywhere is 'in the air' (so to speak) in the context of the ongoing I-Club investigations. It's therefore something that is inside the gun (as the spring turned out not to have been). Daheesha locates the air ('there's air in here'), makes a temporal link* *to pushing in the piston ('when you push it'), demonstrates the movement of air ('the air goes tshoo'), and claims air as a causal agent ('so … it pushes'). But she does not explain how or why air pushes. This too is a kind of black-box explanation.*

Some might prefer to think of Daheesha's entire second attempt as two different attempts. We think of them as one because she deftly marks her 'air explanation' as a continuation, not a new beginning (as each of the other attempts include). Moreover, we think of this move as a demonstration of her brilliance as a Discourse acquirer. She is evidencing change and development right before our eyes. Marking her shift as a continuation of the same kind of explanation indicates (to us) that Daheesha is thinking deftly on her feet, minimizing her mistake by adjusting her explanation quickly in response to her peers, changing her candidate causal mechanism (from the spring to air), but ***not*** *her discourse mode. That is, she indeed did learn from her first attempt that a causal, and not a procedural, mode was called for. In spite of the fact that both causal agents (spring and air) are ultimately found to be inadequate, they represent a radical shift in scientific genre from a temporally based (then, and then, and then) procedural account to a timeless mechanistic account.*

Daheesha signals this shift in genre using contrastive emphasis on HO:W it works at the outset of her second attempt. And, in addition, this shift is marked by a number of discourse features which contextualize her talk as 'causal explanation' rather than 'procedural account,' explaining the internal, causal mechanism of the spud gun:

- *Contrastive stress on HOW it works…;*
- *Timeless '**When**' ('When you do X, Y happens');*
- *Present tense verbs of causality: What it **does** is…. That's what **makes**…;*
- *Conclusory **So**..*

These features are bolded in the transcript excerpts below:

> Okay // this is called a s– … a spud-gun // Okay // … now **ho:w** it works/ … Yeah // Okay // and then **what it does** is that/**when** you push it/like this/ … there's a little/there's air in here/ … **so** like **when** you push it/you can feel like air comin' out of this/ … and then/ … and **that's what makes** the um/.. **when** you scoop the potato/**that's what wh–makes** it fall/just like.. **so when** you push it/the air/goes tshooo/and it **pushes** [*laughter*]

This second attempt, then, is a different kind of scientific genre – an account of 'how IT works' rather than 'how YOU work IT.' Interestingly, in this second attempt, Daheesha's talk is halting, with more false starts, hesitation, use of fillers, repetition, and syntactic infelicities:

> …so like when you push it/you can feel like air comin' out of this/and then/ … and that's what makes the um/.. when you scoop the potato/that's how–what .. makes it fall/just like.. so when you push it/the air/goes tshooo/ and it pushes..

A number of the kids still don't like her account.

BILL: But she – she – you still ain't usin' the air-puppies and the wa:ll // how when you push it // ==

ARIEL: She said this // she said the air pushes out *this* //

BILL: ==The wall comes up and squishes 'em in/so they run into the wall more often and the potato'll go out //

YAMARIS: So there's less / … room for them to run around (and bump into the wall) /

STUDENT: Yeah //

YAMARIS: You didn't say anything about that //

DAHEESHA: Okay // …

Daheesha's third attempt

DAHEESHA: Okay // … well // the *wa:ll* // right now the air-puppies are havin' space // … now that/now I put it in/[*looks down and pushes the red piece into the black piece*] and they don't have no space so they – so they're like [*high pitch, hands moving wildly*] Oh let's get out of here // So they .. push outta this hole thing // and they all sss-[*flying motion*]

BILL?: Thing? // Peanut?

DAHEESHA: (fly) out // Yeah/it flies outta the peanut //

STUDENTS: [*lots of laughter*]

T: Okay // so/[*laughter continues*] uh … what do you think? //

STUDENTS: Better/better /

Comments: Daheesha's third attempt can be thought of as opening up the black box, using the preferred air-puppies tool, contextualized as both story and causal mechanism. Daheesha is no longer using a formal science register as she did in her first attempt ('You do this by inserting the piston into the chamber…'). In contrast, she uses elements of African-American English Vernacular prosody (viewers of the video have commented that she has shifted into 'preaching mode'). Her talk takes the form of a performed narrative (Wolfson 1978), with elements such as direct speech or performed dialogue, use of the conversational historical present tense, animated hand gestures, evidencing fluency and an air of confidence. Beyond invoking the air-puppies tool, she animates the air-puppies, bringing them to life with a high-pitched, frantic voice. The technical-sounding vocabulary or black-boxed scientific terms (used in her first two attempts) drop out of her account altogether, but other elements of causal explanation ('so,' present tense verbs) remain.

Daheesha's I-Club peers find her account to be improved ('Better. Better. Better.') But some note that it is still not perfect. (Someone says, 'Better, but no cigar'). One student points out that molecules cannot want, or decide, or plan anything. The teacher reinforces this point, saying 'They're running around so much // but they're running in a sm-smaller space // so they're gonna hit the walls more // … whether they want to or not/doesn't matter.' Daheesha, without a pause, finishes the teacher's explanation, saying, 'They're gonna *have* to.'

Daheesha's peers vote that her account is good enough to warrant a turn at shooting the spud-gun. Beaming with joy, to cheers and applause, she knocks down a record number of the stacked film canisters.

What caused Daheesha to shift to other forms of explanation?

In the course of this segment, we see evidence of guided knowledge construction in the transformations of Daheesha's discourse. Daheesha gives three different, and progressively better, types of explanations. She begins with a scientific-sounding (i.e., pseudo-scientific) procedural account. As she shifts into causal explanation, her discourse is marked by dysfluency: elements of non-intentional, causal explanation are combined with black-boxed causal agents (a standard move in school science) that do not really explain anything. In her last attempt – making use of the Air-Puppies Model – she provides a more fully mechanistic explanation, combining narrative and causal modes with elements of African-American performance style. Other students' comments serve as scaffolds and models, exemplifying a new set of contextualization cues (Gumperz 1982), which she immediately takes up, enabling Daheesha to move from a procedural account ('how *you* work *it*') to an account focusing on the internal causal mechanisms ('how it *works*').

This episode lasts all of 5 minutes and takes place on the 6th day of the I-Club. At this point, the students have only had 3 days of exposure to the air-puppies tool. This is the first time that this student has been asked to use the tool to give a scientific account of an air-pressure phenomenon – with the additional requirement that she do it as a solo, public presentation! Her performance is accomplished in a short time with relative ease (three tries, no hesitations). Remarkably, it is her peers who convince her that she is using an ineffective discourse mode initially. With their sustained assistance she moves beyond her initial view of what scientific discourse is (her pseudo-scientific account using stilted syntax and black-box science-babble) to give her explanation in terms of the canonical physics of air pressure. It is her peers who conscientiously evaluate her performance(s), warranting her third attempt a success only when she understands (and *displays* her understanding) that there cannot be intentionality involved, that things happen the way they happen because they *must* happen that way, not because molecules decide or want to do something or go somewhere. But it is the teacher, in the background, who has established the norms for the activity, and the criteria for a good explanation. And it is the teacher, who has provided the air-puppies tool to the students, and guided them (using Accountable Talk) in its use.

The case of Daheesha illustrates how the I-Club practice positions students as thinkers, theorizers and critics – and how, in the process, participants take up new discursive moves that first presume and then ensure membership, identity and competence. In our research on the Investigators Club, we are concerned with the 'architecture of intersubjectivity' (Rommetveit 1974; Sohmer 2000), – that is, how a shared world is established – through guided knowledge construction. The I-Club is structured as an apprenticeship because learning is irremediably social. Competence entails skillful use of tools, *and* the ability to signal to others, by contextualizing your actions appropriately, that you are using the right tool in the right way at the right time.

In the Investigators Club, students are invited to make use of their embodied knowledge and home-based ways of speaking, in a range of talk formats (such as partner talk, whole-group, position-driven discussions, and student presentation/critique sessions). At the same time, they are introduced to a new set of discursive norms and forms (Accountable Talk), to help them take up and work with a set of symbolic tools (the Air-Puppies Model). These tools give them purchase on a complex set of relationships that make it possible to re-see the world – in this case 'seeing' invisible forces the way physicists do, as pushing rather than sucking. This is why we talk about 'seeing' science in new ways in addition to 'talking' science in new ways. Here, successful learning can be conceptualized as a function of the Discourse itself coming to be shared, assumed, and serving as a carrier of intelligence as it increasingly speaks through the I-Club members. Who or what is doing the guiding? Clearly, it is an amalgam of the talk, the tasks and the tools.

In most classrooms, matters are more complex than our 5-minute example might suggest. Norms for respectful and equitable talk need to be *established*, not assumed, and carefully socialized (over a period of months) in order to ensure participation by all, and for all. Teachers need deep knowledge of the intellectual domain in order to recognize and then scaffold learners' initial, ill-formed forays into successful enquiries. Sequences of rigorous, generative tasks must be cultivated and treasured – as they are in the Japanese practice of 'lesson study' – so that teachers do not have to 'reinvent the wheel.' Teacher judgments about which talk format is best at any particular point are not easy to develop rules of thumb for. When is partner talk appropriate? When is it likely to waste time? When is it time for student presentations? What characterizes questions that launch effective position-driven discussions? Are they different in mathematics vs. science vs. language arts? To complicate matters further, generative tools like the Air-Puppies Model are hard to come by. (We have yet to find one that works as well in supporting students to 'see' density, for example.)

There is much to learn about how talk, tasks and tools work in concert. We see the analysis of the Investigators Club as a first step in examining the component pieces, which, although analytically separable, are inextricably linked in the practice and process of guided knowledge construction.

Notes

1. The 'Air-Puppies Model' which is described later in the chapter is a useful representation of the canonical, but unusable 'ideal gas law' (PV=nRT).
2. The wall-on-wheels can move to the left or to the right, but is constrained so that it always maintains its orientation perpendicular to the long walls of the room.

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Chapter 8

Implementing technology-facilitated collaboration and awareness in the classroom

Roles for teachers, educational researchers and technology experts

H. Ulrich Hoppe, Reuma De Groot and Rakheli Hever

This chapter is based on the experience of implementing collaboration and awareness-oriented technologies in real classroom settings. The perspective is both comparative and integrative since the authors have been originally involved in different projects with different orientations and are now working together in a new project with a common approach. Based on these experiences over several projects, we offer insights regarding the relationship between the development and implementation of new technologies and classroom practices, or, when looking at the main actors, the interplay between teachers, educational and technology-oriented researchers.

Our starting points for this discussion are three already completed projects: the NIMIS and SEED projects, conducted by the COLLIDE group of Duisburg University, and the DUNES project, headed by the Kishurim group of the Hebrew University of Jerusalem.

Following these separate stories, we describe the Argunaut system, currently being developed by a team including both the COLLIDE and Kishurim groups. The Argunaut project has introduced a new type of teacher involvement in the design process in the form of 'design workshops'. These have turned out to be a very promising instrument for collaboration between teachers, pedagogical experts and technicians. At the end of our chapter we try to relate our joint experience and insights within the present pedagogical research framework.

Background: the NIMIS, SEED and DUNES projects

NIMIS and SEED: computer-integrated classrooms as enrichment of traditional classrooms

The work of the COLLIDE research group at the University of Duisburg-Essen (www.collide.info) was from its beginning in 1995 based on the assumption that the notion of 'learning environment' should be given a much wider definition than the conventional one, which assumes that the learning environment resides on one or more computers. The group's notion of the concept included in addition the spatial and organizational surroundings, social constellations and external requirements on the learners beyond a singular learning experience. In this more integral view of learning environments, we also conceive new and different roles for teachers. In a traditional classroom-based learning environment, the teacher is a central actor. Because of the limited scope of attention, teachers often lose opportunities to select good student contributions or 'helpful mistakes' for further discussion in the classroom. Teachers' classroom behaviour could be improved by technological support for routine supervision processes as well as facilitation of individual and group work using technology, leading to improved flexibility and reactivity to, for example, supporting students with special needs and promoting student creativity.

Technology, in this sense, does not only provide specific additional learning opportunities but it has a central function in the coordination and integration of classroom activities, thus improving the richness, directness and cohesion of educational interactions. Technology also allows for archiving and retrieval functions that facilitate reuse, sharing and exchange of results between teachers and students from different learning groups.

Integrative types of technology potentially provide an added value also to learning scenarios grown from a pedagogical tradition (and not originally defined around new technologies). The classroom is such a grown scenario, which can be enriched with different types of supporting technologies without primarily redefining its basic way of functioning. In a *computer-integrated classroom* or CiC (Hoppe, Baloian & Zhao 1993), a mixture of traditional (or natural) forms of communication and media may coexist with digital media, which can offer new ways of serving existing classroom functions as well as new functions altogether. We have used the term 'digital mimicry' to characterize interactive digital media functions that mimic traditional forms such as the use of a large, pen-based interactive display instead of a chalkboard (Hoppe 2004). Digital mimicry offers a good opportunity to extend grown professional skills towards new media. The added-value functions of the digital representation, such as easy storage, retrieval and multiple reuse, can be gradually explored and adopted.

The European project NIMIS (1998–2000) adopted ubiquitous computing technologies, particularly supporting pen- and finger-based interaction, starting at the first grade (see Figure 8.1 for an illustration of a NIMIS classroom). Ubiquitous computing was combined with speech technology to support the acquisition of reading and writing skills following a method called 'reading through writing' (Lingnau, Hoppe & Mannhaupt 2003). A special design challenge for the NIMIS environment was to support users-learners who did not (yet) have full reading and writing skills. This was achieved by introducing a new visual desktop with very intuitive visual tools for archiving, sending messages and integration of peripherals (scanner, camera) to archiving and retrieval. It has several groupware functions for synchronous and asynchronous cooperation. The flow of information and ownership of data were major challenges in designing a CiC for early learners.

The NIMIS classroom was inspired by the vision of bringing the ubiquitous computing technologies that had been originally designed and developed for more general purposes (cf. Weiser 1993) to a concrete learning setting, specifically early learning (first graders). The researchers, who lacked specific expertise related to primary school pedagogy, could capitalize on the expertise of the

Figure 8.1 NIMIS classroom scenes.

cooperating teachers, who proposed the concrete pedagogical methods used, particularly the 'reading through writing' approach (Reichen 1991).

As a consequence of the technology support, a change in the teachers' distribution of classroom work could be observed. Since feedback that could originally only be provided by the teacher to each individual student was now generated by the computer environment, the teacher could now focus more on students with special needs and on activities involving the whole group (Lingnau et al. 2003).

As a follow-up to the NIMIS project, the COLLIDE group has participated in another European project, SEED (IST-2000-25214), in which new forms of user-interactive media in the classroom were explored in a participatory approach with high school teachers. The SEED approach was based on the premise that computerized media would not be used to introduce new content but to achieve a richer and more integrated form of media use in accordance and synergy with the teachers' grown teaching styles and curricular needs. In these learning scenarios, computerized tools supported a range of representations including pen-based free-hand annotation as well as different 'visual languages' for concept mapping and modelling. Modelling activities were, for example, related to stochastic experiments or dynamic systems and included simulation capabilities.

In the SEED context, teachers used the collaborative learning and modelling environment CoolModes (Pinkwart 2003) to set up different group scenarios in their classrooms. The setting up of the environment was time consuming and only the more experienced teachers were able to do it on their own. Thus, there was a general demand for setting up classroom networks with flexible grouping and archiving/recording mechanisms to ease the task of setting up and managing the group work. This gave rise to the implementation of an 'ad hoc session manager' (Kuhn, Jansen, Harrer & Hoppe 2005). In this context, the CoolModes framework is used with two different intentions: On the one hand, it is used by the students to perform their collaborative modelling task, and on the other hand, it is used by the teacher in order to specify and orchestrate the group work. At run time, the teacher has the possibility to model and control (start/stop) sessions, see how many clients are currently connected with the session and the unique name of the session.

DUNES: designing and orchestrating new pedagogical approaches for collaborative argumentation

The projects presented above used tools that facilitate existing practices by mimicking prevalent teaching and learning functions with technological tools that have new affordances. Another approach for the use of technology in the classroom is to design technological tools that do not only facilitate common learning activities, but also aim at promoting desired pedagogical outcomes and practices such as argumentation, dialogism and critical reasoning. Such an approach was adopted in a considerable number of CSCL and CSCA projects (e.g. Scardamalia & Bereiter 1996; Stahl 2006, chapter 6; Andriessen, Baker & Suthers 2003; Schwarz & Glassner 2007; Asterhan, Schwarz & Gil, submitted).

This approach was taken by the Kishurim group of the Hebrew University (De Groot 2001). Originally, the group tried to develop pedagogies to promote collective argumentation and critical group thinking (e.g. De Groot 2002). The group recognized the necessity to elaborate technologies for these purposes, and subsequently coordinated the projects DUNES (IST-2001-34153, http://dunes.gr) and ESCALATE (SAS6-020790, http://escalate.org.il).

To address the needs arising from the field, three tools were developed within the framework of the DUNES project: (1) the Digalo tool for synchronized argumentative discussions; (2) the Oasis, a web portal supporting the design of argumentative activities; and (3) the 'Shared desktop', a synchronized communication platform.

The Digalo e-discussion tool was developed in order to promote educational argumentative discussion. The rationale was that the digital representation of ongoing discussions would help students and instructors to refer to past actions, to evaluate them, and to build on these actions in the elaboration of new argumentative moves. Moreover, if visual traces of argumentative moves are made more apparent via use of suitable technology, it may lead to cognitive off-loading and make it possible for students and teachers alike to better understand the argumentative discussions taking place.

The Digalo tool enables synchronous, textual talk through mediation of geometrical shapes that represent different dialogical moves (ontology). A user has to choose a particular contribution shape from a fixed set of options (e.g. argument, claim, question, explanation), write his/her contribution to the discussion in the shape and link it to one or several contributions in the discussion map (Schwarz & Glassner 2007). Figure 8.2 presents an example of a Digalo map.

The development of the Oasis portal started with the growing conviction that argumentative activities cannot be separated from the overall sequence and structure of a learning unit. As a result of the joint work with teachers, the

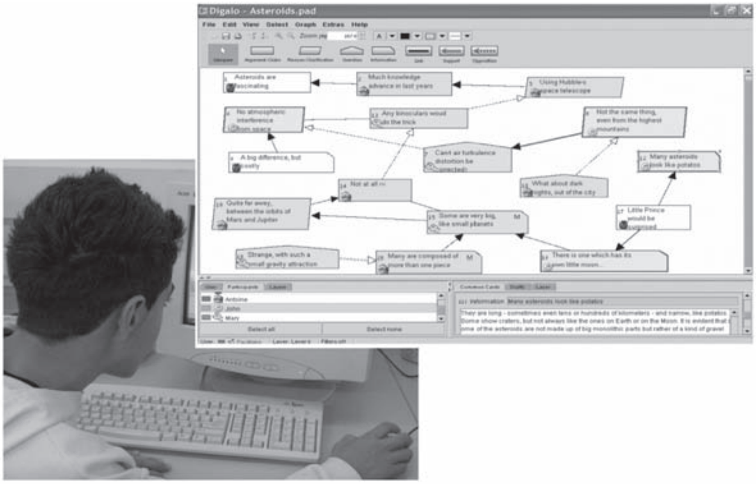

Figure 8.2 Work with a Digalo discussion map.

Kishurim group created a template for designing teaching units, or ‘cases’, which promote argumentative activities in various learning domains through planning a sequence of varied learning activities (including Digalo-based argumentative discussions). Beyond the sequencing and structuring of activities, a ‘case’ also typically includes resources such as texts, previous discussion maps and other media. The idea is that each teacher uses the general template of cases to design his or her own teaching units. To facilitate this process, the Oasis portal was created. This portal supports the design of teaching units through a ‘case’ template, the easy import, storage and linking of different types of learning resources and integration of the discussion environment within the learning scenarios. Via this portal, teachers were also given access to a repository of successful examples of cases, as designed by other teachers (Börding et al. 2003).

The Argunaut project and system

The projects described above presented two different approaches to the development and integration of technological tools in the classroom: On the one hand, ‘digital mimicry’, the development of tools that support existing practices; on the other hand, the promotion of desired pedagogical practices through the development of tools.

The Argunaut project team involves researchers and developers from both the COLLIDE group and the Kishurim group, and merges their respective approaches to the design process. The aim of the Argunaut project (IST-2005-027728, www.argunaut.org) is to provide teachers and other moderators of e-discussions (such as those held in the Digalo environment) with a computerized tool and its associated pedagogical methodology, in order to support and increase their effectiveness and thereby the quality of the monitored e-discussions (De Groot et al. 2007).

The system would facilitate moderation of multiple e-discussion environments by helping the moderator understand what is going on in the discussions and furthermore, allowing the moderator to intervene in a way that can facilitate better discussions without disrupting the peer interaction of the discussants. Figure 8.3 illustrates these goals.

The Argunaut system incorporates: (1) two e-discussion environments (Digalo and FreeStyler, a further development based on CoolModes); (2) user- and session-management capabilities; and (3) the moderator's interface (MI).

The MI supports the moderators' awareness of important occurrences in several ongoing e-discussions concurrently taking place, a situation that is prevalent in small-group discussion in one classroom. This is done through constantly updating visualizations summarizing important characteristics of the ongoing discussion. The design of the indicators and visualizations incorporates feedback from experts as well as the projected end users (primarily teachers), covering a variety of dimensions. The MI also offers a mechanism for defining alerting rules, so that the moderator can choose to be alerted when certain conditions are met, for example, long periods of inactivity from a user, and the appearance of moderator-defined keywords in the discussion text.

In addition to more 'shallow' or superficial indicators, such as participation, activity categories and social network analysis, 'deeper' indicators (e.g. claim and reason, reasoned opposition, question–answer pairs) are available to the moderator. These indicators are the fruit of joint work from pedagogical researchers and machine-learning experts ('deep loop'). They are incorporated into the system using an AI web-classifier module. The results of these classifications are presented on the discussion-map display, at the request of the moderator (to avoid

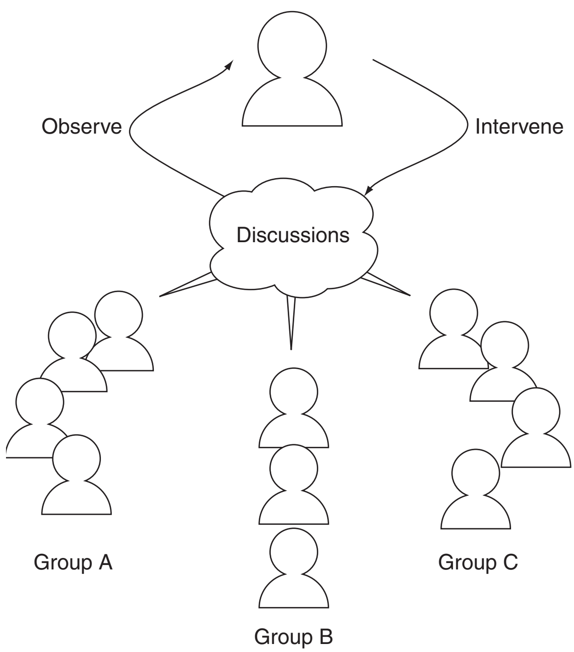

Figure 8.3 Argunaut moderation schema.

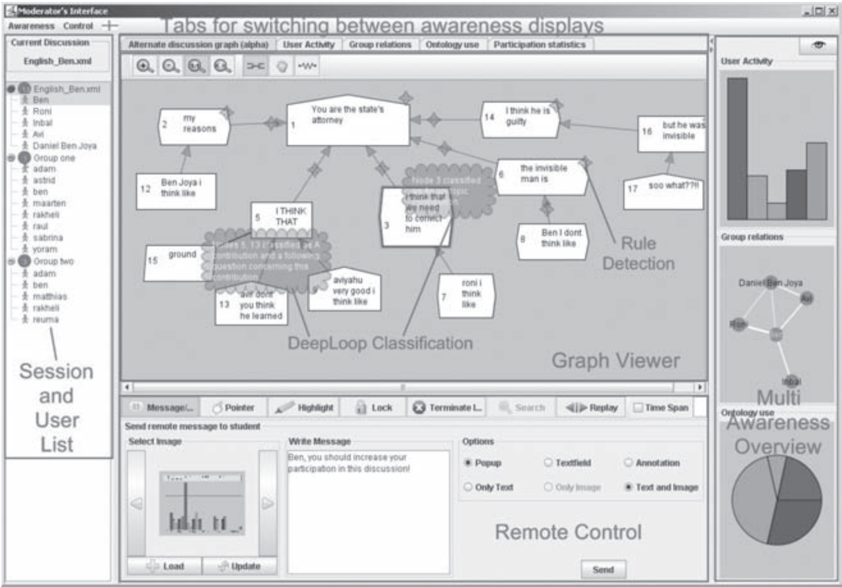

Figure 8.4 Argunaut's Moderator's Interface.

over-loading the moderator during a synchronous discussion by presenting all the available information). In Figure 8.4 a sample view of the MI can be seen, including some 'shallow' awareness visualizations and a display of some rule-detection results.

The MI provides the moderator with intervention tools (Figure 8.4, bottom part) to address issues of which the moderator may become aware with the help of the awareness tools described above. The interface for this is called the 'remote control', because it has the potential to remotely control the discussants' e-discussion environments on their own computers (typically in a school setting, these would be the students). Thus, the moderator can send pop-up messages containing text and/or images, highlight discussion objects on the discussion graph/map, attach annotations to discussion objects, remotely point, and more. This can be done for a single discussant, a selected group of discussants, or all discussants.

The Argunaut approach to involving teachers in the design process: evolution from the previous projects

The development of technological tools and environments to support learning – such as those presented above – does not ensure the success of their integration and implementation in a classroom context. In order for these tools to be integrated, they must be appropriated by the students, and, perhaps more importantly, by the teachers, who are perceived as the agents of change in their classrooms. It is our belief that these processes require strong motivation and active participation on the part of the teachers, often including adaptations and changes of their teaching styles and pedagogical beliefs. Changing teachers' pedagogical practices and beliefs is not a trivial matter – it takes time, effort and willingness. In our experience, this can be facilitated with the help of pedagogical researchers who can support the teachers' work with the new technological tools in their classrooms and provide opportunities for joint reflection and feedback on classroom practices.

The Kishurim group has gained some experience with this type of teacher–researcher interaction when introducing the Digalo tool and DUNES oasis to several Israeli schools (Schwarz & De Groot, submitted; De Groot & Schwarz 2006), with the purpose of promoting practices of argumentation and critical reasoning. The design process of the tools was accompanied by a teachers' training course, which focused on relevant pedagogies. Throughout this course, relevant concepts were introduced, discussed and negotiated with the participating teachers. This process led to the emergence of new practices by the teachers, on the one hand, and an adjustment of theoretical concepts by the researchers, on the other.

Another crucial element when it comes to introducing new technologies to teachers is their own involvement in the technological development process. It is our belief that if the tools are developed *in conjunction* with the teachers, their suitability to the teachers' needs and the teachers' motivation to use them in their classrooms would increase.

The SEED project, also described above, did not aim at curricular reform but operated on the basis of the given curriculum with a focus on maintaining and possibly enhancing teaching styles and practices. That is, the active appropriation of these new media in the everyday classroom. For this purpose, a specific type of participatory approach was adopted ('complementary action design'; cf. Lingnau, Harrer, Kuhn & Hoppe 2007), which included actively involving teachers in designing and adapting the tools that they would use. The first step in this process was to select a group of teachers who showed initial willingness to work with new technologies and to familiarize them with the general collaborative modelling platform, CoolModes. These first demonstrations and exercises showed them how the tools could be used in the classroom. They were then encouraged to try and use these tools in their classroom, with full technical support from the COLLIDE group. Based on these first experiences, some teachers articulated new ideas for using these and similar tools in specific course sequences. This led to the conception and development of new domain-specific CoolModes plug-ins in a variety of areas including probability experiments and statistics, genetics (in biology), and for simulating SMS discussions in German literature lessons. In this process, the teachers acted as co-designers: they came up with the ideas for plug-ins and then tested them and evaluated their usability.

In the framework of the Argunaut project, teachers, pedagogical researchers and technological experts are working together in a sort of 'cross-boundary lab' (Engeström, Engeström & Suntio 2002) that facilitates ongoing discussions and the creation of a mutual vocabulary. Such ongoing discussions, often negotiations on best practices, enhance the teachers' work as well as the researchers' understanding of pedagogical difficulties (Schwarz & Glassner 2007; Schwarz & De Groot, submitted).

Within this framework, three different roles in the development process are made explicitly distinct: (1) *pedagogical experts* act in the centre of the project, envisioning new educational scenarios and mediating between technology and educational practice; (2) *technological experts* suggest and implement solutions for the pedagogical scenarios by adapting and partially further developing up-to-date technologies with a basis in distributed systems and artificial intelligence; (3) *experienced teachers* test and adopt these solutions by enacting the novel scenarios and give early feedback on design decision and usability issues. The basic relationships and roles in this triangle are depicted in Figure 8.5.

Figure 8.5 also shows the central role of the pedagogical experts as initiators and mediators. 'Design workshops' that involved the three groups enabled the sharing of early externalizations of design ideas in the form of non-operational mock-ups or of prototypes. Pedagogical experts acted both as mediators and as data collectors to evaluate these design decisions. In the preparatory phase of a design workshop, possible and/or reasonable system features are defined and characterized and their proper presentation to the teachers is elaborated. Once a prototype with some of these initial features is available, the process can commence. Pedagogical experts organize hands-on activities on a set of system functionalities for first trials and discussions with the teachers. They discuss with the teachers how these functionalities meet actual needs and how they fit in with classroom practice. As a follow-up, teachers are asked to use the tool with their students in the classroom. Teachers' experience and insights concerning these functionalities are then articulated and transferred to the technical experts to be incorporated in the further development of the tools.

In general, the role of the pedagogical experts is to initiate and maintain an iterative process of presenting educational concepts and their possible realization with technological tools to the teachers, relying on a clear understanding of the technologies on a functional level. Thus, they act as a 'bridge' between the world of teaching and the world of software design.

Let us take, for example, an excerpt from a focus-group discussion held in one of the Argunaut design workshops, after the teachers' first experience with using an early pre-release version of the Argunaut system for moderating a discussion. The teachers and their trainer (a teacher working in conjunction with the Argunaut development team) sat together with the pedagogical researchers and openly

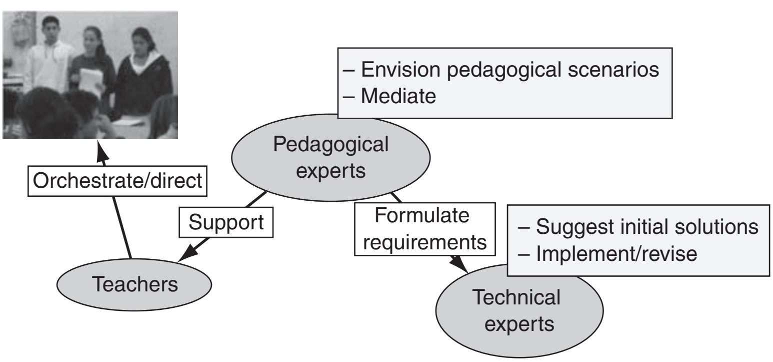

Figure 8.5 Roles in the Argunaut tool development process.

shared their impressions regarding the tool. They were asked to elaborate on both good and bad points.

TEACHER B: With me, it is the opposite [of what 'Teacher A' said before]. I enjoyed the graphics very much and the immediacy of seeing what was going on. It's another form of … It immediately gave me an overview of who is present, who is talking … less the content, but more about who is participating and who is not. I see this more as a tool that I can use after the lesson is over. I see this more as a thing for an analyst that looks after each lesson how each of people responded, what they said, who did what, etc., and this person would be in ideal position to give answers to each student about their performance in class, even via email, on the very same day. I think it would have a very strong effect on them [the students] to see that they immediately get a response [from the teacher], some attention, because they always say, 'you don't respond to me, you don't see me in the classroom'.

PED. EX. A: You mean that … But as it [the tool] is at the moment, it doesn't give you the picture per student.

TEACHER B: It can give it, it can. You can take … there are … You have the graphs.

PED. EX. A: There are the graphs.

TEACHER B: Yes.

PED. EX. A: Without the content.

TEACHER A: But you already have the content in the map itself [the Discussion Graph tab in the Moderator's Interface].

TEACHER B: Yes.

TRAINER: But that's a lot of work.

PED. EX. A: This reminds me of something I heard before from other teachers last year [who worked with Digalo before Argunaut]. They said, 'If I only had [the option], in one click, to get an output of all the things a [particular] student said…' […]

PED. EX. B: Ah, yes, another teacher also said this in the previous [Argunaut] design workshop.

TEACHER B: Yes, yes, I looked for something like this as well. It would add a lot.

PED. EX. A: In order to give feedback after [the discussion].

TEACHER B: Right, right, and to see how they …

PED. EX. B: And do you join this, [Teacher A's name], do you also think such a thing is necessary?

TEACHER A: In principle, I don't have a problem just to have the [Digalo discussion] map in front of my eyes and go over it, student by student. In general, I don't really see this as a problem.

TRAINER: To be able to get an output, [Teacher A's name], it's a lot. You can really see the development of the child [over the course of the discussion].

TEACHER A: But you can't see who he responded to.

TRAINER: It doesn't matter; you can see how he writes. You can see how he writes.

TEACHER A: It depends on the purpose of your analysis.

In the above excerpt (transcribed from a video recording and translated from the original Hebrew), we see how insights from one of the teachers ('Teacher B') are explicitly linked by pedagogical experts to technological functions such as the option to export awareness features post-discussion and send them as reports to the students, and the need to present the teacher with reports per student, rather than per group only. These insights are also related to practices of incorporating both the Digalo discussion tool and moderation via Argunaut in the context of a real lesson. The questions of the pedagogical experts, as well as those of the trainer, supported the teachers' reflections on their practices and led them to find links between their current experiences with a specific technological tool and general pedagogical needs and practices in their classroom. Following this discussion, several decisions were made regarding the design, including the addition of an export function for many of the awareness tabs in the moderator's interface and the addition of three specific awareness display tabs, which allow the teacher to see the discussion content arranged in different ways and in different context (e.g. sorted in columns per student along a vertical axis).

The example above shows how pedagogical experts can come up with formalized and concrete answers to design questions, while at the same time retaining an anchor in the practice environment run by the teachers. Teachers can contribute to this process also by pointing out good and bad examples of support functions and tools usage. Indeed, the Argunaut design workshops allowed for a pedagogy that takes into account both teachers' needs and possible impacts on students' learning as well as the affordances of the tools and artefacts. This new pedagogy focuses on cooperatively exploring the potential of the new technologies rather than prescribing the implementation beforehand.

An example of this can be seen in the teachers' use of the intervention options offered by the Argunaut system. A moderator can send a message to a discussant or a group of discussants in several ways, for example, as a pop-up message or as an annotation note attached to a discussion contribution. The researchers did not prescribe how the teachers should use these functions, yet an emerging pattern of use was discovered. Moderators tended to send pop-up messages to entire groups of discussants, focusing on general comments such as urging them to participate, or to link their contributions to others. Annotation notes, on the other hand, were more often sent to single students, and usually carried more specific messages (related to the content of the contribution they were attached to). The teachers often used the annotation notes in conjunction with another intervention: highlighting the contribution in order to draw the discussants' attention to it.

Summary: on the relationship of technological and pedagogical innovation

The projects (re)visited in this article combine technological and pedagogical innovation from their very inception, i.e. already in their initial definition of objectives. They all aim at enriching and enhancing classroom practices and they all see the role of teachers as central in this respect.

One starting point for improvement in teachers' capacity to incorporate technology in their lessons is to overcome certain deficits of information management and information flow (regarding their pupils' performances) in the classroom. The NIMIS project has demonstrated how networked interactive media can facilitate these processes. By partially automating feedback in the specific 'reading through writing' process, teachers were relieved from routine tasks and could focus their attention more on special learner needs. This is very similar to Argunaut's rationale of supporting the moderation of e-discussions. Also here we expect a shift of the teachers' attention and activities towards specific learner needs and creative opportunities.

It is also common among the projects that teachers have assumed an active role in the design and development process. In NIMIS, primary school teachers proposed and propagated the specific pedagogical methods used in the project; whereas in SEED, secondary school teachers adopted a given platform to provide new interactive and collaborative representations in specific domains. In the Digalo and Argunaut context, a set of elaborate awareness tools and corresponding communicative practices were developed to facilitate the teachers' pedagogical practices in the context of e-discussions. This facilitation was carried out in a process of intensive co-design of learning activities and tools involving teachers' and researchers' co-work in the Kishurim group at the Hebrew University, Jerusalem.

Our experience shows that teachers can be more than just the 'judges' who would accept or not the innovation put forward by educational researchers and technical experts. However, we need to be realistic about the scope of this claim: We should not expect teachers to *invent* the new technologies. Our experience shows that they rather tend to be conservative with respect to technological innovation as such (or 'for its own sake'). Their cooperation and appropriation of the new concepts will depend on convincing prospects of a practical added value also for the possible use of the tools in classroom. Once convinced, teachers will be active and critical adopters and co-designers in the development process. The ensuing interaction of pedagogical and technological ingredients will transform practice and yield new outcomes beyond the original blueprints.

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Part 3

The role of argumentation and dialogue in transformation of knowledge

Chapter 9

Intersubjective and intrasubjective rationalities in pedagogical debates

Realizing what one thinks

Michael Baker

Introduction and theoretical background

Contemporary research on collaborative learning (CL) lacks the deep integration between theories of learning and theories of communicative interaction that is required for understanding the contexts and processes by which knowledge is interactively elaborated.

Until quite recently, CL research has been dominated by the attempt to extend cognitivist theories of human learning, centred on the individual, to the study of learning in groups (Dillenbourg, Baker, Blaye & O'Malley 1996). However, many of the learning processes that these theories propose do not readily correspond to genuine interactive phenomena. For example, results concerning the self-explanation effect (Chi, Bassok, Lewis, Reimann & Glaser 1989) were obtained with respect to individual problem solvers (with experimenter prompting). Although it has been conjectured that this phenomenon can also occur in CL situations (e.g. Ploetzner, Dillenbourg, Preier & Traum 1999), actually finding such explanations (*qua* individuals' expressions of their problem-solving processes) in communicative interactions is problematic. Explanations can rarely be analysed as discrete segments of interactions: they are usually processes underlying extended sequences. Explanation is an interactive contextual reconstruction, rather than an expression of problem-solving processes that occurred in an individual's mind (Baker 1999). A second example of lack of correspondence between postulated learning processes and interactive processes can be seen in the case of the focus of the socio-cognitive conflict paradigm (Doise & Mugny 1981) on the incidence of verbal conflicts, rather than on their interactive contexts and (possibly) associated argumentative processes (cf. Mevarech & Light 1992).

Of course, not all theories of CL are cognitivist. However, theories such as Cultural-Historical Activity Theory (e.g. Leont'ev 1981; Engeström 1987) and situated learning (Lave & Wenger 1991) suffer from a different problem in this context: the link between theory, model and (interactive) data remains to be established. Although interactive processes such as 'dialogue', 'appropriation in social interaction' and 'negotiation with the situation' are referred to by these theories, precise models (explicitly derived from these theories) of how such phenomena can be identified and analysed in interaction corpora are in their early stages of elaboration (but cf. Wells 2002).

Finally, the major theories of communicative interaction (dialogue, conversation, talk) were of course not specifically elaborated in order to understand learning in interaction. The now classical distinction introduced by Levinson (1983), between 'DA' (Discourse Analysis) and 'CA' (Conversation Analysis) is probably still useful here. DA approaches, commonly based on varieties of speech-act theory (Austin 1962), see dialogue as the expression, exchange and recognition of individuals' mental states; these theories are not primarily concerned with changes in the structure of the (propositional) objects of those states, i.e. with learning. CA approaches see interpersonal interaction as the *locus* of the enactment and re-creation of social practices. Neither is it obvious in this case to create links between the main aspects referred to (such as 'face-saving', 'turn-taking') and learning processes (but cf. the notion of learning as 'interactive meaning making', proposed by Koschmann 2002, from a CA approach).

In summary, there is a need in CL research for both integration of theories of learning and of interaction, and a need for making clear links between any such integrated theory and empirically validated models of interactive processes. Analysing interactions in detail can reveal learning processes not predicted by learning or dialogue theory (as will be described below). The analysis of such processes could in turn have implications for the elaboration of an integrated theory of CL. In other terms, case studies of interactive learning can provide the new and precise phenomena for which theories must account. This is what I attempt to illustrate in what follows, for the case of a specific type of argumentative interaction: (computer-mediated) pedagogical debates. The problem of understanding the relation between cognitive changes in individuals in relation to interactive processes is here couched in terms of the relationship between two 'rationalities': intrasubjective (concerning individuals, before and after interaction) and intersubjective (concerning dynamic relations between individuals during interaction).

Dialectics, rationalities and collaborative problem solving

There is a growing literature on the role of argumentative interactions in CL (for syntheses, see Andriessen & Coirier 1999; Andriessen, Baker & Suthers 2003). This singling out of one type of interaction for special attention is understandable given the fact that its very raison d'être is to attempt to establish what should be accepted and believed, by exploring logical and conceptual foundations of views (see Asterhan & Schwarz this volume), reflecting on and explaining them.

One specific approach to argumentative interaction analysis, termed 'pragma-dialectics' (Barth & Krabbe 1982; van Eemeren & Grootendorst 1984), aims to bring out the pragmatic and logical dimensions of argumentative moves in dialogue games (games that have rules, legal and obligatory moves, and clear means for determining outcomes). Intersubjective rationality, in this case, bears on what must be and has been publicly accepted or conceded in the argumentative dialogue game, and as a function of it. Intrasubjective rationality concerns the changes that occur in individuals' views (arguments, values, opinions) outside the dialogue itself, as a function of its intersubjective rationality. Trognon (1993) has pointed out that learning in-and-from dialogue (cf. also Trognon & Batt

2003)[1] must be distinguished from the kinds of cooperative learning that can occur well beyond the dialogue itself. To that extent, intersubjective rationality can be seen as relating to learning in dialogue, and intrasubjective rationality seen as individual learning, although it operates within dialogue it can best be analysed outside it, given that by definition, dialogue involves mutual influence and co-construction of meaning. By rationality, I just mean a coherent relationship between values, beliefs, reasons, opinions and goals, aiming for what is achievable, having reasons for opinions, attempts to avoid contradiction, not having opinions if one only has reasons against them, and so on.

As will be described, the specific dialectical analysis presented here (based on the approach presented in Baker 1999, 2002, 2003) reveals on one hand, an inexorable intersubjective rationality in students' dialogues, and on the other hand, a quite different and surprising intrasubjective rationality.

A straightforward relation between the two types of rationality referred to previously would be where refuted proposals are no longer believed, successfully defended proposals are believed, and inconclusive argumentative outcomes lead to no change in belief: but then would anyone imagine that human beings are quite so straightforward…? Supposing that such a completely general theory of these kinds of changes in belief could be elaborated (e.g. Harman 1986), it would have to be strongly tied in to what is at stake in the situation, to individuals' characteristics and to the nature of the referent being discussed (Golder 1996). For example, claims concerning putative facts that would not make an important difference to people's lives are not discussed in the same way as claims about what should or should not be done in cases that touch upon high economic, ethical and personal stakes.

When adolescent students are trying together to solve exploratory school science problems that go beyond their present degree of understanding, in such situations personally important ethical issues are rarely at stake, and – as a principle of pedagogical design – the students' knowledge is assumed to be in the process of elaboration. This means that such dynamically evolving knowledge can and will not usually give rise to firm conviction of the kind that underlies adversarial argument; rather, it will produce a cooperative exploration of a dialogical space (Nonnon 1996), in which 'friable' (my term) beliefs may be expressed and quite quickly dropped, where students may argue against an idea they only very recently proposed themselves (Baker 2002).

Across several situations of cooperative solving of exploratory scientific problems by secondary school students, I showed (Baker 1996, 2002, 2003) that the relation between inter- and intrasubjective rationalities is most often characterized by *weakening of conviction*: if students were in favour of, or undecided with respect to, an intermediary problem solution, the argumentative dialogue led them to become undecided or else to reject that solution, respectively. This was in fact expressed by one of the students in the corpus analysed in de Vries, Lund & Baker (2002): 'since we debated it, that means that it can't be right'. Once doubts are raised in a situation where 'no-one really knows', confidence is easily eroded. Furthermore, such erosion of confidence is of course associated with counter-argumentation: a single counter-argument is often sufficient for students to put aside a possible solution, whereas many positive arguments may be required for a doubtful proposal to be collectively accepted (Miller 1987). Notwithstanding, in all these cases, the students' changes of attitudes were expressed during the dialogue itself (using a specially designed Computer-Mediated Communication interface); so it seems quite possible that changes in personal opinions (intrasubjective rationality), outside the dialogue itself, could be quite different from those occurring under the stringent constraints of coherence imposed by intersubjective rationality.

In this chapter I consider students' debates about a somewhat different kind of school problem, one that does in fact impinge upon students' everyday opinions about what should or should not be done on a societal level. Their debates concerned the question of whether or not the production and use of Genetically Modified Organisms (GMOs) should be authorized (in France). This debate, on a societal and personal level, touches on fundamental issues such as health, solving problems of hunger in the Third World and what is 'natural' in terms of human beings and the environment. As well as scientific, economic and environmental viewpoints on this question, students also have deep-seated personal views at stake here: how, therefore, will their intrasubjective rationalities function in this case, with respect to the intersubjective rationality of the debate?

The analysis of a specific (CMC) debate on GMOs presented below shows how one student, who initially expressed a neither-for-nor-against opinion about the question being debated, came to realize more clearly what she herself thought. In the dialogue, all her *pro* arguments were refuted, and she accepted several counter-arguments to her view. But this did not 'tip the balance' in her mind to being against, neither did she retain her initial opinion; to the contrary, the debate forced her to reflect, and realize what she in fact thought. Beyond the dialogue itself, she 'tended towards being in favour', while nevertheless recognizing the existence of counter-arguments, whose validity was nevertheless not definitively proven (the debate concerned GMOs, whose effects on the biosphere are not yet known). This is the opposite process to the weakening of conviction mentioned earlier: this student became in favour of a view given that her arguments for it were precisely refuted. That is a rather subtle and surprising change in view to be accounted for in theoretical terms.

In what follows, I present the educational situation, the changes in views before and after debate, and then attempt to explain the latter in terms of dialectical characteristics (arguments for and against theses, together with argumentative outcomes of sequences) of the debate itself. In the penultimate section, I mention some limits of a dialectical approach to argumentation analysis: in corpus analysis, one often finds things that one was not initially looking for. The limits concern phenomena relating to discourse genres, and their roles in intrasubjective rationality, seen from a dialogical perspective (Bakhtine 1977; Wertsch 1991). The specific relation described here between intrasubjective and intersubjective realities constitutes a new datum for integration of theories of learning and of interaction; the proposal for articulating dialectical theory and dialogism concerns integration of theories of the elaboration of cognition in and by dialogue.

The situation

The CHAT interaction, together with the students' pre- and post-debate texts, that will be analysed below (translated by the author from the original French) was recorded as part of work of the EU-funded SCALE project,[2] by the CNRS-Lyon team (Baker, Quignard, Lund & Séjourné 2003). The learners were secondary school students (17 years old), specializing in socio-medical studies. Using a CHAT system, they were asked to debate in (friendship) pairs the following question: 'should the production of Genetically Modified Organisms (GMOs) be authorised or not?'.

The 6-hour teaching sequence, elaborated in collaboration with the teacher, was organized in four phases:

1. *Training*, on fundamentals of argumentation (arguments, opinions and theses) and on use of the DREW[3] Computer-Supported Collaborative Learning tool (Corbel et al. 2003);
2. *Preparation* for debating: students were given information to read (around 12 pages of text) containing information and viewpoints about the GMO issue, that were carefully balanced in terms of social actors (Research Ministry, Greenpeace, grain producers, citizen organizations,) and *pro/contra* arguments across different epistemological viewpoints (scientific, economic, environmental, health, ethical); students were given a table to use for taking notes, containing cells for *pro* and *contra* arguments with respect to each social actor and epistemological viewpoint; they were asked individually, during 30 minutes, to write a short text (that we term a *pre-text*) of around two-thirds of a page expressing and arguing for their personal opinions on the question;
3. *Debate* in dyads using a CHAT tool, at a distance (opposite ends of a large computer room, with partners separated by a curtain); the debate lasted approximately 45 minutes; students were asked to synthesize the main points of agreement and disagreement during the last 5 minutes;
4. *Consolidation* of what was learned from the debate: students were asked to take their original (computer-typed) texts and update them so that they better reflected their argued opinions, 'in the light of the debate'. They had 30 minutes in which to do so.

The case study and its analysis

Comparative analysis of pre- and post-texts

We asked students to revise their initial texts after the debate because asking them to write a new text would not have been acceptable from the teacher's point of view: why ask the students to write a completely new text on a topic when they have already written one? It is of course questionable as to whether the students' texts 'truly' reflect 'what they really think' (cf. Edwards, 1993), and of course they do not, entirely (supposing that the question of what they 'really thought' is meaningful). We viewed them simply as authentic productions in a genuine pedagogical setting (with their teacher present, of course, who would

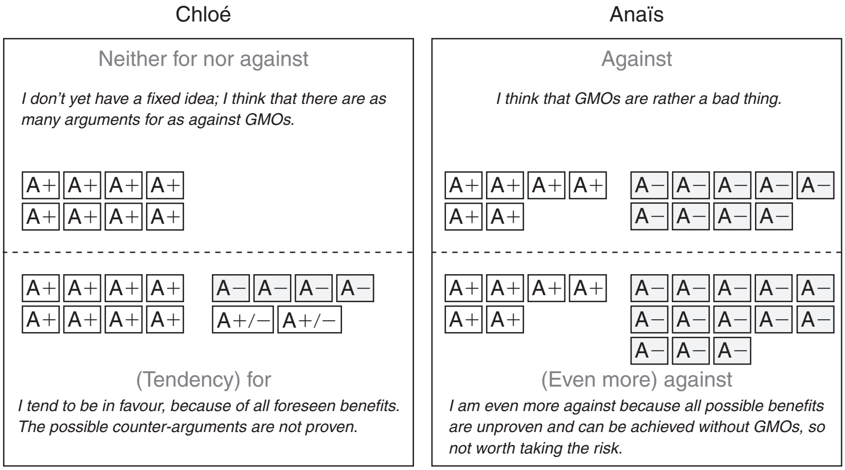

Figure 9.1 Comparative analysis of arguments and opinions expressed in students' texts written before the debate and subsequently revised in the light of it.

mark their work), and as such as providing ecologically valid yet necessary fragmentary indications of what students thought.

The two 17-year-old girl students whose work is analysed here have been renamed 'Chloé' and 'Anaïs'. Figure 9.1 presents a comparative analysis of opinions and arguments expressed in students' texts, written individually before the debate and subsequently revised (again individually) in the light of that debate.

In her text written before the debate, Chloé only wrote arguments in favour of GMOs: for example, that they could improve nutritional quality of foods, enable production of new vaccinations, solve problems of famine in the Third World, reduce use of polluting pesticides and enable more stable supply of commodities in economic terms. In her post-text, she basically added counter-arguments against GMOs, for example, that they could penalize macrobiological agriculture, lead to new allergies, that there was a risk of unpredictable damaging effects on the biosphere, and that it was not a good thing to tamper with 'Nature'.

Anaïs argued mostly against GMOs, before and after the debate, while conceding certain possible positive effects of them. For example, she wrote that while quality of some foods could be improved, they would lose their taste; Nature should not be tampered with given that positive effects had not been proven; nearly all supposedly beneficial effects could be obtained without GMOs, so why take the risk of using them?

In her final text, Anaïs added more counter-arguments, some of which were refined versions of her previous ones.

From Figure 9.1 it can be seen that:

- *Chloé* expresses a neutral opinion about GMOs in her pre-text ('I don't yet have a fixed idea, I think that there are as many arguments for as against GMOs'), while expressing only arguments in favour of them ($N = 8$). In her post-text, her opinion is a concessive 'for': 'the few arguments against must nevertheless be taken seriously, but my opinion tends towards accepting them [GMOs].' This change of expressed opinion is associated with addition of four counter-arguments to her text, and two conditional arguments ('in favour, provided that…').

- *Anaïs* was against GMOs from the start – 'I think that GMOs are a bad thing' – and remained so after the debate, while conceding the existence of *pro* arguments: 'I'm still against GMOs, even though there could be some progress for medicine.' She adds more counter-arguments to her text at the end.

Explaining changes by analysing the interaction

To what extent and in what way is the intersubjective logic of the interaction (a debate) between the students responsible for the intrasubjective changes discussed above (shown diagrammatically in Figure 9.1)?

Explaining why students acquired certain arguments appears relatively simple. As the first dialogue extract, shown in Table 9.1, illustrates, the reason why Chloé added counter-arguments to her text was simply that she conceded them all.

For example, Chloé added the counter-argument 'could lead to cloning humans' to her text at the end of the debate because she conceded it in the dialogue. In these cases of conceding counter-arguments, the link between inter- and intrasubjective rationalities seems direct: a conceded counter-argument is added to the individual's view. This does not necessarily imply that the argument is genuinely or deeply believed.

Furthermore, all of Chloé's own *pro* arguments were refuted by Anaïs, as the dialogue extract shown in Table 9.2 illustrates.

As for Anaïs, as we have seen, however briefly, she has refuted all of her partner's *pro* arguments, so there is no reason for her to accept them, and she does not.

Later on in the dialogue, the students agree on an 'argument from ignorance', or 'argument from precaution': nothing has been proven either way that the good or bad effects of GMOs will actually occur (see the third dialogue extract in Table 9.3).

Table 9.1 First dialogue extract

Line	*Time (hh:mm:ss)*	*Locutor*	*CHAT message*
46	09:44:03	Chloé	but tell me i think you're against so explain why to me will you?
47	09:44:26	Anaïs	because it's bad for the human organisms
48	09:44:55	Chloé	answer me
49	09:45:11	Anaïs	and if we start with plants in 10 years at least it will be human beings' turn
50	09:45:38	Chloé	to be modified?
51	09:46:02	Anaïs	yeah sure maybe we'll even be cloned
52	09:46:19	Chloé	yes it's true but ya know i am totally against cloning any individual
53	09:46:33	Anaïs	so am i of course

Table 9.2 Second dialogue extract

Line	*Time (hh:mm:ss)*	*Locutor*	*CHAT message*
41	09:41:04	Chloé	there'll be a better production thus less famine
42	09:41:35	Anaïs	yeah but if it's bad for the organism, then it comes down to the same thing
43	09:42:13	Chloé	it will maybe permit us to create vaccinations against mucovicidose and i think that that is maybe a good thing
44	09:42:58	Chloé	there'll be – pollution and this is essential if we don't want to die
45	09:43:00	Anaïs	yeah but they can create it without making all food and the rest genetically modified

Table 9.3 Third dialogue extract

Line	*Time (hh:mm:ss)*	*Locutor*	*CHAT message*
54	09:48:07	Chloé	why are you against GMOs? Isn't there a single positive argument in your opinion?
55	09:48:33	Anaïs	phhh maybe but nothing has been proved
56	09:48:46	Anaïs	for the vaccinations nothing has been proved
57	09:50:08	Chloé	it's obvious that these are nothing but hypotheses at the moment but imagine just one instant if it worked don't you think that it would be a great step for mankind?
57	09:50:08	Chloé	yeah but they can succeed otherwise until now how have we done

This is crucial in explaining Chloé's concessions of counter-arguments in her final text, since thanks to this global 'who knows?' argument, she was able to minimize the importance of such counter-arguments. In the case of Anaïs, *mutatis mutandis*, supposed benefits are not proven, so this gave her a further confirmation of her refutation of *pro* arguments.

In summary, Anaïs' change in opinion seems quite straightforward: she refuted the *pro* arguments, and agreed that they were not proven anyway. She was against before the debate, and so has no reason to change her own view as a function of the dialogue, other than to consider it to have been reinforced.

The most interesting change takes place in Chloé's view. Before the debate she said she had no firm opinion either way, yet only expressed *pro* arguments: it appears that she was really *pro* GMOs but did not realize or recognize it. After the debate she took counter-arguments into account, and in some sense ignored the refutation of her *pro* arguments. She was able to do this because she had a general defence of the type 'no one really knows'. The debate, however, made her stop 'sitting on the fence' and recognize that she was in fact in favour of GMOs: refutation of her view did not make her *against it*, but rather made her recognize that she was *in favour* of it.

This seems intriguing, and worth exploring. Suppose that the opinion of a person, X, concerning an important question is not clear (for example, the question debated in schools, cited in Tozzi 2000: 'supposing medical research made it possible, should men be allowed to bear and give birth to children or not?'). Suppose X states that she has no firm opinion on the matter and debates the question with Y; yet X proposes only arguments in favour (here, allowing men to give birth to children), and concedes that Y has refuted all of them. One might expect the undecided X to therefore become against the issue discussed. But suppose – as we saw here – X therefore becomes in favour? What does that say about the possible relationships between intra- and intersubjective rationalities, about the influence of others' arguments on what we think? Does argumentation make any difference to what people think about questions that are important to them? If it does not, then,... why argue? I shall not try to answer these far-reaching questions here. Simply, I would like to propose that any answers to these fundamental issues put at stake the very idea, in CL research, that discussions between students can change what they fundamentally think in relation to the specific characteristics of interactions themselves.

Limits of the analysis: from the dialectical to the dialogical

There is something missing from the dialectical analysis that has just been presented,[4] however operational it might appear to be in explaining intersubjective rationality. That 'something missing' concerns what was said in the debate but was *not* added to the texts at the end; it concerns the dialogical dimension of *discourse genres* (Bakhtine 1977; Wertsch 1991) relating to school and to adolescents' everyday speech and experience, rather than the content and logic of arguments, the outcomes of debates.

In the early part of the debate, the students appeared to be largely repeating arguments they had read in the previous text providing information on social actors' views on GMOs: in Bakhtinian terms, this is 'ventriloquating' the school discourse genre. This can be seen from the second dialogue extract (Table 9.2), where one student seems to be simply listing such arguments.

Once these had been refuted, the debate got off the ground and the students moved on to discussing the topic in terms of a more personal discourse genre (see the fourth dialogue extract in Table 9.4), mentioning body piercing and makeup (these are two adolescent girls of 17 years of age).

While discussion of body piercing and makeup might not at first sight appear to be relevant to learning about GMOs in school, this change of discourse genre is in fact important for conceptually based learning. What the girls are touching upon here, but are not discussing in any depth, is the whole pedagogical aim of the teaching sequence: arriving at a better understanding of the concept of *Nature*.

It is possible that the girls did not take this discussion into account in their final texts precisely because they thought it was not part of the school discourse genre. Yet, this was a 'missed opportunity' (Baker & Bielaczyc 1995) for deepening conceptual understanding that a teacher could possibly have built on.

Table 9.4 Fourth dialogue extract

Line	*Time (hh:mm:ss)*	*Locutor*	*CHAT message*
94	10:08:12	Chloé	look it's like body piercing in the beginning everybody was against it but then people changed their minds
95	10:09:16	Anaïs	yes that's a fashion it's not the same this is nature that's on the line and the human organism
96	10:09:48	Chloé	i am for j300% in the only case that it doesn't cause any problems but they have to be sure 600%
97	10:10:21	Anaïs	no i'm against 1000
98	10:10:32	Anaïs	%
99	10:10:51	Chloé	you put make-up on though so that's not natural it's more or less the same
100	10:10:56	Anaïs	i am for
101	10:11:11	Anaïs	no it doesn't go into the organism*
102	10:11:34	Chloé	we gotta stop so see ya big kisses bye

Cooperative learning, in these terms, can be theorized as a problem of achieving a new coherent discourse genre that integrates yet distinguishes the nature and situational relevance of two others: school and personal everyday life.

Concluding discussion

Although it is of course not possible to generalize from a single case study, the analysis presented here provides an opportunity for discussing the extent to which alternative theories of learning and interaction can account for its results. This is what I shall discuss in conclusion.

Clearly, people do not drop their beliefs just because their arguments in favour of them have been refuted. When they are not clear about what they think, refutation of their *pro* arguments can in fact make them understand that they are really *in favour* of their intersubjectively refuted standpoints.

This suggests a three-fold relationship between inter- and intrasubjective logics in argumentative interaction:

1. In the first instance, the intersubjective logic of dialogue and acceptance (Hamblin 1971; Cohen 1992) requires that individuals at least *concede the hypothetical validity* of arguments contrary to their own views.
2. In the second, it is usually possible, in any domain that is by hypothesis debatable, to find *strategies for minimizing the import* of views that are contrary to one's own.
3. In the third, the two previously mentioned processes could lead to a *zoom into awareness* of one's own view. This can be seen as a type of *knowledge restructuring from collective reflexive activity*.

Whether argumentative interaction involves refutation or defence does not always appear to be what is most important. What does seem to be important is that – almost irrespective of the dialectical characteristics of the argumentative interaction – the interaction is *intensive* and *stimulates reflexion*. Such reflexion can enable students to *realize what they think*. We should also be wary of simplistic and bipolar analyses of attitudes as either for or against. For example, we have seen more subtle and complex attitudes, such as 'tending towards acceptance while recognizing contrary views that are nevertheless not yet completely proven'. With respect to pedagogical objectives, overcoming entrenched for/against positions, gaining understanding of opponents' views that are nevertheless relativized, knowing what one really thinks, achieving greater argumentative coherence and an opinion with more *nuance*, can all be seen as certain degrees of 'progress', or collaborative learning.

But the point of argumentation-based cooperative learning is not necessarily to change students' beliefs or other attitudes, but rather to get them to broaden and deepen their views, to make them more reasoned and reasonable, to enable students to know of and understand others' views, to reflect upon them and (sometimes but not always) respect them as worthy of debate. We have seen that is not necessarily (counter-)argument that makes beliefs change: so what does or might? Probably, we need to get beyond arguments, opinions and theses in order to address this question, and consider underlying *value systems*. For instance, in the case study analysed here, we can discern a general 'ecological save-the-Earth' ideology (I mean the term 'ideology' in a non-pejorative, purely literal sense of a *logos*, or rational system of ideas and values), as well as a 'scientific progress is intrinsically a good thing' ideology. Value systems do not change because of a few exchanged arguments, but for other reasons relating to forms of life, which will not be discussed here. Even in this case, the point of dialectical educational situations is perhaps not necessarily to change values and ideologies at all, but precisely, to encourage students to understand, respect and take others' views, values and feelings into account, to accord each other *ethical consideration* (Allwood, Traum & Jokinen 2000), from a more clear and coherent personal viewpoint.

It also seems necessary to go beyond analysis of arguments in another way, looking at discourse genres and the social settings in which they are anchored. We have also seen that adolescents' everyday-life discourse genres can contain potential for deepening conceptual understanding. To that extent, argumentation-based pedagogy is not only for a *bourgeois* intellectual elite, but rather for all students from all *milieux*, whose everyday-genre communication in school can be taken as providing potential for scaffolded learning.

In introduction, I stated that CL research requires deeper theoretical integration. I have presented a datum for such theoretical development, and discussed why a cognitive-linguistic theory (pragma-dialectics and revision of cognitive attitudes) needs to be extended to integrate a dialogical theory of discourse genres. Detailed analysis of further case studies is required, within an inductive approach, in order to establish the right experimental field for theorization.

But the explanations and interpretations of interactive phenomena I have proposed here, turning on the essentially cognitive notion of *reflexion* leading to realization of what one thinks, are certainly not the only possibilities in this case. It would also be possible to explain Chloé's position of being in favour of GMOs in terms of research in social psychology on the phenomenon of *polarization of attitudes* in groups[5] (Moscovici & Zavalloni 1969): Chloé's attitude could have moved to 'for' because the attitude of her interlocutor, Anaïs, was (almost dogmatically) that of 'against'. Essentially the same phenomenon has been described in linguistic theories of communicative interaction (e.g. Vion 1992), but in terms of *reciprocity of interactive roles*: speakers who occupy certain roles in interactions (such as 'opponent', in a debate) implicitly constrain their interlocutors to adopting the 'remaining' or 'corresponding' roles (such as 'proponent'). Thus, Chloé was in some sense 'forced' into defending GMOs simply because Anaïs so adamantly opposed them. And yet, I do not think that these two latter explanations are alone sufficient: if Chloé had been genuinely against GMOs, she could have said so, she was not a priori forced to be in favour of them, unless, in the literal sense of the term, she was simply playing a (dialogical, educational) 'game' that had no genuine relation to what she thought (cf. the role of devil's advocate). Social dynamics may constrain people to adopt certain roles and attitudes, but this does not obviate the intra- and intersubjective requirements and obligations for reflexion, justification and … argumentation.

Rather than deciding to choose between these alternative theoretical approaches to explaining how attitudes and ideas are transformed in communicative interactions, another possibility is to search for integrating them into a new theoretical approach: that is what I have at least argued for in this chapter. Such a research programme represents a major and stimulating challenge for CL research; and yet it can not be carried through successfully while attempting to bypass some of the most fundamental problems in social sciences, concerning the relations between language and thought, the cognitive and the social, the individual and the collective.

Acknowledgements

This chapter is partly based on work carried out with Arnauld Séjourné (IUFM du Pays de la Loire), published in Baker and Séjourné (2007), that is more far-reaching than the summary analysis presented here. I would like to thank the students who participated in this study, and to take this opportunity to remember their teacher, Mme Anne-Marie Chevalier, colleague and friend, tragically deceased in 2002. The data analysed here has been taken from a corpus collected in collaboration with colleagues of the SCALE project (Annie Corbel, Jean-Jacques Girardot, Philippe Jaillon, Kristine Lund, Matthieu Quignard and Xavier Serpaggi) who participated in the empirical study.

Notes

1. For these authors, learning in-and-by dialogue is analysed as 'a process by which a speaker integrates in the set of his propositions an inference that he has constructed using a "thesis" of his interlocutor as an hypothesis' (Trognon & Batt 2003, p. 403, my translation).
2. SCALE: (IST-1999) (Internet-based intelligent tool to Support Collaborative Argumentation-based Learning in secondary schools) (www.euroscale.net; http://drew.emse.fr).
3. Dialogical Reasoning Educational Webtool: http://drew.emse.fr. DREW contains a large variety of CSCL tools, including argument graphs and structured CHAT; they are not our concern here.
4. For our purposes here, the fact that this is spoken French written down in a quasi-SMS or MSN style will not be discussed.
5. I am grateful to Prof. A.-N. Perret-Clermont for having pointed out this possibility to me.

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Chapter 10

Transformation of robust misconceptions through peer argumentation

Christa S. C. Asterhan and Baruch B. Schwarz

Research on argumentation and the transformation of declarative knowledge

Argumentation has become an increasingly popular topic of investigation within the psycho-educational research community, especially so in the last decade and a half. Based on distinctions made by leading theorists (Baker 2003; van Eemeren et al. 1996; Walton 2006), argumentation is defined in this chapter as a social activity in which interlocutors attempt to strengthen or weaken the acceptability of one or more ideas. This goal is achieved by engagement in reasoning.

Research on argumentation as an instigator of learning can be roughly divided in two different but intrinsically related venues: The first concerns the effect of argumentation on thinking and reasoning. A number of studies have shown that peer argumentation improves subsequent individual or collective thinking and leads to more balanced or elaborate standpoints on the particular topic of discussion (e.g. Baker, this volume; Goldberg, Schwarz & Porat 2008; Kuhn, Shaw & Felton 1997; Schwarz in press; Schwarz, Neuman, Gil & Ilya 2003; Voss & Means 1991; Wegerif, Mercer & Dawes 1999; Zohar & Nemet 2002). The second concerns the effect of argumentation on the transformation of declarative knowledge. However, although argumentation is often associated with the activity of knowledge construction (e.g. Andriessen, Baker & Suthers 2003; Duschl & Osborne 2002), empirical research on the benefits of argumentation on learning has commenced only recently.

First indications that argumentative dialogue may improve declarative knowledge were reported by Teasley (1995) and Jimenez-Aleixandre (1992). This claim was further supported by evidence from qualitative analyses and field studies (e.g. Baker 2003; Fernandez, Wegerif, Mercer & Rojas-Drummond 2002; Mason 2001; Schwarz & Linchevski 2007; Schwarz, Neuman & Biezuner 2000; de Vries, Lund & Baker 2002). Whereas descriptive data may provide insights into processes of emergent learning, they cannot provide conclusive answers to questions concerning causes for improvement. Does argumentation, for example, lead to better understanding or are individual differences in intelligence, skill, knowledge or experience responsible for both the engagement in argumentation and for better understanding, as suggested by Means and Voss (1996)? To answer such questions, experimental designs are needed.

This approach presupposes that argumentation can be isolated and manipulated as an independent variable in order to study its effect on learning. As with the majority of psychological constructs, argumentation cannot be *directly* manipulated. Therefore, one can only compare the effects of designs that are thought to foster argumentation. This is justified if there is a way to make sure that these designs actually foster more argumentative activity than comparable settings without such elicitation. Another possibility is to adopt a correlational approach and to compare conceptual change and quality of talk; that is, to correlate between kinds of dialogue characteristics and learning. Recently, we combined both approaches with a set of experimental and non-experimental investigations into the role of argumentation in knowledge transformation in the topic of evolutionary theory (Asterhan & Schwarz 2007a, 2007b, 2008a, 2008b). The type of knowledge transformation we chose to focus on concerns conceptual change.

Conceptual change: a particular form of knowledge transformation

Domain-specific theories of cognitive development propose that innate mental structures guide learning in early childhood by actively seeking and assimilating different inputs (Carey & Spelke 1994, 1996; Gelman & Brenneman 2004). Whereas some of these structures foster the accumulation of more sophisticated knowledge in a domain, early learning can also interfere with the understanding of complex scientific constructs that children are confronted with in formal instruction. Extensive research has shown that children's (and adults') naive theories concerning constructs such as evolution, force and astronomy are not only different from, but incommensurable with the scientifically accepted ones (e.g. Carey 1992; Chi 2008). The radical reorganization (Vosniadou 1999) that is required in these knowledge representations has traditionally been referred to as *conceptual change* (e.g. Carey & Spelke 1996; Chi 2005). Some of these misconceptions, also referred to as *naive theories*, *everyday concepts* or *intuitive concepts*, are difficult to uproot even with extensive formal instruction (e.g. Limón 2001). They are often very adaptive in and compatible with everyday experience and are sustained by ambiguous language use. In addition, Chi (2005, 2008) has suggested that the robustness of certain misconceptions can be attributed to the fact that students often misinterpret one kind of process, the *emergent* type, for another, namely *direct* processes. A direct process is, among others, characterized by the fact that it has a clear beginning and end, a sequence of distinct actions that are contingent and causal, and an identifiable, explicit goal. Emergent processes, on the other hand, are uniform, simultaneous and ongoing and have no clear goal (Chi 2005; Ferrari & Chi 1998). According to Chi, conceptual change in these instances requires a lateral re-categorization to an ontologically different and often lacking conceptual category, that of emergent processes.

Natural selection is an example of an emergent process (Ferrari & Chi 1998). However, most students frame evolution as a direct process: For instance, it is often regarded as a process that serves a certain purpose or goal (e.g. *becoming* better adapted). Moreover, all individual members within a population are considered to develop new characteristics as a result of and in response to changes in the living environment. Similar to other robust misconceptions, naive theories of natural selection have consistently been found to be extremely difficult to uproot, even following extensive formal instruction on the subject (e.g. Bishop & Anderson 1990; Brumby 1984; Jensen & Finley 1996).

The set of studies that we discuss in this chapter focus on whether, when and how peer argumentation may foster conceptual change on scientific topics that have been known to be notoriously difficult to teach. The topic we chose for these studies concerns natural selection. The learning tasks in these studies were designed within the socio-cognitive conflict paradigm, according to which collaborating peers are either confronted with anomalous data or contradicting views and/or are paired with peers who have different views (Limón 2001; Mugny & Doise 1978). Elsewhere we have argued that peer argumentation combines a number of social and cognitive processes that have either been identified or proven to foster concept learning within such task settings (Schwarz & Asterhan in press). However, a causal relation between peer argumentation and conceptual change had not been established yet. This was the goal of our first experimental study.

A dyadic study: consolidating peer collaboration gains through argumentation

The first experimental study tested the effects of instructions to conduct an argumentative discussion on different measures of conceptual understanding in a dyadic setting. Seventy-six undergraduates from the social sciences and humanities each watched a 20-minute excerpt of an instructional movie on evolutionary theory. In the excerpt several examples of animal evolution were described. It also contained a detailed explanation of how Darwin's theory accounts for evolutionary change in the particular case of a bird species called 'Darwin's finches'. Following, students were randomly assigned to dyads and were instructed to collaboratively explain a newly presented case of evolutionary change (i.e. the evolution of webbed feet of ducks). Half of the dyads were instructed to engage in peer argumentation on their respective explanations. After at least 30 *s* of discussion, they were also shown a short excerpt of a critical discussion of four turns between two (hypothetical) subjects which, they were told, had participated in the experiment a year earlier:

X: Then the ducks had to change their feet so that they could swim. The area was flooded with water, and because of the new environment webbed feet developed.

Y: What do you mean 'developed'? How did that happen?

X: Hmmmm. In the beginning they did not know how to swim. But slowly they learned to do it and that caused some sort of development in their feet. I mean, webs developed between their fingers. And that's how it was passed on to the next generation.

Y: Well if that were true, then Olympic swimmers should also develop webbed feet, since they also swim all day long?!

The discussion in the excerpt modelled a critical discussion on the ducks item without actually revealing or hinting at the correct solution. Control dyads were merely instructed to collaborate. Individual evolutionary understanding was assessed as the quality of the explanatory schemas they used to explain newly introduced evolutionary phenomena on three separate test occasions: prior to, immediately after and a week following the dyadic collaboration phase. In addition to this measure of conceptual understanding, we also assessed the number of discrete Darwinian principles that students explicitly mentioned in their written responses.

When controlled for pre-test performance and other variables, delayed post-test explanations of individuals in the argumentative condition reflected superior conceptual understanding compared to those of control students. Furthermore, the pattern through which this advantage was attained revealed that students in both conditions improved their conceptual understanding immediately following the intervention. However, students who were merely instructed to collaborate lost this temporary gain, whereas students in the argumentative condition retained the same level of performance at the delayed post-test. The improvement in conceptual understanding as seen in the explanatory schemas that students applied could not be attributed to an increase in the number of discrete Darwinian principles they produced in their explanations: Students in both conditions showed immediate gains on this measure which disappeared on the delayed post-test a week later. Potential intervening variables, such as whether students arrived at the Darwinian solution *during* the interaction and the length of their discussions, were not found to be dependent on condition.

Taken together, these findings seem to suggest that the differences in conceptual understanding may be the result of different levels of processing during or after the intervention phase. The conjecture that the difference between the two conditions may be attributed to superior processing in argumentation was further explored in two different ways: First of all, we analysed the dyadic dialogues in an attempt to identify the characteristics that distinguished between dialogues that were followed by conceptual change and those that were not. Second, we examined whether the patterns of change could be replicated in a follow-up experiment in a more rigorously controlled design that further isolated the engagement in *dialectical* argumentation. Manipulation checks showed that all the experimental dyads engaged in argumentation. However, some of the dialogues were characterized by *one-sided* argumentation, in which students only produced reasoned arguments that strengthened the acceptability of a certain explanation. In dialectical argumentation, on the other hand, both weakening and strengthening arguments are proposed (Asterhan & Schwarz 2007a). We will first present the second experimental study and then discuss the findings from the dialogue analyses.

Scripted argumentation directed at and prompted by a peer

The design and procedure of the second experiment were almost identical to the first one, except for the fact that in both conditions a peer confederate played the role of one of the student participants. Following the movie excerpt, participants in both conditions were asked to write down their individual answer to the 'webbed duck feet' transfer item. In the experimental condition, the student and the confederate were each assigned a different role: a 'reader' who would read aloud a sequence of structured questions given to them and the 'respondent' who would answer these questions. The role of the reader was invariantly assigned to the peer confederate. The task scenario in the experimental condition was designed to ensure that participants engaged in dialectical argumentation in a controlled design, while preserving the perceived equal-status, peer-collaborative nature of the first study. First, participants were requested to read aloud their answer to the 'webbed duck feet' question. They were then asked to discuss the strengths of that solution, to criticize it, and to discuss whether it explained the change that occurred to the ducks' feet. Following, the confederate was requested to read 'her' solution aloud, after which the participants were asked to discuss that solution according to the previous steps. In the control condition, the subject and the confederate only read aloud their solutions to each other, without discussing them further and performed a filler task to control for time-on-task. Thus, students in both conditions were prevented from conducting a natural dialogue, interacted with the same confederate whose behaviour was controlled for and the additional solution was always presented as being the confederate's, who personally read it to a student.

So as to ensure uniformity of exposure to another explanatory schema (i.e. the confederate's), while preserving a minimum difference between that and the student's explanatory schema, two different answer sheets to the duck item were prepared for the confederate. Each contained a solution according to an explanatory schema that was qualitatively different, but belonged to the same schema category. The solution that was read by the confederate as her own was thus contingent on the participant's explanatory schema (see Asterhan & Schwarz 2007a, for further details). In sum, the conditions were identical on factors such as social facilitation, actual exposure to an alternative view, the nature of this alternative solution and the personification of viewpoints. They differed only in engagement of dialectical argumentation.

The results showed that students who were instructed to engage in scripted dialectical argumentation about their own and another person's solution showed greater conceptual gains than control students. They were also more likely to have attained conceptual change (Asterhan & Schwarz 2007b). Thus, the advantage of elicitation of argumentation observed in collaborative dyadic situations was replicated in a situation of scripted argumentation directed at and prompted by a peer, a design that isolated dialectical argumentation from the interactional features of the collaboration.

Similar to the findings from the first (dyadic) experiment, students in the argumentative condition preserved the conceptual gains obtained during the intervention. However, students in the control condition did not show improvement on any of the tests. This suggests that control subjects' temporary gains in the dyadic study derived from the peer interaction per se, and not from the movie they saw. When students were exposed to the same misconception but not allowed to discuss each others' solutions, such temporary gains disappeared.

The combined results of the two experimental studies first and foremost provide important experimental support for the assertion that eliciting argumentation promotes conceptual understanding in science in a socio-cognitive conflict learning paradigm. Even a meticulously designed task meant to cause cognitive conflict did not lead to lasting cognitive gains, unless the students were specifically instructed (dyadic study) or scripted (confederate study) to engage in dialectical argumentation. Second, our findings particularly emphasize the importance of delayed assessment, especially in the case of dialogical argumentation, since its potential benefits may not become apparent at immediate test occasions. Whereas peer collaboration by itself was found to have a positive effect on conceptual understanding, these gains proved to be merely temporary and disappeared in delayed post-tests.

Dialectical argumentation requires explaining oneself and justifying one's standpoints, as well as considering and evaluating alternative solutions. We suggested that the advantage of argumentation for concept learning is achieved through superior cognitive processing. This interpretation was indirectly supported by two findings from our experimental studies: (1) mentioning the Darwinian account during the discussion was not related to learning gains; and (2) the particular pattern in which the advantage of argumentative conditions was achieved in both experiments (preserved gains versus loss of temporary gains or no gains). To further progress in the understanding of the role of dialectical argumentation in concept learning, we then analysed the dialogues from the dyadic study to identify characteristics of the dialogues responsible for concept learning.

Explanation development and dialectical argumentation as two activities with a different impact on learning

In spite of the mean effects of instruction for dyadic argumentation on conceptual understanding, not for all experimental subjects conceptual gains were identified and not all experimental dyads engaged in a dialectical argumentative discussion, as they were instructed to (Asterhan & Schwarz 2007a). This difficulty was expected since arguing about scientific issues is difficult to sustain (Baker 2003; de Vries et al. 2002). On the other hand, the results of this dyadic study indicate that proper instruction often yields 'productive' dialogues, in the sense that these dialogues were more likely to be followed by a resilient change in conceptual understanding by at least one of the dyadic partners.

Following, we subjected the conversations of the first study's experimental dyads to detailed dialogue analyses, in an attempt to identify dialogue and interaction characteristics that predict learning from interaction. Two complementary analyses of different granularity were developed (Asterhan & Schwarz in press): the micro-level scheme assessed the nature of students' dialogical moves within the interaction and focused on moves that referred to the epistemic status of an idea (i.e. different argumentative interlocutory moves) and those that developed ideas (i.e. different moves that introduced new information to the discussion, such as those that develop or expand on preceding contributions). Dialogical moves that were assessed included, among others, claims, reasoned challenges, reasoned rebuttals, reasoned supports, simple agreements, concessions, elaborations and requests for information.

The complementary macro-level coding scheme was intended to capture interpersonal and socio-cognitive features of the interaction *as a whole* that could distinguish between gaining and non-gaining dyads. The following characteristics of dyadic collaboration were assessed: (1) *interpersonal repartition* of the different solutions that were mentioned during the interaction; (2) whether the discussants reached *closure* by the end of the discussion; (3) whether they discussed the *central and crucial issue* of how the change occurred; (4) whether they *equally contributed* to the discussion; and (5) whether the argumentative structure of the interaction as a whole was *dialectical* (students proposed more than one solution from which they feel obliged to choose, or the dialogue contains a single proposed solution that is both contested as well as defended) or *one-sided* (students only discussed why a proposed solution is correct).

The dialogue features of gaining dyads (i.e. at least one student achieved conceptual change) were compared with those of non-gaining dyads (i.e. none of the students achieved conceptual change) and revealed the following (Asterhan & Schwarz in press): the dialogues of gaining dyads contained a larger number of dialogical moves that reflect dialectical reasoning (i.e. reasoned challenges, simple oppositions, concessions and reasoned rebuttals) than those of non-gaining dyads. In contrast, gaining and non-gaining dyads equally engaged in consensual processes of explanation development and validation (i.e. providing reasoned support for solutions, simple agreements, elaborations of explanations). The importance of dialectical engagement for conceptual change was further emphasized by the macro-analyses: the dialogues of all gaining dyads were characterized by interpersonal repartition of solutions within the dialogue (i.e. the different solutions proposed during the interaction were represented by different persons) and by a dialectical argumentative structure.

In light of the experimental findings discussed earlier, the finding that engagement in dialectical argumentation predicted conceptual change was expected. However, the fact that engagement in consensual processes of knowledge construction did not relate with learning gains is surprising. In fact, the literature on explanation- and elaboration-based dialogue has extensively shown its benefits on learning (e.g. Coleman 1998; King & Rosenshine 1993; Neuman & Schwarz 1998; Van Boxtel, van der Linden & Kanselaar 2000; Webb, Troper & Fall 1995). We therefore continued to explore the relation between engagement in consensual explanation development and learning gains on the individual (instead of the dyadic) level.

Even though collaborators often commonly construct and sustain a shared problem-solving space, it does not inevitably follow that the knowledge that has been constructed during the interaction is perceived in the same manner by all participants, nor that individual ideas are similar following collaboration. Likewise, it is possible that different dyadic partners benefited differently from the interaction. For example, whereas analyses on the dyadic level did not detect a relation between consensual processes of explanation development and conceptual change, self-generated engagement in processes of consensual explanation development may be beneficial to individual learning (Chi, de Leeuw, Chiu & Lavancher 1994). Observing a collaboration partner doing the same thing, on the other hand, may not lead to gains (Webb et al. 1995). To disentangle self-generated dialogue moves from the partner's, we then conducted several regression analyses in which a single person's learning gains were predicted by the frequency of each dialogue move enacted by this person him/herself, as well as those by his/her collaboration partner. The results mirrored the previous ones: Individual learning gains were solely predicted by the extent to which that particular person actively engaged in dialectical argumentation him/herself.

These intriguing findings open a new avenue of research into the potentially different roles of consensual development of explanations and of critical-dialectical argumentation in learning through peer dialogue. It is possible that processes of consensual explanation and elaboration are only beneficial for learning that involves assimilation and conceptual learning of the enriching type. However, they may not be sufficient for the radical reorganization in knowledge structures that is required for certain particularly robust misconceptions. Needless to say, these directions should be further explored in future research.

The double-edged sword of socio-cognitive conflict: dialectical argumentation or interpersonal harmony?

Taken together, the combined findings of the three previously presented studies emphasize the importance of dialectical argumentation in knowledge transformation of the conceptual change type. To some extent, this reflects the literature on cognitive conflict (Piaget 1985) and socio-cognitive conflict theory (Mugny & Doise 1978). According to these, the confrontation of different cognitions in combination with the equality in status are what make peer-collaboration settings particularly powerful, since this is assumed to induce high levels of cognitive conflict which, in turn, are thought to be crucial for conceptual change to occur.

However, the match between these landmarks of developmental theories and the studies described here is far from perfect. First of all, a distinction should be made between cognitive conflict as a description of a learning mechanism and as a paradigm of instructional design. As shown in the experimental studies, *designing* learning tasks according to a socio-cognitive paradigm by presenting students with anomalous or contradicting information did not prove to be sufficient. Even when the exposure to another view was controlled for (i.e. in the confederate study), only those students that were elicited to engage in dialectical argumentation gained from the intervention. Thus, contrary to common assumptions, this seems to suggest that it is not the interpersonal pressure of the peer settings, nor the contradicting views that the learner is exposed to, but the actual engagement in dialectical argumentation that is responsible for deep conceptual gains. This may in turn be facilitated by the social settings of peer collaboration.

Second, in alignment with socio-cognitive conflict theory (see also Baker 2003), the interaction of all the gaining dyads was characterized by interpersonal repartition of explanations, that is: each discussant proposed at least one explanation that differed from his/her partner's. However, such a characterization still needs further clarification. Do the 'confrontations of cognitions', as suggested by socio-cognitive conflict theory, imply a confrontation between persons or only between the ideas represented by them? Results from our dialogue analyses provide some first suggestions: Whereas gaining dyads posed a larger number of dialectical moves, dialogues of gaining and non-gaining dyads contained an equal number of moves that reflect consensual explanation construction and validation. Moreover, further qualitative analyses (Asterhan & Schwarz 2009) showed that when the interaction was only characterized by critical-dialectical argumentation in a competitive, non-constructive atmosphere, students did not gain from the interaction either. Vice versa, when learners only engaged in consensual co-construction, they merely developed and consolidated their misconceptions. Even the meticulous design of this task, in which the students' conceptions were confronted with the scientifically accepted concept and students were instructed to engage in critical, argumentative dialogue, failed to cause some dyads to take a critical stance (Asterhan & Schwarz in press).

This seems to indicate that dialogue that combines dialectical-critical argumentation and consensual construction of knowledge is a particularly powerful one. The question is however: How is this, or can this be, accomplished? They may easily be perceived as two extremes of the same continuum. Being critical towards the ideas that have been proposed by another person may easily be interpreted as a personal attack on that person or as threatening group harmony, even if the intentions were neutral. In fact, peer-collaboration literature has a long history of contrasting these as two opposing accounts of learning: at one end, Piagetian theory that emphasizes the elicitation of cognitive conflict by peer disagreement (e.g. Piaget 1985), and at the other, Vygotskian theory that conceptualizes learning as a process of internalizing socially constructed, consensual products (e.g. Vygotsky 1978).

In a recent study, we explored how some dyads managed to maintain the delicate balance between engaging in critical-dialectical argumentation while preserving interpersonal harmony (Asterhan & Schwarz 2009). The protocols of these dyads showed that the episodes of dialectical argumentation were characterized by a pleasant and constructive atmosphere, not by interpersonal conflict or antagonism. Some students employed sophisticated techniques, such as spontaneous role-playing, in order to critically challenge different ideas without explicitly attacking the other or his/her views. Arguably, this would have allowed them to critically explore different perspectives, but preserve a productive and constructive atmosphere of collaborative problem solving and reach a better understanding, therefore.

In the following section, we will present two excerpts from one particular dialogue between two female students (A and B) to illustrate how students can engage in co-constructive, yet dialectical argumentation on each others' explanations.

An example of dialectical, yet co-constructive argumentation in peer dialogue

Prior to the instructional intervention, both students explained evolution in Lamarckian terms, according to which individual members of a species developed new traits and passed these on to the next generation. During the first part of their interaction (i.e. on the warming-up item in which they were required to explain a phenomenon that was also explained in the movie), student A attempted to construct a Darwinian explanation. At the start of their conversation on the target item (the evolution of the webbed feet of ducks), A and B each presented their respective solutions for the phenomena. Whereas A's explanation for the evolution of webbed feet included several clear Darwinian characteristics, the explanation proposed by B was of the Lamarckian type. However, this incongruence did not surface to the conversation; rather, A and B even overtly express agreement. The introduction of the modelling dialogue by the experimenter then led these two discussants to juxtapose their two different explanations and engage in dialectical argumentation:

[The experimenter gives them the excerpt and they read it aloud]
A 8: Difficult. . . . I think that–
B 9.1: –If it is a matter of survival, then, ehhh, evolution will not occur, because swimmers, they do not need it for their survival.
B 9.2: It is something that nature feels that something has to happen, or the creature feels that something has to change for it to survive, only then will the change take place. That's amazing!
A 10: The question is, ehh, whether the change is biological or not? I mean, one survives and the rest does not, so one develops–
B 11: –You mean like a mutation?
A 12: Cause there are all kinds of animals. By chance one kind is well adapted to the new situation and that kind survives and continues itself.
B 13: Yes.
A 14: I do not understand how he understood that – suddenly he says that they developed webbed feet.
B 15: So how do you think it happened, that it happened overnight?
A 16: That one by chance had something similar to webs. He survived, and . . . the webs just developed [evolved], became more sophisticated.
B 17: And what if–
A 18: –Not something out of nothing!
B 19: And what if no one had it?
A 20.1: Then they would not have survived.
A 20.2: How could they have survived?
B 21: Maybe they just developed it somehow?

In this excerpt, the two discussants overtly discuss the incongruence between their respective views and challenge each others' ideas. What is striking, however, is the fact that the conversation is not antagonistic in nature and that we did not detect any expressions or signs of discomfort throughout the entire protocol.

Rather, the respective explanations are challenged 'from within', by exploring the validity and explanatory strength of the proposed explanation, and not by head-on confrontations. Disagreements are settled by requesting clarifications rather than by expressing opposition: For example in turn 10, A does not openly disagree with B's explanation, but instead asks her whether an account of intentional change can also account for 'biological' changes, such as the change in webbed feet. In turn 14, she formulates her doubts as a lack of understanding from her part ('I don't understand how he understood that – suddenly he says that they developed webbed feet'). Then, in turn 15, discussant B requests A to clarify her viewpoint ('So how do you think it happened, that it happened overnight?'). She attempts to follow the line of reasoning proposed by A and asks her local questions that test the limitations of A's explanatory schema (see turn 19). This 'what if…' stratagem is also adopted later on in the conversation, as shown in the next excerpt:

B 36: Do you think that if there was a person that had a bit of webs between his fingers, a human creature, then, ehhmm, and if we were to let this human being reproduce, then he would continue this, or that it would come off in the next generations?

A 37: No. Of course, when all the rest of the people would not have it and he would be the only one to exist, he is the basis for the reproduction, so obviously it, ehhh, his offspring would be like him. He is made of the same materials in fact.

B 38: But maybe these things, when they are at the start of their development, say something like, theoretically, a man with webs between his fingers, maybe they will develop only when they are needed for survival. And when they are not needed then they will go extinct with the following generations.

A 39: Development [evolution] just like that–

B 40: –by accident, by accident, mutation. Like, haven't you ever seen someone with a problem, say something a bit different–

The 'what if…' scenario presented by B in turn 36 once more tests the limitations and implications of A's explanatory schema. This pushes A to explicitly refer to an important aspect of evolutionary theory that had not been introduced in the conversation until then: heredity. Whereas B seems to have accepted some aspects of the variability principle (turn 38), she continues to try to convince A of the possibility that evolution is an intentional, teleological process ('they will develop only when they are needed for survival'). A's reaction in turn 39 refers to the fact that traits cannot simply develop 'just like that', an issue that they had resolved and agreed upon earlier on in the conversation. This, in turn, causes student B to introduce yet another important concept, that of random mutations.

Their constant challenging in a gentle manner pushed them to articulate the different principles of evolutionary theory. These short excerpts show how B challenges the Darwinian-type explanation that is initiated by A in a non-antagonistic manner, whereas A seems to be able to consolidate and articulate this new explanation exactly as a result of B's critical questions. The students juxtaposed their explanations and, in spite of their disagreements, attempted to understand their respective lines of reasoning and were open to each others' comments. The conversation is characterized by a pleasant communicational style that avoided overt personal confrontations that may have led to social inhibition and premature closure of the discussion. The interlocutors created a shared space in which they could critically reason on the arguments and articulate new arguments without denying the authors of the replaced arguments: ideas seemed to be perceived as a common property to be explored collaboratively.

Concluding remarks

The studies we reviewed here and the protocol excerpt we presented illustrate the potential of dialectical, yet co-constructive argumentation for triggering conceptual change. The occurrence of dialectical, co-constructive argumentation in the A–B dyad was certainly influenced by the instructions, task design and script they had been given. But it did not occur in all dyadic interactions. For this type of peer-to-peer dialogue to occur, it is important that students (and educators) understand that when they are invited to be dialectical and to discuss their (or others') differences, this does not necessarily imply that they have to persuade each other in a debate-type 'win–lose' competition. When the situation is perceived as competitive and its participants as opponents, learners are likely to merely engage in attempts to refute the other's explanation and to prove the superiority of their own explanation over others. Insights from qualitative protocol analyses reveal that learning gains should be expected to be less substantive in this case (Asterhan & Schwarz 2009). One may argue that in order to convey more clearly that being dialectical does not imply interpersonal competition, collaboration scripts should be more explicit and clearer on this issue. However, the two experiments we have conducted suggest that the effect of argumentative design is often difficult to predict. More generally, the studies we reviewed suggest the variability of behaviours in peer argumentation.

Finally, in addition to its potential benefits for conceptual change, encouraging dialectical, co-constructive argumentation is important in its own right. From an educational point of view, we would *like* students to engage in collaborative endeavours in which they critically consider and integrate different perspectives. For teachers, the more prevalent reason for implementing argumentative discussions in their classroom may in fact be the expectation that these activities will teach their students to become civilized, rational and empathic discussants. So, even though argumentative design may not cause *all* people to transform their misconceptions, it may turn them into better citizens.

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Chapter 11

Commentary on the chapters by Baker and Asterhan and Schwarz through the lens of commognition

Anna Sfard

The two bodies of work presented in the last two chapters are unified by the assumption that a dialogue in which interlocutors confront, explain, defend and modify their views is highly conducive to learning. Inspired by this assumption, I will now try to engage Michael, Christa and Baruch in the very activity on which they themselves focus in their studies: I will invite them to dialectic argumentation on their own ideas. More specifically, I will take a constructively critical look at this research, trying to find out what it has already achieved and what it yet needs to deliver. To make it into a truly dialectic argumentative exercise, I will discuss the epistemic status of the researchers' interpretations, expressing my agreement with some of them and challenging some others. My main claim will be that the discourse on argumentation, developed as it is, is in need of operationalization. I will follow with a proposal of such a discourse and will then ask whether the change in the way of talking brings any new insights about their data and whether it is likely to have an impact on pedagogy.

What we learn from the studies on learning-through-argumentation and what needs additional attention

The main tenet of the research exemplified by the studies by Baker and by Asterhan and Schwarz is that argumentation may generate learning. Indeed, it is reasonable to assume that argumentative interactions are occasions for changing, or at least clarifying and solidifying, individual viewpoints (Baker) and may be particularly helpful at those junctures in learning where the required change is far-reaching and difficult. In this latter case, the expected learning is often called *conceptual change* (Asterhan & Schwarz).

The studies by Baker and by Asterhan and Schwarz bring quite a number of insights on the issue of learning through argumentation. Both teams look closely at the nature of interaction and at the results obtained. While doing so, they gauge the change in students' views (Baker) and try to identify those features of argumentative dialogues that make these dialogues most conducive to substantial learning (Asterhan & Schwarz). Both studies are insightful and bring important conclusions, likely to inform the practice of teaching and learning. Baker's study shows that argumentative interactions, in which one's arguments evoke interlocutors' rebuttals, may 'broaden and deepen' this person's beliefs rather than overthrowing them. The study also shows with clarity that a discussant whose arguments are grounded in values rather than in rational considerations is unlikely to change her position, whatever the nature of interaction. Asterhan and Schwarz's study brings new understandings about the nature of successful argumentation: only those cases of argumentation which are both dialogical and non-confrontational seem to help the participants in resolving difficult problems and in usefully reorganizing their knowledge. To verify these results, the researchers formulate and test a pedagogical advice on how to improve the practice of learning-through-argumentation. Their research is thus of an immediate practical importance.

All this said, there are aspects of the two studies that could benefit from further elaboration. Thus, for example, neither of the two chapters presents an explicit operational definition of learning. True, in one of the chapters we are told that the change due to occur as a result of argumentation is in *thought*, *attitude*, or *beliefs* (Baker). Operational definitions of these latter terms, however, are missing. In the other study (Asterhan & Schwarz), the reader is informed that a *conceptual change* has taken place as a result of the argumentative dialogues, but neither the term *concept* nor what counts as a change thereof are explained. Without the explicit definitions, the recipients of the research report cannot judge its results and implications by themselves. Moreover, having an operational definition of learning is crucial for our understanding of *how* and *why* the learning occurs (or fails to occur) in interaction; this, as opposed to knowing merely *that* it occurs.

In what follows, I engage in the exercise of supplementing what is missing. Later, I return to the two studies to see whether the proposed conceptualization of learning leads to additional – either complementary or alternative – interpretations of their data.

Commognitive operationalization of the notion of learning

Below, I begin with operationalizing the notion of *thinking* and then show that the proposed definition imposes particular understandings of the other key notions, such as *learning*, *socio-cognitive conflict*, etc. For reasons which will be explained in a moment, I call the resulting perspective *commognitive*.[1]

Thinking

According to Vygotsky, all the uniquely human patterned activities, such as writing or cooking, are learned by individuals through participation in collective performances of these activities. There is no reason to assume that the origins of thinking, perhaps the most human of human activities (at least in the form in which we know it), are any different from those of all the others. At a close look, the best candidate for the collective activity that morphs into thinking through the process of individualization is *interpersonal communication*. It seems, therefore, that human thinking can be regarded (defined, in fact) as the individualized form of the activity of communicating. Indeed, it is self-communication – a person's communication with oneself. This self-communication does not have to be in any way audible or visible, nor does it have to be in words.

According to this definition, thinking stops being a self-sustained process separate from and, in a sense, primary to any act of communication, and becomes an act of communication in itself, although not necessarily interpersonal. To stress this unity and to be able to speak about thinking and communicating as one category, I combined the terms *cognitive* and *communicative* into the new adjective *commognitive*.

Discourse

Commognitive activities, to be effective, must follow certain rules, the primary source of which is in *historically established customs*. There are many types of commognition, differing one from another not only in their rules, but also in the objects they refer to and in the media they use. Individuals may be able to participate in certain types of communicational activity and unable to take part in some others. The different types of communication that bring some people together while excluding some others will be called *discourses*. Given this definition, any human society may be divided into partially overlapping *communities of discourses*.

Different discourses are made distinct by a number of interrelated features: their *keywords* and the way they are used; the *visual mediators* with which participants of discourses identify the object of their talk and coordinate their communication; and their *routines* – the well-defined repetitive patterns in interlocutors' actions, characteristic of a given discourse. Of particular relevance to the issue of argumentation is yet another discursive feature: the set of *narratives* that the given discourse community *endorses* and labels as true. Narrative is any text, spoken or written, which is framed as a description of objects, of relations between objects or activities with or by objects. Terms and criteria of endorsement may vary considerably from discourse to discourse, and more often than not, the issues of power relations between interlocutors would play a considerable role.

Discursive Learning

The adjective *discursive* narrows the present debate to changes in commognition, as opposed to many other types of learning (e.g. learning to drive or to play a musical instrument). For example, learning mathematics means modifying one's present discourse so that it acquires the properties of the discourse practiced by the mathematical community. Such change may be attained by a straightforward addition – by extending the vocabulary, by developing new routines or by producing and endorsing new narratives. There are also certain special points in the development of discourses when the change expresses itself in the transformation of some substantial features of the discourse rather than its mere extension. Whatever the nature of the discursive change, it is this discursive change that the commognitivist has in mind while speaking about learning. Within the commognitive framework, therefore, asking what the participants of a study have yet to learn becomes equivalent to inquiring about required transformations in students' ways of communicating. This conceptualization allows giving a clearer answer to the question about learning which was left somewhat vague in the two presentations: 'In the case under study, what kind of change was supposed to occur as a result of learning?' In Asterhan and Schwarz's research, the intended target was to introduce the students to the Darwinian discourse on evolution. In order to evaluate the results of learning, therefore, one would need to examine the change in the students' use of keywords, in the rules according to which they substantiate their claims and in the narratives about evolution they eventually endorse. In Baker's study, the learning was expected to bring a change in the narratives about the admissibility of GMOs, endorsed by the students.

Commognitive perspective on research on learning-via-argumentation

Since the authors of the preceding chapters do not provide explicit definitions of *thinking* and *learning*, it is difficult to tell whether and how much their use of these terms differs from the ones proposed above. There is, however, some indirect evidence for a possible divergence. It can be found in all those places where the commognitive reinterpretation of Baker's and Asterhan and Schwarz's data seems to be bringing something new, something that was not said by the researchers themselves. In what follows, I show that at least three issues can benefit from the additional commognitive analysis: (1) the issue of relation between communicating and thinking (and this last term includes the phenomena that occasion Baker's talk on *beliefs* and Asterhan and Schwarz's references to *concepts*); (2) the issue of the relation between communication and learning; and (3) the question of how the principle of learning-through-argumentation could best be implemented in the classroom. The commognitive reinterpretation, it must be stressed, complements rather than contradicts what was said in Baker's and Asterhan and Schwarz's chapters.

Conceptual difference: disappearance of the thought–discourse dichotomy

As one of the main conclusions from his studies, Baker states that the collaborative interaction with a friend 'forced [one of the participants] to realize what she in fact thought'. This statement, as well as the author's frequent references to 'knowing what one really thinks'[2] imply a strong dichotomy between two activities, that of thinking and that of communicating, with the latter being concerned with conveying to others, or even to oneself, the products of the former. From this dualistic perspective, thinking has a greater stability than communication: the process of thinking, which is implied to be hidden and often inaccessible to the thinker herself, may remain basically the same even when what is being explicitly communicated changes. This dualistic language is clearly incompatible with that of commognition.

Renouncing the thinking–communication split is not a trivial matter. This dichotomy is too deeply entrenched in both our everyday and scientific discourses to be easily removable. If commognitivists argue against this vision, it is because they find it detrimental to research. For one thing, the assertion that a participant of a study 'came to realize more clearly what she herself thought' is inherently untestable, since nobody can check, including the thinker herself, what it was that she thought, as opposed to what she had said, in the past.[3] In addition, the thinking–communicating dichotomy may lead to logical entanglements. In dualist discourse, special place is reserved for the noun *thought*, which serves as an explanatory construct. Whatever is seen on the surface is taken to be caused by this *product* of the undercurrent activity of thinking. What appears to be an explanation, however, is likely to be a tautological statement. Indeed, the 'thought-behind-words' purported to be the independent cause for the discursive actions is nothing more than reification of certain properties of these actions. No value is thus added by this kind of 'explanation'.

Once we reject the idea of the 'real thought' as the hidden factor that guides Chloé in her argumentative activity, how can we account for the fact that in a seemingly paradoxical way the girl strengthens her support for GMOs after being exposed to numerous counter-arguments? A commognitive account of the episode offered below will address this question explicitly.

Difference in interpretation of phenomena: commognitive rather than cognitive or socio-cognitive conflict

To make the case for dialogical argumentation, Asterhan and Schwarz help themselves with the idea of *cognitive* and *socio-cognitive conflicts*, two pivotal constructs of the (recently updated) theory of Conceptual Change. Whether prefixed with 'socio-' or not, the cognitive conflict is defined as one that occurs when the learner is exposed to contradicting views. According to conceptual-change theorists, this type of conflict has the power to stimulate conceptual change, one that expresses itself in a transformation of the former knowledge rather than just its simple extension. This is the most substantial type of learning, alas also the hardest to achieve. Dialectical debate would typically provide arguments for and against each one of the contradicting views, thus creating grounds for an informed choice and for the resolution of the conflict. This is how conceptual change would eventually occur.

It is only too tempting to view this latter thesis as grounded in the assumption that any two narratives sounding as mutually contradictory are also mutually exclusive, and that there is a common criterion for deciding which of them should be rejected and which one endorsed and labeled as true. Indeed, the law of non-contradiction, according to which of any two proposition that remain in direct contradiction only one can be true, is considered to be among the basic principles of rational argument. This principle does seem to be in force when, for example, the contradiction is about the chemical composition of a certain substance or about the solutions of a given equation. I now wish to claim, however, that this does not always have to be the case. What is known from literature as *cognitive* or *socio-cognitive conflict*, the commognitive researcher is likely to reconceptualize as *commognitive conflict* – as the phenomenon that occurs when seemingly conflicting narratives are coming from different discourses – from discourses that differ in their use of words, in the rules of substantiation, etc. Such two discourses are usually *incommensurable*, that is, they do not share criteria for deciding whether a given narrative should be endorsed. Unlike in the case of conflicting narratives coming from the same discourse, two narratives that originate in incommensurable discourses cannot automatically count as mutually exclusive even if they sound contradictory. Although using the same words, they may, in fact, be referring to different phenomena. To sum up, in the situation of commognitive conflict, the law of non-contradiction may no longer be in force. Operationalization of the notion of conceptual change as a transition to an incommensurable discourse has several theoretical and practical entailments. In this section, let me focus on its implications for research.

To begin with, the notion of commognitive conflict makes us aware of the possibility of apparent contradiction that cannot be resolved by a direct rational argument. While analyzing argumentative dialogues we need to keep in mind that discussants who argue for or against a given claim may not necessarily be participating in the same discourse. Even one person may be facing two conflicting narratives, which, nevertheless, appear to her equally endorsable because, unknown to herself, this person is moving between incommensurable discourses. Commognitive conflict seems to be a much more common phenomenon than suspected by even the most experienced of discourse analysts. Argumentative dialogues should thus always be scrutinized for the possibility of incommensurability of the seemingly contradicting claims.

To be sure, the task is far from easy. Only too often, commognitive conflicts are mistaken for direct disagreements, subject to the law of non-contradiction. One of the reliable indicators of commognitive conflict is the resilience of the contradiction, its refusal to go away even when the possibility of an error in reasoning has been eliminated. Commognitive conflicts just cannot be resolved by a simple argumentation. A narrative that is unacceptable in one of the incommensurable discourses may be fully endorsable within the other. Let me now go back to our two studies and show that here also, the disagreements that are expected to be resolved through logical argument may be arising from a commognitive conflict – from a confrontation between incommensurable discourses.

In Michael Baker's study, Chloé and Anaïs cannot agree on whether genetic modifications should be promoted or banned. I wish to claim now that the students' inability to arrive at an agreed position and in particular the fact that Chloé's initial position solidifies rather than weakens as a result of Anaïs's counter-arguments signal the possibility of commognitive conflict. Indeed, a closer look at the dialogue reveals that the two girls, without realizing, are participating in incommensurable discourses, differing in their endorsement routines (the routines of substantiation) and in the use of the words *nature* and *natural*, which are pivotal to Anaïs's argument. Let me elaborate, beginning with the routines of endorsement.

In defending the use of genetic modifications, that is, in trying to show the endorsability of the narrative 'The production of GMOs should be allowed', Chloé puts forward a variety of narratives that speak about beneficial effects of such use – see, for example, her claim that with GMOs 'there'll be a better production thus less famine' (41) and that it may 'permit us to create vaccinations against mucovicidose' (43). These substantiating narratives, therefore, provide new information (cannot be simply derived from what was endorsed before) and entail the claim about the desirability of GMOs as their logical consequence. No such argumentation is found in Anaïs's substantiation of the opposite narrative 'The production of GMOs should not be allowed'. Anaïs's only argument, which the girl repeats time and again, is that 'GMOs are bad for the human organism' (47; see also 42). No substantiation for this claim is provided in spite of Chloé's recurrent requests (46, 48, 54) and counter-arguments. Thus, whereas for Chloé the narratives about the desirability of GMOs' effects on humans cannot be endorsed without a substantiation grounded in previously endorsed factual narratives, for Anaïs these are primary atomic truths that stand on their own and from which all the other claims on GMOs are to be derived.

Additional difference between Anaïs's and Chloé's discourses is in their use of the terms *nature* and *natural*. These words are crucial to their debate because of Anaïs's tacit reliance on the endorsed narrative 'non-natural (or nature-altering) is bad'. For Chloé, *natural* seems to be a simple opposite of *human-made* or *human-modified*. This is what transpires from her vision of body piercing and make-up as antithetical to nature (94). It is clearly not so for Anaïs, who explicitly says that fashion-following cannot count as nature-altering (95). What can count as such, however, is never explicitly explained and is only vaguely hinted at in statements about factors that 'go into the organism' (101).

To sum up, whereas for Anaïs the answer to the question of GMOs' permissibility constitutes a *moral belief*, that is a narrative about bad and good that is endorsed without substantiation and constitutes a basis for all other endorsements, Chloé derives her position on genetic modifications from other, already endorsed narratives regarding the utility of GMOs.[4] Anaïs can thus be said to be participating in the *discourse of morality*, whereas Chloé's discourse is that of *utility*. The difference in the keyword use and the rules of endorsement tacitly guiding moral and utilitarian discourses makes the girls' attempts to convince one another inherently futile: Anaïs's moral objection to GMOs is 'atomic', and thus irrefutable (note that she rebuts Chloé's factual claims simply by expressing her disbelief in any information that seems to contradict her stance – see e.g. 57), whereas Chloé remains unimpressed by Anaïs's rebuttals because they are not the kind of arguments she would accept as a proper substantiation. In fact, it may well be because of these ineffective rebuttals that Chloé's initial position solidifies rather than weakens. Inadequate arguments would often backfire.

Asterhan and Schwarz's study can be seen as providing an opposite example, one of the successful resolutions of a commognitive conflict. The conflict, this time, is between the discourse of evolution that speaks about *phylogenic* change, and the discourse on individual development that can be called *ontogenetic*. The community of the phylogenic discourse is represented by the researchers and by the experts on evolution who created the instructional movie. The ontogenetic discourse is sampled in the brief excerpt of simulated conversation on the emergence of duck's webbed feet. The incommensurability of these two discourses finds its most striking expression in the use of the words *development* which, for the participants of the simulated conversation, refers to a change in particular individuals, whereas for evolutionists it refers to a change visible only at the level of entire species and occurring as a result of inter-individual processes. The two types of change differ also in their timescales. The claim that 'ducks developed webbed feet' is interpreted in ontogenetic discourse as referring to changes in particular ducks, happening within the confines of individual lifespan and genetically transferred to the offspring (such a claim must, of course, lead to contradiction with commonly endorsed narratives about reality; in the present case, it clashes with the fact that the swimmers do not develop webbed feet).

In the conversation between students A and B, quoted in Asterhan and Schwarz's chapter, it is interesting to see how the transition from the ontogenetic to phylogenic discourse on development happens gradually, through subtle molding of the word use. The ontogenetic discourse is represented by student B, whose initial statement 'creature feels that something has to change for it to survive' (9.2) implies not only that the change occurs in an individual, but also that it is intentionally caused by an agent – possibly the individual himself. B's latter statement about a person 'with a bit of webs between his fingers' who, while reproducing, would pass this feature to the next generation (36) is, once again, the evidence of his thinking about development as something that happens to an individual. The inherent ambiguity of the nouns *creature* or *duck*, both of which may signify either a category or an individual member of the category, makes it difficult to diagnose the presence of two different discourses. On the other hand, this blurriness may also help the interlocutors in the imperceptible transition from the ontogenetic discourse that talks about separate individuals to the phylogenic discourse that uses the term *duck* as the name of the species, that is, a single encapsulated entity with a history much longer than that of an individual bird. This change, which indeed seems to have taken place in the conversation between the students, is driven by B's interlocutor. Student A, whose discourse appeared to be phylogenic from the very beginning, does struggle to find a substantiation for the claim 'Ducks developed webbed feet', but this struggle seems to be happening already *within* the phylogenetic discourse rather than being a result of inter-discursive clashes.

The commognitive interpretation of my colleagues' data considers aspects of argumentative interactions that did not get comparable attention in Michael's and Christa and Baruch's studies. The commognitive analysis, therefore, provides an additional point of view rather than directly competing with what was said before. In the next section, I address the question of practical implications of this additional interpretation.

Difference in pedagogy: how much can be attained in collaborative learning, through argumentation that does not involve an expert discursant?

The ultimate aim of studies such as those by Baker and by Asterhan and Schwarz is to inform educational practice. Both teams of researchers focus on those properties of dialogues that seem particularly conducive to learning, and both of them bring much valuable insights into what makes dialectic argumentation effective. Commognitive approach adds an advice of its own. Let me mention just two pedagogical principles that arise from the commognitive claim that learning–teaching interactions may sometimes be occurring across incommensurable discourses.

The need for an exposure to the expert discourse

At those special developmental junctures where further learning requires transformation of the existing discourse rather than its mere extension, facing the learner with commognitive conflict does not seem to be a mere option but rather a necessity. In other words, the learner needs an interaction with an interlocutor who is already an insider to the new discourse. Indeed, since the new discursive rules are a matter of historically established customs rather than of an externally imposed necessity, the student cannot be expected to invent these rules on their own, whatever the nature of their interaction. In Asterhan and Schwarz's study, the learners were exposed to the expert discourse with the help of educational movie on evolution. Student A was clearly able to pick up some of the leading characteristics of this discourse, and student B followed through dialogue with A. Baker's study shows that the exposure to the incommensurable discourse may not, in and of itself, be sufficient for the transition to happen. On the contrary, if not handled with care, the resulting commognitive conflict may solidify into a barrier to rapprochement. It may even deepen the rift, as it did in the case of Chloé and Anaïs. To prevent this from happening and to help in turning commognitive conflict into an opportunity for learning, yet another need of the learner has to be considered.

The need for meta-discursive negotiations

According to the commognitive approach, one should not expect a true commognitive conflict to resolve itself through dialogic interaction, on the force of rational argument. The belief in such straightforward controversy dissolution would have been justified if the conflicting narratives were coming from the same discourse. However, when the required change is that of the *ways* of communicating, more than just logical reasoning within the confines of one's own discourse may be necessary. An explicit conversation about the use of words and about the types of arguments that each of the interlocutors regards as proper in the given context may be vital for further progress. To put it differently, those who try to overcome a controversy should always monitor the conversation for the possibility of commognitive conflict. And thus, Chloé and Anaïs could have done better if they tried to reach an explicit agreement on how the words *nature* and *natural* should be understood in the context of the debate on GMOs. Another helpful action would be to discuss the epistemic status of their endorsed narratives and the type of substantiation each one of them regards as convincing. To sum up, dialectic argumentation, to be successful, needs to be happening at the discursive and meta-discursive levels in parallel. Only too often, those who engage in dialectic argumentation must negotiate not only the products, but also the tools of knowledge production.

Postscript

I would not end this commentary without 'turning the discourse on itself' and taking advantage of the fact that this present chapter, when put together with

Baker's and Asterhan and Schwarz's chapters, constitutes an almost paradigmatic example of a dialogical argumentation. Indeed, in this response to my colleagues I objected to thought–communication dichotomy and proposed a non-dualistic alternative – a discourse in which thinking is defined as a particular case of the activity of communication. Obviously, this latter discourse is incommensurable rather than incompatible with those of the previous two chapters. My conversation with the authors of these chapters may thus be seen as an attempt to overcome what Asterhan and Schwarz call *socio-cognitive conflict* and what I preferred to call *commognitive conflict*.

Being such an obvious case of dialogic argumentation, this present debate asks to be examined through the lens offered in the previous chapters. Above all, it is natural to try to compare its results to those obtained by Baker and Asterhan and Schwarz in their studies. More specifically, one should ask whether this direct encounter of incommensurable discourses has brought about any significant learning. Well, let me see. The first round of our argumentative dialogue took place during the Jerusalem Workshop in February 2007. Since then, the two chapters underwent revisions and my present text, written in April 2008, is a reaction to these revised versions. Although the new texts are visibly different from the previous drafts, the difference is not what might have been expected by the reader of my commentary. As it turns out, for all the non-confrontational dialogical argumentation that took place in the course of these past 14 months, I do not seem to have had much success in enticing my colleagues into changing their way of talking about learning and argumentation! Having read the previous two chapters, however, I am neither surprised nor disappointed. I am not surprised, because, as concluded by Michael Baker, it is only rarely, if at all, that an exercise in dialectic argumentation makes people change their minds in a radical way; rather, it helps them to rationalize their own position. And I am not disappointed because it was never my intention to present the commognitive discourse as *the* proper way of talking, to be used by all. Rather, my aim was just like the one that has been guiding Michael in his studies on argumentation: 'to broaden and deepen [the discussants'] views, to make [these views] more reasoned and reasonable, to enable [participants] to know of and understand others' views, to reflect upon them and … respect them as worthy of debate.' All this, I hope, was indeed attained in this conversation.

Notes

1. For a more detailed presentation of this perspective, its origins and relation to other similar frameworks see Sfard & Lavie 2005; Sfard 2006, 2008.
2. See, e.g., the following expressions: 'one student, who initially expressed a "neither for nor against" opinion about the question being debated, came to realize more clearly what she herself thought', 'she was really pro GMOs but did not realize or recognize it', 'It is of course questionable as to whether the students' texts "truly" reflect "what they really think" (cf. Edwards 2003), and of course they do not, entirely'.
3. Michael Baker seems to be aware of this difficulty when he hedges his statements on 'the real thought' in multiple ways: 'It is of course questionable as to whether the students' texts "truly" reflect "what they really think"' (note the quotation marks surrounding the italicized expressions, and then the remark in the brackets). Interestingly, these reservations do not prevent him from elaborating on the thought-behind-words idea in his interpretations.

4. Some of Chloé's more basic narratives are primary too, and thus can also be called beliefs. However, the girls differ in what they endorse as a belief and what they require to be rationally substantiated.

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Chapter 12

A dialogue on dialogue and its place within education

Rupert Wegerif, Paolo Boero, Jerry Andriessen and Ellice Forman

In this chapter we explore the role of dialogue in education. Academic papers do not only communicate through their explicit content, they also communicate through their form. By convention, for example, regardless of the number of authors, papers will express a single coherent point of view only acknowledging apparently different perspectives to dismiss them or to integrate them into the synthesis, or 'contribution to knowledge', that is usually put forward as the purpose of the paper. This chapter breaks that convention by taking the form of a dialogue. This experiment with form is important to explore, in a self-reflective way, the nature and role of dialogue. From a dialogic point of view the purpose of education is not only to impart knowledge but also, more importantly, to draw students into dialogue. Similarly this chapter aims not to produce an authoritative synthesis of the state of knowledge on dialogue in a few bullet points at the end but rather to draw readers themselves into a space of dialogue in a way that communicates, through its form, some of the intrinsic motivation and significance of the process of dialogue.

While unusual, this method has precedents in the history of science. It was once common to present ideas in the form of dialogue between competing points of view. Galileo, for example, wrote his seminal work *Dialogue Concerning the Two Chief World Systems*, in the form of a dialogue between three parties: one arguing for a Copernican world-view, one expounding the views of Ptolemy and a third, neutral party open to being convinced by the best argument. In this chapter we try to return to this dialogic form as a means for advancing science, but with a dangerous twist. It was clear which side of the argument Galileo favoured, but for this chapter we embarked on a genuinely open-ended exploration of the question of dialogue in education without knowing in advance where this would end. What follows are some edited excerpts from our email exchanges.

Rupert: kicking off with a dialogic theory of education

Dialogue features quite often in educational theories but almost always as a means to an end, the end of 'the construction of knowledge' in one form or another. I want to suggest that dialogue should be treated as an *end-in-itself*. On the one hand this is quite a straightforward idea, the idea that the process of dialogue embodies the kind of 'skills' that are important for thinking and learning and so a capacity to engage in dialogue should be an aim of education. When I present the idea of 'dialogic education' to audiences of practitioners I say that it is more than teaching *through* dialogue, it is teaching *for* dialogue and this formulation seems to make sense and be easy to apply. On the other hand though, if we think through what it really means to treat dialogue as an *end-in-itself*, I think that we have to develop a dialogic ontology that offers a new and challenging theory of education.

So what is 'dialogic'? Often this term is used for anything pertaining to dialogue but it is of more value when used as a technical term contrasting to 'monologic'. Bakhtin repeatedly points out that meaning requires a dialogue and does not exist outside of dialogue (e.g. Bakhtin 1986, p. 162). His point is that meaning is not just there in things already but is always an answer to a question. Dialogue therefore does not aim at complete agreement, if we were somehow able to reach complete agreement and coincide with our interlocutor in a dialogue then there would be no need for further dialogue and the flow of meaning would cease. In other words, meaning always implies difference and multiplicity.

When we think of a dialogue we think of people in conversation and then we think of the dialogue as a relation between them. But what if we try to think of the relation coming first? It is only in the context of dialogues that identities are defined. Aristotle's words become an Aristotelian position on the solar system only in dialogue with Copernicus: I become me and you become you, only in the context of dialogues. Yet the dialogic principle itself, because it is the source of identity in this way, does not have a clear identity or location of its own (Wegerif 2007).

The concept of dialogue as an end-in-itself or 'dialogic' can do useful work within educational theory. This useful work is pointing to a dimension of development that is really important and readily recognized intuitively but is often overlooked simply because of the difficulty of thinking it and of representing it. I understand this as the dimension of growth from having a fixed identity position towards identification with 'dialogic' or the 'space of dialogue'. Growth in the direction of dialogue as an end-in-itself is easy to see in the quality of responses towards anything which is new and 'other'. To be more dialogic is not necessarily to be more productive in constructing knowledge but it is to be more open to other voices, more able to question and to listen and so more able to allow new unanticipated meanings to emerge.

Jerry: two challenges

Rupert, you talk (in your monologue) about confronting and bridging perspectives, as the impossible objective of a dialogue. My conjecture would be that if people could be clearer about their objectives, something might be bridged. Otherwise, dialogue sounds like people in trenches throwing ideas to each other. To what extent are we considering the other, and is that a relevant extent to explore (and qualify) as researchers?

Where is the moment that some understanding arises? How can that be described in dialogic terms? Is it that: (two) individuals become a dialogue, as you seem to be suggesting, rising above their personal concerns, without individual reflection or thinking? You point at a way of being in the activity of dialogue. Can this be described empirically, or shall we stick to the hard-to-pin-down fine-sounding abstractions of philosophy?

Paolo: making sense of the 'gap' of dialogue in the context of mathematics education

In reading Rupert's contribution, I am more interested in its potential implications in mathematics education (my field of expertise) and related problems than in its theoretical features; by this way I can understand what is its 'meaning' (e.g. Wittgenstein: the meaning of an expression is inherent in its use), and thus if it is of interest for me or not.

First of all, I think that we should consider the mechanisms of those dialogues where 'the gap between…' can work as 'the context of meaning' (I will call them 'productive dialogues'), and the educational choices that allow to develop a productive dialogic style of work in the classroom. By analysing peer interactions in the mathematics classroom we can see that in many cases no meaning comes into being because the task was not well-chosen, or perspectives are too far, or the tension is not constructive, or the necessary guide or mediation (by the teacher or a more competent peer) is lacking. As an example, consider the following excerpt (eighth grade):

TEACHER: How many decimal numbers there are between one and three?
MONICA: We cannot count them.
STEFANO: Yes, we can count them: 1.1; 1.2; 1.3; up to 2.9.
MONICA: But you should also count 1.01, 1.02, and so on.
STEFANO: OK, thus you can count them.
[long silence, end of the dialogue].

The fact that students propose different perspectives is necessary but not sufficient, and the long-term analysis of classroom activities shows how experiencing too many sterile dialogues can have negative effects on students' (and teachers') engagement in further dialogues. Productive dialogues can be intentionally promoted by the teacher by establishing an appropriate didactic contract and assuming a suitable mediating role in the classroom, related to the mechanisms of production of meaning (Consogno, Boero & Gazzolo 2006; Boero & Consogno 2007).

A second point concerns the fact that important parts of our intellectual life develop as solitary adventures, where productive inner dialogues take place. In a Vygotskian perspective, intra-personal dialogue is one of the desirable follow-ups of inter-personal dialogues, thus it is important to understand how internalization takes places and what educational conditions can favour it.

In our solitary inner dialogues, the 'other' can be a real potential interlocutor (for instance, a colleague), or a figure created by us as a virtual interlocutor. Productive intra-personal dialogues need both kinds of figures (whose dialogic functions are different). How can classroom dialogues prepare students to generate and host such figures? In some teaching experiments concerning the ‘voices and echoes game’ (see Boero, Pedemonte & Robotti 1997; Boero, Chiappini, Pedemonte & Robotti 1998; Garuti, Boero & Chiappini 1999; Garuti & Boero 2002), some evidences of genuine inner dialogue have been identified when students had to echo dialogues taken from the history of sciences (in particular, Galileo’s dialogues).

Ellice: two questions

Jerry, you referred to monologue and dialogue. Do you mean one voice (speaker) versus two voices (speakers)? Or do you make the distinction in a way closer to Bakhtin who argued that multiple speakers could end up in monologue if a single speaker’s voice (or perspective) dominates. In contrast, dialogic speech is responsive to the other by affirming the listener’s agency and could occur even with one speaker (in terms of one’s inner speech).

Like Paolo, I am also interested in the potential of ‘productive dialogue’ in mathematics and science education and understanding how to contrast them with ‘sterile dialogues’. Are ‘sterile dialogues’ similar to Bakhtin’s monologues – situations where (in Bakhtin’s terms) authoritative speech drowns out internally persuasive speech? How can teachers productively scaffold these genuine dialogues?

Rupert: reflection on the process of this dialogue

In trying to respond now to the dialogue after only four messages I find that I already have about 16 different threads of meaning to choose from. I find it interesting that there are already so many strands to this dialogue. As I understand it, I made one point, then there were responses to that, each taking a different perspective in relation to it, sometimes two or more different perspectives, and each also responding not only to the initial message but also to the messages that followed it. I think that this fertility is somehow intrinsic to the nature of dialogue. The strands of meaning in a dialogue always multiply and fly beyond any attempt to limit them in advance. This fertility seems related to the fact that there is never a simple transmission of a perspective or a predictable response. As Bakhtin emphasizes, we cannot simply passively receive the words of others, even if we try to do so, but we must inevitably understand with our own ‘answering words’ which are always unique because they are shaped by our unique situation and history.

Rupert: response to Paolo

I really like the idea that productive dialogues can be defined as those in which ‘the gap between’ is also ‘the context of meaning’. Bakhtin defined dialogue, against mere conversation, as a shared enquiry in which answers give rise to new questions, writing: ‘If an answer does not give rise to a new question it falls out of the dialogue’ (Bakhtin 1986, p. 168). This neatly inverts the common idea that dialogic enquiry is seeking consensus. When we reach unity there is no more dialogue and where there is no dialogue there is only what Bakhtin calls 'systemic cognition' by which he means the mechanical processes of machines and nature with no reflection and therefore no intrinsic 'meaning'.

If 'falling out of the dialogue' because of reaching consensus defines one limit of productive dialogue – the limit when the gap is not great enough – it follows, as you say Paolo, that the other limit is when the gap is too great so that the dialogue does not start up in the first place. The extract you show is a clear illustration of the common situation that when there are not enough links or hooks or connections between people the dialogue does not start up. Starting a dialogue can then seem like trying to start a fire with damp twigs on a windy day.

This account of the two sterile sides of productive dialogue fits rather well with the idea of dialogue occurring in a kind of 'zone of proximal development'. If a problem is too easy – too coincident with current level of development in Vygotsky's terms – then the zone does not open up, but if the problem is too hard then equally the zone does not open up. This effect was explored using the problems of Raven's non-verbal reasoning test, a set of problems in five sets which are graded for difficulty and get progressively harder within each set. We analysed the talk of children around these reasoning test problems and found that they only engaged in productive dialogue (what we called 'exploratory talk') when the problems were just right, not too hard and not too easy (Fernandez et al. 2002).

Paolo: mechanisms of productive dialogue

In a long-term study of construction of knowledge in a mathematics classroom, we found three kinds of mechanism at work in the use of language that led to increased knowledge construction during classroom discussions 'orchestrated' by the teacher (Bartolini Bussi 1996).

1 Evolution of a personal interpretation of the situation

One student's interpretation of the problem situation is enriched and integrated by the interventions of some schoolmates who propose other interpretation(s) of the same situation, up to the full apprehension by the first student and his/her relevant contribution to the solution of the problem in the classroom discussion.

2 From a situation to the opposite one, to a wider perspective

Students' contributions put on the table two opposite situations related to the task (for instance, one case fits the conditions of the task, while the other escapes them). This contributes to construction of knowledge by offering a wider perspective for a discourse that embraces both cases and allows a jump in conceptual construction and reasoning.

3 From single cases to generalization

Students propose some similar cases related to the task, then a collective process of induction takes place by considering common features of the evoked cases. A general statement is the outcome of the process.

Here, for example, is a transcript extract illustrating mechanism 2. The task (Grade 6) consists in the production and validation of a conjecture concerning common divisors of two consecutive natural numbers.

ROSY: In case of divisibility, the remainder is zero.
LORENA: While in case of non-divisibility, the remainder cannot be zero.
DANIELE: In the case of two consecutive numbers … [*long silence*]
FRANCESCA: In the case of one number and the following one … [*long silence*]
IVAN: In the case of the following number, we move from remainder zero to remainder one, so the following number is not divisible by that divisor.

In this example (like in the other examples reported in Boero & Consogno 2007) we can observe how 'productive' dialogue depends on a specific kind of interaction (I could call it constructive opposition), and develops through an evolution of the expressions used by students, which is related to their effort of understanding. '*In case of … the remainder*' is the key expression that allows moving from a situation to the opposite one, and then to a linguistic expansion that embraces both cases and allows finalizing reasoning. Note also how Francesca contributes to the debate by transforming the expression '*Two consecutive numbers*' (coming from the task) into the expression '*one number and the following one*', which allows Ivan to 'see' the transition from '*remainder zero*' to '*remainder one*'.

Ellice: teaching for disciplinary dialogue

I agree with Rupert that we want to teach *for* dialogue (not merely *through* dialogue) but I would also want to modify this statement as: we want to teach *for disciplinary dialogue*. In the sciences, where argumentation among and between groups of investigators is necessary for scientific progress, there is another partner in the dialogue: Nature (Pera 1994). Inscriptions based on experiments or observations serve to ground debate by forcing the parties in dialogue to submit to its material agency (Pickering 1995). In classrooms which attempt to engage students in simulated scientific inquiry and debate, students need to both construct logical arguments and ground them in material evidence. Thus, productive dialogues, in Paolo's terms, require a didactic contract among participants, mediated by the teacher that affords the opportunity to assume the dialogic roles of constructor and critiquer of both theoretical constructs (e.g. scientific models) and accepted methods of generating and evaluating evidence (e.g. experimental design; replication) (Ford & Forman 2006).

In a transcript from the MUSE (Modeling for Understanding in Science Education) Evolutionary Biology Project (Stewart, Cartier & Passmore 2005), several students (ages 16–18) debated each other's grant proposal at the end of the 9-week course. The topic of the grant proposals was to study sexual dimorphism in ring-necked pheasants: Why the trait of bright coloration in male pheasants is adaptive despite its selective disadvantages (salience to predators). Matt was a member of a team of four students that had just finished presenting their proposal. Vicky was the first classmate to critique their proposal, using the criteria of experimental design (e.g. that this design fails to control all but one of the key variables in the Darwinian model). [The two variables focused on in this design are the selective advantage of a particular trait AND its heritability.] Matt admitted that their design is not ideal but had been modified for financial (practical) reasons. Several other classmates, including Mike, then questioned the study's ability to collect sufficient data to evaluate the heritability of this trait (e.g. studying enough generations of pheasants to assess the coloration of off-spring from brightly coloured versus dull-coloured male pheasants). Matt was forced to admit that they would need to follow several generations of pheasants (a topic not explicitly addressed in their proposal). Mike finished this exchange by pointing out that one of their red-headed classmates has two parents without red hair, thus making an analogy between inheritance of traits in pheasants and in humans (diSessa 2007).

Rupert: the 'third party' in every dialogue

Ellice really takes things forward for me by pointing out the importance of 'Nature' as a partner in science dialogues. If dialogues are 'shared enquiry' as Bakhtin writes, then they must be shared enquiry about something and this something is like the 'third party' in the dialogue. It is natural to think that this third party is some sort of thing, like a subject domain, but I am sceptical about that. I am not sure what 'Nature' is but here it seems to play the role of the ideal of the truth of things as seen from a God's eye point of view. I think that Bakhtin refers to this with his concept of the super addressee, which is the ideal of a third party or witness to every dialogue who has a capacity to understand fully what is meant by an utterance even when the specific addressee cannot understand it due perhaps to his or her limitations (Bakhtin 1986, p. 126). This ideal of an unsituated perspective is understood by Bakhtin as a projection of situated dialogues, it is a situated-unsituated kind of perspective, and this can be seen in the way it varies from being sometimes the perspective of 'God' to being 'the eventual consensus of the community of scientists', or the 'judgement of the future'. While the super addressee is not real, i.e. it is not really 'Nature' as seen by 'God', it seems to be a necessary ideal in every shared enquiry, drawing the participants out from partial or self-interested views towards ideals of 'truth' and/or 'justice'.

Ellice: response to Rupert's 'the "third party" in every dialogue'

To me, the gap between my account about the role of Nature in scientific dialogues and your response to my account is quite large. You wrote that Nature 'seems to play the role of the ideal of the truth of things seen from a God's eye point of view'. That connection to a divine perspective would naturally create a disturbance in our developing notion of dialogue, so I think it would be important to respond to it. To try to narrow the gap between your account and mine, I would like to elaborate my account of the agency of Nature. Nature never speaks directly in scientific dialogues but it expresses its agency through data patterns depicted in inscriptions (texts, models, graphs, charts) (e.g. Latour 1987). That is, each scientific team produces its own version of Nature in its inscriptions, which serve as the evidence presented to advance their scientific argument and respond to the anticipated arguments of other scientists. The example that Latour presents is that of the double-helix model of Watson and Crick versus the previous three-chain helix proposal of Linus Pauling. Nature's agency was expressed only through the empirical evidence that could be used to support or refute the validity of one helix inscription versus the other. As we know, the double helix model fit the data that Nature provided about the behaviour of DNA better than other opposing models. Thus, Nature is a dialogic partner in scientific practice, not a perfect vision of divine truth from the super addressee.

The implications for educational practice are that students also need to grapple with the measurement and depiction of data patterns (a rare event even in laboratory activities) and to engage in scientific arguments with their classmates of the validity of alternative data inscriptions of a common dataset. Their dialogues would need to include the evidence that supports their inscriptions as well as anticipation of their peers' use of evidence that would refute their claims.

Jerry: on productive dialogue and the importance of the ethical dimension

Boero and Ellice have already referred to the idea of 'productive dialogue', but how do we define this and what makes it possible?

Shotter (1997) describes a type of dialogue pervasive in public debate, in which participants, perhaps aiming for certainty and clarity, each present their own viewpoint, as an individualistic achievement, without any commitment to the others, as if in a monologue. Those who are enclosed within their own hearts will not be able to take their responsibilities with others seriously (Shotter 1997, quoting Alexis de Tocqueville). Such a language of individualistic achievement and self-fulfilment often makes it difficult for people to sustain their commitment to others (Bellah, Madsen, Sullivan, Swidler & Tipton 1985). By its very nature, such a form of rational argumentation prevents the constructing of a shared version of things: it is not dialogical. It is as if two separate spaces of ideas independently exist without touching each other. What does it take for two participants to enter into a dialogue, in which spaces interact and a new space emerges in which dialogical thinking can be observed? And what does such dialogic thinking look like, how do we define it relative to different activities and practices?

Every utterance must be regarded as primarily a response to preceding utterances of the given sphere (Bakhtin 1986, p. 91). Every utterance starts with the preceding utterance. Both speakers and listeners must be sensitive to the intervention of another voice, occurring at the interactive moment. For those sensitive enough, this is a genuine source of tension (Andriessen, Baker & van der Puil in press). Dialogic speech is responsive to the other by affirming the listener's agency. In such a dialogue, agency subsists only and entirely in the ongoing, living interactive activity between people, as formative influences at work in time (Shotter 1997). It is precisely in such interactions, at the moment of indeterminacy and ambivalence, when one person has finished acting and the other must respond, in this 'interactive moment' (Shotter 1997) in which one person makes contact with, or constructs a link with another, that everything of importance occurs. Life is manifested at this boundary, where the gap is filled by a new utterance.

It is interesting to consider the importance of 'ethics' to productive dialogue. Allwood, Traum and Jokinen (2000) discuss the main features of 'ideal cooperation' as being cognitive consideration, joint purpose, ethical consideration and trust. Cognitive consideration is needed for shared understanding, joint purpose is required for moving into the same direction. Ethical consideration is about not forcing each other, not preventing the other to pursue something, and making it possible for the other to engage in successful rational action. Trust refers to the certainty of the others sharing the same considerations.

These criteria are gradual, not only with respect to the dialogue, but also with respect to the temporal dimension of the activity: trust develops, or is broken down by certain contributions during dialogic activity. The following fragment illustrates two dynamic aspects of an educational dialogue: tension/relaxation, and the discrete nature of forward movement. The fragment essentially shows dialogic thinking in an educational setting.

In the fragment we see a group of four students (14–15 years old) involved in an 8-week self-assigned project of making a movie about the human senses, to use in the biology class. Each sense will be illustrated with short movie clips and comments. They have weekly meetings of 50 minutes. This is the seventh session; hence there is quite some tension from time pressure. During previous discussions several proposals have been put forward and rejected. While not all previous discussion was collective, the following discussion clearly is. It revolves about finalizing part of the film script illustrating the concept of 'temperature'.

1 (R): I think the script we are making together is far too complicated.
2: [. . .]
3 (R): You want a beach . . .
4 (I): Yes that's what we want.
5 (R): We all want things that are too difficult to do. I mean, at the newsstand we also could not . . ., at the garbage can, look these are nice ideas! We are now discussing temperature, so, ehhm, then she can pretend burning her hand, or someone can stand in front of a sun-thing. . . .
6 (K): Or just the heater.
7 (I): Yes, I thought so too.
8 (J): Yes, but now we are starting all over again, and I wonder how useful it is to start all over again!
9 (I): But we are not! We are discussing a different script!
10 (J): So: why don't we just start filming?
11 (I): Because we have not yet all agreed on what to do with temperature.
12: [*silence*]
13 (I): I now have written down all ideas I ehm, have. . . .
14 (J): Let us now look at what we can do with each of those ideas.
15 (K): I think heater or steam.
16 (J): Turn higher, yes, heater, what do you think I?

17 (I): Yes, I think heater is OK, but steam as well, maybe that gives a better idea of temperature.
18 (R): But how could we do steam?
19 (I): Well, we have a water cooker here.
20 (R): Well, that is hard to film, for starters. . . .
21 (J): Why?
22 (R): Steam from a water cooker, difficult to see, but maybe still . . .
23 (J): We are stuck again!
24 (K): I do not think so, we are nearly there, we just have to choose between heater and steam!
25 (J): Like we did an hour ago.
26 (K): I do not agree, then it was just one person who wanted it, now we are discussing convenience of the idea.

First, why is this collective thinking? Briefly, because there is display of trust: participants freely object to each other's ideas, but not only that: objections are met by rebuttals. This already requires cognitive consideration, joint purpose as well as ethical consideration. All of these are shared by all four participants. Second, the driving force of the dialogue is in its social dimension: the pattern of tension/relaxation which is displayed by agreements and objections. Especially the rebuttals in (9) and (25) display relaxation by reformulation of an objection into a different objective. Third, these rebuttals also display changes in group perspective, because (which is not shown) they are taken up by all group members. This is one manner of learning, seeing things in a different perspective.

Rupert: the shift into productive dialogue: mechanisms and ethics

Some of the profound implications of dialogic for education in general were brought out by the findings of a series of empirical studies of an intervention programme, 'Thinking Together', which focused on teaching children how to talk together more effectively in small groups (Mercer 2000; Wegerif and Dawes 2004). To assess the quality of talk we asked groups of children to talk together around a kind of reasoning test, Raven's non-verbal Standard Progressive Matrices already mentioned above: a test that correlates well with academic achievement. We found that unsuccessful group talk failed in mainly two ways, through each individual identifying with their own self-image in the dialogue and so trying to impose what they saw as their position on the others or through individuals identifying with a sense of group identity and uncritically agreeing with each other in order to avoid any disruption to what was felt as group solidarity. In more successful group talk, talk that helped them to solve the reasoning-test problems together, the most obvious difference was that individuals were able to change their minds, to question their own positions and to ask for help when they did not know the answer. An example is given below. The full transcript is published in Wegerif and Dawes (2004, pp. 37–39), so I just reproduce some short extracts here:

Transcript extract 1: pre-test initiation and challenge

TARA: Square and diamond, it's 2.
PERRY: No it's not.
TARA: It is 2.
PERRY: No it's not.
TARA: It is.

In the pre-test Tara, a girl, initiates with a suggestion; Perry, a boy, rejects it and they move into a dispute. This disputatious approach continues and eventually Perry imposes his own solution, number 6, against the opposition of two girls, Tara and Keira, by grabbing the pencil and writing down his answer in the space provided.

Transcript extract 2: post-test initiation and challenge

TARA: That has got to be a diamond, a square with a diamond with a circle in that one, number 6, do you agree?
PERRY: No, what do you mean?
TARA: OK, no it's got to be square.

In the post-test, three months later, after 10 or more lessons focusing on using the ground rules of exploratory talk, the same group respond to the same problem quite differently. When Tara suggests number six she does so with a question asking if the others agree, Perry then asks her politely to clarify her reasons and, in the act of reflecting on her claim, Tara changes her mind. The talk continues for some time exploring different alternatives. The video also shows long pauses with the group all leaning forwards towards the problem sheet with concentrated expressions. Eventually Tara sees the correct answer and tries to communicate this to the others.

Transcript extract 3: post-test, sharing the solution

TARA: Look, that's got a triangle, that's got a square. Look. That's got a square with a diamond with a circle in, that's got a square with a diamond in and that's got a square with a circle in so that's got to be a square.
PERRY: I don't understand this at all.
TARA: Because, look, on that they've taken the circle out yes? So on that you are going to take the circle out because they have taken the circle out of that one.
PERRY: On this they have taken the circle out and on this they have taken the diamond out and on this they have put them both in, so it should be a blank square because look it goes circle square.

After Tara tries to explain her vision, Perry admits that he does not understand her in a way that invites her help. Tara then tries again using the phrase 'taking the circle out'. Perry suddenly seems to see the answer. His eyes light up and he

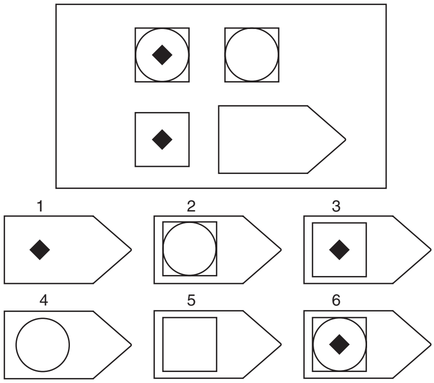

Figure 12.1 The problem*.

shows signs of excitement. He then repeats Tara's words 'taking the circle out' with energy and animation to express his new understanding.

This illustrates a change that was found more generally. The more successful post-test dialogue exhibited many examples of children apparently arguing against their own positions, admitting their ignorance, asking for help and changing their minds. This suggests a different kind of identity-in-dialogue crucial to reflection and creativity, an identification not with any bounded image, an image of self, or group, gender, ethnicity, etc., but an identification with the space of dialogue itself as a vantage point from which voices can evaluate and criticize even their own previously asserted position.

The key change to observe therefore, in the direction of more effective problem-solving dialogue, is not only the natural language mechanisms that Paolo refers to, for example, here use of explicit forms of language such as 'because' or 'do we agree?', but also the ethical development that Jerry refers to which is seen here in the ability of Perry in the post-test to humbly admit that he did not understand, to invite Tara to explain her solution to him and then to adopt her words as if they were his own with pride. In general, across many examples, improvements in the quality of shared thinking are accompanied by children being able to listen to others, change their minds and argue against their own initial positions. This observation led me to propose a general direction in the development of more effective reflective thinking dialogues away from self-identification and towards an identification with the dialogue itself. This is where I kicked off this dialogue with the theory of a dimension of development into dialogue but here I have returned to show where that theory came from in the interpretation of productive classroom dialogues.

Paolo: coming back to the importance of the teacher's role

Thanks to Rupert's example, we can see how evolution of dialogue towards a productive dialogue can take place. In an educational perspective, I am very interested in the issue of the conditions (on the students' side and especially on the teachers' side) that allowed evolution to take place.

Following the evolution of verbal interactions in several primary school classrooms over a long period of time (from one to five years) and comparing situations of positive evolution with situations of failure, it seems to me that three kinds of conditions could be relevant for success. First of all, conditions inherent in values, in the 'ethics' of verbal exchanges (not only cooperative construction, but also constructive opposition). Second, mastery of natural language: precision in producing and understanding verbal expressions, quality of verbal expressions (mastery of linguistic connectives and specific terms related to the object of the discussion). Third, influences depending on the out-of-school socio-cultural environment. The role of the teacher is crucial: the first and second kind of conditions strongly depend on his/her mediation! By intervening on the first and second kind of conditions the teacher can even succeed in contrasting possible negative influences coming from students' socio-cultural environment: when students develop the linguistic skills needed for productive dialogue and learn to interact in a productive way, they can take real pleasure in constructive classroom dialogues.

Jerry: what is the gap about, and what is development about?

As we draw the dialogue to a close, my summary of main events focuses round the *gap*. Rupert talked about the gap between voices in dialogue, as a gap between perspectives, and as a source of tension in dialogue, as a thing, with an outside and inside which only derives its meaning from within the dialogic. Dialogic points to the context of meaning, as a point of reference rather than as an activity, it seems. It is the point where education should grow to: into being dialogic. Paolo remarks that such tension alone is insufficient for productive dialogue to emerge. Moreover, he refers to inner dialogues with virtual interlocutors. Rupert says we need dialogues occurring in a zone of proximal development. Paolo describes three mechanisms of productive language use: evolution of personal ideas, opposition and generalization. My own contribution also refers to a gap, described as an interactive moment, a source of tension, where two individuals make contact. My discussion stresses the social dimension: it basically says that development in dialogue is by development of trust and shared thinking. Finally, Rupert presents a closed problem-solving task, to illustrate features of dialogic thinking, displaying both productive language use and development of trust. He sees this as moving towards identification with dialogue itself.

So, now, from my position it is interesting to ask what develops: individuals becoming increasingly dedicated and precise in taking up and developing each other's ideas (even in monologue), or learners becoming more dialogic, that is learners knowing how to handle questions, misunderstandings, differences of opinion? Is there a difference or are we talking about the same gap?

Rupert: final questions

From the beginning, this dialogue, when it shot out in many directions at once, has explored the field of dialogue in education. In doing this it has touched on some of the issues that anyone investigating dialogue in education should be aware of: the role of the discipline area, the distance between participants, the importance of trust, the role of preparation and of key linguistic strategies and more. This could be seen as a 'widening' movement. There has also been some deepening. I think we have made progress together on understanding some of the key factors involved in generating productive dialogues in educational settings, for example.

However these notions of 'widening' and 'deepening' have a different meaning in a living dialogue from that found in a review paper. In any living dialogue, like the one recorded in our chapter, widening and deepening are driven by disagreement and misunderstanding as well as by agreement and understanding. The result then is not a flat surface, like the spreading out of a concept map on a two dimensional table. The genre of academic monographs does not make it easy to acknowledge fissures opened up by the creative tension between incommensurate points of view. However, living dialogues are spurred by these tensions. A dialogic form of writing might not be the most efficient way to communicate a consensual body of knowledge but it might be a good way to draw the reader into the creative space of dialogue that always precedes and surrounds areas of settled consensus in any science.

Jerry, in his contribution above 'what is the gap about?', suggests that our dialogue remains within a gap of different conceptions of development: on the one hand, a vision of dialogue contributing to individual development and on the other hand, a vision of 'learners becoming more dialogic'. While these visions may not be incompatible in a classroom context, they remain, nonetheless, very different visions of development (Wegerif 2008). Another clear fissure in our text opens up between myself (Rupert) and Ellice on the role of Nature in scientific dialogues. Perhaps these open questions define the field of dialogue in education better than any set of provisional answers.

There is already a long tradition of writing that communicates its message through dialogue, like Galileo's famous dialogue on two world systems that we referred to at the beginning. One motive for organizing this chapter as a real dialogue was to explore a further question: is it possible to write in a way that not only communicates a message through dialogue but also communicates the message of dialogue itself? This is a question that only our readers can begin to answer: so, dear reader, what do you think?

Bakhtin: last word on last words

There is neither a first nor a last word and there are no limits to the dialogic context (it extends into the boundless past and the boundless future) (Bakhtin 1986, p. 170).

Note

* This is a parallel version of the problem created with the agreement of John Raven, holder of the copyright to Raven's reasoning tests whose support for this research is gratefully acknowledged.

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Part 4

Methodologies for studying transformation of knowledge in classroom interaction

Chapter 13

A methodological framework and empirical techniques for studying the travel of ideas in classroom communities

Geoffrey B. Saxe, Maryl Gearhart, Meghan Shaughnessy, Darrell Earnest, Sarah Cremer, Yasmin Sitabkhan, Linda Platas and Adena Young

Powerful methods for analyzing the travel of ideas are essential for understanding the process of learning in classroom communities. In this chapter, we argue that a genetic perspective provides a useful methodological frame. We describe specific empirical techniques for the study of the travel of ideas, and illustrate our approach with findings from recent investigations of lessons on integers and fractions for fifth grade.

We begin with a summary description of a classroom exchange in a classroom that supports inquiry-oriented mathematics instruction.

Students in Ms. Jones's classroom begin their math lesson by working on the problem: *How many equivalent fractions can you name for Point A on the number line?* After a brief class discussion of the range of answers, Ms. Jones sends students to small groups to explain their thinking and try to reach consensus on one solution. In one group, students argue about different ways of re-partitioning the number line to generate names of equivalent fractions. In another group, students share ways of multiplying the fraction by one (2/2, 3/3, etc.) to generate equivalent fractions. As Ms. Jones observes groups at work, she notices that some students are changing their thinking, while others persist with their original solutions. Ms. Jones re-convenes the class and orchestrates a discussion, building upon students' ideas to guide students toward richer interpretations of equivalence, as she sets the stage for a future lesson on estimating and comparing points on a number line.

A guiding principle of inquiry-oriented mathematics instruction is that student thinking is a resource for teaching and learning. Teachers pose problems, solicit students' solutions and orchestrate discussions to guide students to make their mathematical ideas public and reflect on relationships among their ideas. In the social context of classroom lessons, students' ideas may be taken up or rejected, valued or devalued, and interpreted in various ways, and, in this process, students' mathematical ideas become elaborated and often transformed. In this chapter, we argue that investigating the emergence, travel and transformation of ideas is essential to understanding learning and teaching in inquiry-oriented classroom communities.

Our chapter introduces a methodological framework and specific empirical techniques for the study of the travel of ideas, and we illustrate our approach with findings from recent investigations of lessons on integers and fractions for fifth grade. We begin with the principles that guide our design of lessons that support whole-class and small-group discussions. Next, we present our framework for investigating the travel of mathematical ideas during the lessons, with a particular focus on the constructs we are using to understand processes of change: (a) students' learning over the course of a lesson through short-term *ontogenetic processes*; (b) students' moment-to-moment construction of representations and communications through *microgenetic processes*; and (c) students' propagation of ideas in local interactions in classroom communities through *sociogenetic processes*. The third section illustrates our approach with an analysis of the travel of ideas in a single classroom lesson. We conclude with reflections on our methods and the challenges and prospects for future work.

Design and investigation of inquiry lessons on integers and fractions

Integers and fractions are challenging topics, and our goal is to design a curriculum unit that engages diverse students with the mathematics in sustained and coherent ways. Our approach to lesson design builds on socio-constructivist assumptions about meaningful learning: In the context of mathematical activities where students work with others and independently, students develop understanding as they produce, coordinate and adapt representations (number lines, area models, Hindu–Arabic arithmetical procedures) to serve mathematical functions (subtract negative numbers, compare fractions) in communicative and problem-solving activities. Lesson sequences should support students' efforts to (a) extend their own sometimes idiosyncratic forms of mathematical representation for integers and fractions to serve new and important mathematical functions, and (b) incorporate new forms of representation valued in school to serve and extend mathematical functions that they already know. Thus we view learning as shifts in the relationships that students construct between mathematical forms and mathematical functions.

Our principles for lesson design are grounded in our socio-constructivist assumptions about the ways that mathematical ideas travel and are transformed.

1. *Lessons should target core mathematical ideas.* Elementary mathematics textbooks in the United States contain too many mathematical topics and representations to be covered in depth, and students cannot be expected to develop rich connections within and across topics and representational forms. We are focusing on core generative ideas and setting aside more peripheral topics and representational forms. For fractions, we consider equivalence to be one core idea and the number line one core representational form. Equivalence is the basis for the principle that any particular representation of a rational number is not *the* number itself, and any rational number can be represented in an infinite number of ways. As a representational form, the number line can support elementary students' understanding of equivalence, order and magnitude of rational numbers; it then becomes a critical tool for secondary school students' later work with the Cartesian coordinate system and mathematical functions.

2. *Lessons should engage all students.* US classrooms are diverse, and we need

pedagogies that support the intellectual engagement of students who vary in mathematical understandings, interests and investment. This principle led us to the six-phase inquiry lesson structure depicted in Figure 13.1, a structure that encourages active participation, reflection and questioning about mathematics (Saxe et al. 2007). The lesson is organized around a Problem of the Day that is challenging and yet accessible – students must choose one answer among several alternatives that represent common patterns of student thinking. The multiple-choice format for the Problem of the Day is an adaptation of the Itakura method originally developed for science lessons (cf. Inagaki, Hatano & Morita 1998). To support student exploration of form–function relationships between fractions, representations and the functions they can serve, students revisit the Problem several times over the course of the lesson through individual work, small-group interactions, whole-class presentations and teacher-led discussions.

Phase 1: Independent work and pre-assessment. Students work independently to solve a problem like the one illustrated in Figure 13.1, 'How many fraction names for point A?'. To provide a scaffold for student engagement, students choose one of five multiple-choice alternatives and then justify their choice in writing. The answer choices are based on previous studies of students' reasoning on similar problems (Saxe, Langer-Osuna & Taylor 2006; Saxe, Taylor, McIntosh & Gearhart 2005): (a) only one fraction name; (b) two fraction names; (c) between 3 and 10 fraction names; (d) between 11 and 20 fraction names; (e) more than 20 fraction names. Students' answers are their initial forays into the mathematics as well as the teacher's initial assessment of the range of ideas in the classroom. The teacher then chooses several students who represent that range to make presentations in Phase 2.

Phase 2: Student whole-class presentations. The teacher invites several students to present their solutions. During the presentations, other students hear the solu-

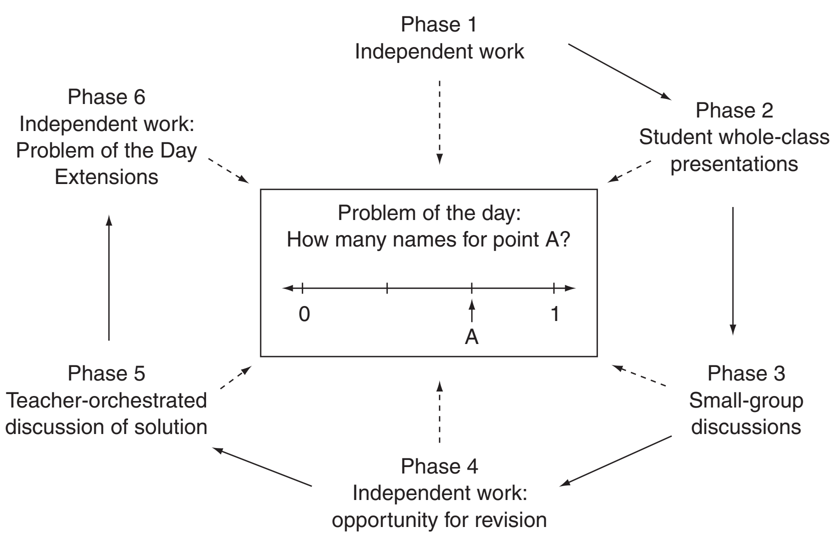

Figure 13.1 An illustration of the six-phase inquiry lesson structure.

tions they themselves had chosen as well as solutions they had considered and rejected, along with their peers' justifications.

Phase 3: Small-group discussions. Students meet in small groups to present their thinking to one another and try to reach consensus on a single solution and justification. The requirement that each student presents as well as listens supports student engagement.

Phase 4: Opportunity for revision. Students revisit their previous solutions to the Problem of the Day. They are invited to explain how their revisions – or their decisions not to revise – have shifted based on the whole-class presentations and small-group discussions.

Phase 5: Teacher-orchestrated discussion. Drawing on students' ideas in student presentations and small-group discussions, the teacher guides students through an exploration of contradictions. The discussion concludes with extended discussion of the correct solution.

Phase 6: Independent work and post-assessment. The class concludes with independent work on two extension problems that parallel the Problem of the Day.

3. *Lessons should be organized as a mathematically coherent series.* What students learn in one lesson should build on prior lessons and support learning in subsequent lessons. In our lesson on identifying a point on a number line with unequal partitions, for example, the answer '¼' (Figure 13.2) was designed to foreshadow the next lesson on relationships between equivalent names such as 2/8 and ¼ for the same point on a number line. Students' work with the non-routine number line with unequal partitions (Figure 13.2) was designed to raise issues about the necessity of equal intervals that students will address again when reflecting on relationships between ¼ and 2/8.

'Travel of ideas' in the classroom

In our approach to analyzing the travel of mathematical ideas in and across lessons, we distinguish between a *methodological approach* and *empirical techniques*. Our methodological approach begins with two framing questions – the what? and the how? of travel.

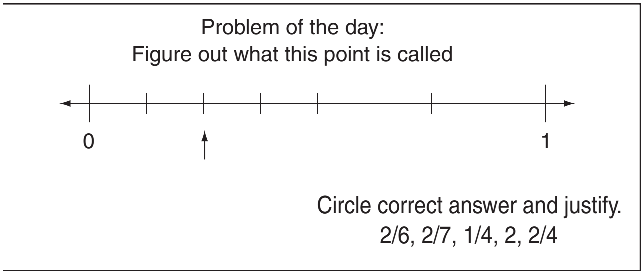

Figure 13.2 Problem of the Day for the lesson preceding the equivalent fractions lesson.

What are the 'ideas' that travel?

The ontology of mathematical ideas is a topic of polemical discussions across epistemological and psychological frameworks. Our account builds upon the work of theorists who locate ideas in the constructive activities of individuals (Piaget 1970; Sfard 2008; Vygotsky 1986): We treat mathematical ideas as emerging and becoming crystallized as students participate in the collective practices of classroom life. The process occurs as students construct connections between representational forms and the mathematical functions that they use those forms to serve in communicative and problem-solving activities (Saxe 1994; Saxe & Esmonde 2005). In the equivalent fractions lesson, the ideas that travel are students' shifting uses of geometrical and arithmetical forms as students conceptualize, discuss and solve problems about fractions.

Consider, for example, the ideas that the lesson supports as students explore ways of answering a Problem of the Day on equivalence. In Figure 13.3, the left column illustrates connections that students might generate between 'two-thirds' and equivalent fractions by constructing *geometric forms* on the number line (e.g., hatch marks, interval lengths); as students explore geometric operations on the line, some may eventually conjecture that it is possible to produce an unlimited number of divisions of the line. The right column of Figure 13.3 illustrates connections students may generate between 'two-thirds' and equivalent fractions through an *arithmetic form*, multiplication of 2/3 by the fractional equivalents of 1; as students explore arithmetic transformations of equivalence, they may come to understand that the series of multiplications can proceed indefinitely. Students may also explore *relationships between geometric and arithmetic forms* – for example, they may observe that every geometric repartitioning of the line corresponds to an arithmetic multiplication of the fraction by the equivalent of 1. Partitioning each distance that is equivalent to 1/3 of a unit into halves to create

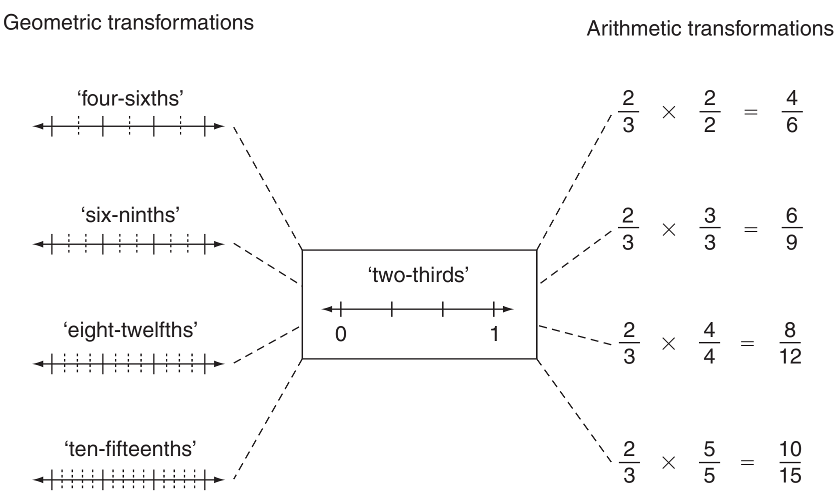

Figure 13.3 Geometric and arithmetic forms used to explore equivalent names for the same point on the number line.

sixths corresponds to the multiplication of 2/3 by two halves (2/2), or $2/3 \times 2/2 = 4/6$; similarly, repartitioning each third into thirds corresponds to the multiplication of 2/3 by three-thirds (3/3), making ninths, or $2/3 \times 3/3 = 6/9$, and so forth.

How do ideas travel?

The collective practices of the classroom support the emergence, reproduction and alteration of mathematical ideas in students' problem solving and discussion. All participants contribute to the collective practices of the lesson – to the lesson's emerging structure, to the use of valued forms of representation and associated functions, and to the social positions of students and teacher (e.g., how students value, devalue or ignore one another's actions and contributions). The activities of individuals are in part constitutive of this structure but also take form in relation to it, and it is in the dynamic between individual and collective activity that we locate 'travel of ideas.' We analyze the travel and transformation of ideas in the activities of individuals through three coordinated strands of genetic analysis: ontogenesis, microgenesis and sociogenesis. Though these strands are integrated in the nexus of activity, their analytic separation is critical to an understanding of learning and development. *Ontogenesis* focuses on shifts in patterns of thinking over the development of the individual; *microgenesis* involves the construction of meaningful representations in activity; and *sociogenesis* entails the reproduction and alteration of representational forms that enable communication among participants in a community. For each strand, we assemble somewhat different but overlapping sets of evidence. As we will show in the section on empirical techniques, our methodological approach is a framework for producing evidentiary claims about the genesis of ideas, rather than a fixed set of distinct procedures for collection and analysis of data for each strand.

Ontogenesis. Individual development of mathematical thinking – its ontogenesis – is marked both by continuity in the individual's ways of understanding the experienced world and discontinuity as the individual structures new systems of understanding out of prior ones. As Langer (1970: 733) has expressed it, 'The central theoretical issue is dialectical: How does a developing organism change qualitatively and at the same time preserve its integrity?' The answer to this question was the life's work of Piaget (1970), Vygotsky (1978), and Werner (Werner 1948; Werner & Kaplan 1962), and our interest in understanding developmental relations between forms and functions builds upon their work when we examine qualitative shifts in student thinking over time. But unlike most treatments of ontogenetic change where the focus is on major time spans of intellectual development (e.g., infancy to early childhood, middle childhood to adolescence), we focus on continuities and discontinuities in students' thinking over the course of a single lesson. Are students creating new functions for forms that they have used previously? Are they incorporating new forms linked to classroom life for functions that they already have understandings? If so, how could the character of such changes be documented and analyzed?

To date, our investigations of the ontogenesis of ideas have focused on shifts in individual students' uses of mathematical forms for particular functions from the beginning to the end of each lesson. As our research proceeds, we will also track shifts in form–function relationships over the course of lesson sequences.

Microgenesis. Microgenesis is the process of moment-to-moment construction of representations as individuals work to turn representational forms into means to serve mathematical functions. In the classroom, representational forms like number lines or fraction words contain no inherent mathematical functions, and mathematical functions like solving rational number or whole number problems can be served by any number of representational forms. Relationships between forms and functions emerge and shift in students' moment-by-moment activities as students appropriate and adapt forms to accomplish local and emerging goals (Saxe 1991). We illustrate some properties of microgenetic processes with examples of students' efforts to identify a fraction on the number line and produce equivalent names for the fraction.

A child working on the Problem of the Day in Figure 13.1 may be coordinating multiple forms and potential mathematical functions to identify a name for Point A. If she conceptualizes the task in terms of fractions, she needs to: conceptualize the geometrical form of the number line as partitionable into three intervals that are three equivalent fractional parts of a whole; conceptualize a register of fraction word forms as a sequence ('zero, one-third, two-thirds') with each successive term representing a magnitude equivalent to the prior term in the sequence; construct correspondences between her conceptualization of the partitioned line and her conceptualization of the lexical forms so that 'two-thirds' comes to refer to the endpoint of the second interval (Point A). Each of these conceptualizations is challenging. Students' idiosyncratic conceptualizations and coordinations will vary, and their understandings will shift in different ways as they participate in and contribute to independent and collective activities over the course of the lesson.

Let us consider two hypothetical students' use of geometric forms to identify additional names for Point A in Figure 13.1. One student inserts a hatch mark between 1/3 and 2/3, counts the hatch marks on the number line in Figure 13.1 starting with 0, and decides that Point A is 'three-fourths' because it is the third hatch mark of four (see Figure 13.4a). This is one microgenetic construction; shortly after, when he notices that some other students are adding additional hatch marks to their number lines, he inserts a second hatch mark to his line, and then decides that another name for Point A can be 'four-fifths.' In these two successive microgenetic constructions, this student has revised the geometric form of his representation, though he has used his new representation for the same mathematical function (assigning values to points based on counting hatch marks). Another student, also using a geometrical form, conceptualizes fractions on the line as equally partitioned intervals, and labels Point A initially as 'two thirds.' When asked to produce another name for Point A, she adds three new hatch marks at the midpoints between the existing hatch marks (Figure 13.4b) and uses them as a means to identify the equivalent name of 'four-sixths' (Figure 13.4b), again coordinating lexical with geometrical forms in the act of quantification. When a tablemate suggests to her that there are infinitely many names for Point A, she resists, arguing 'you can't fit in that many marks on the line.' Thus, while she extended her conception of equal partitions to produce 'four sixths,' she does not appreciate that the physical form can be treated as a representation of the *idea* that the partitioning process can be repeated indefinitely.

Both students used hatch marks as geometric means to identify a new name for the point, but their uses of the hatch marks served different functions – to add one additional mark and recount (Figure 13.4a), or, to partition the line between 0 and 1 into equal intervals and re-compute the relationship between parts and whole (Figure 13.4b). Nonetheless, in both cases, these children have generated new mathematical ideas in their microgenetic constructions.

Our investigations of the processes of microgenesis focus on students' uses of mathematical forms for particular functions. We select cases for analysis based on findings from ontogenetic analyses of change in form–function relations as well as sociogenetic analyses (next) of the case students' roles in communication, uptake and alteration of mathematical ideas.

Sociogenesis. Sociogenesis is the reproduction and alteration of ideas over time in the classroom community. When students are engaged in discussion or working jointly on a problem, they express their ideas in particular ways to help their listeners understand and appreciate what they are saying. Mathematical talk serves both personal and interpersonal functions, and, at times, what students present to others may not fully represent their understanding. At the same time, when students listen to others, the sense they make of what others are saying or writing may not be fully in accord with the speakers' intentions. Communicative and collaborative acts have unintended consequences, and, over time, classroom communities unwittingly sustain or alter ways of talking about mathematical topics and solving mathematical problems as they make efforts to communicate with representational forms (Croft 2000; Evans 2003; Keller 1994; Saxe & Esmonde 2005).

Investigation of sociogenetic propagation of ideas requires analyses of the social processes that shape communication and uptake of representational forms and the functions they serve. We use a suite of methods to document students' perceptions of other students' influence on their mathematical ideas, as well as patterns of communication, uptake and alteration of ideas over the course of the lesson.

Onto-, micro- and sociogenetic processes are intrinsically related in the travel of ideas. In the context of whole-class or small-group discussions, students may use the products of one another's microgenetic constructions, and in so doing, unwittingly participate in the reproduction and alteration of forms and functions in processes of sociogenesis. At the same time, the production and uptake of

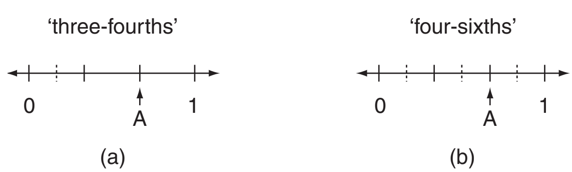

Figure 13.4 The products of two microgenetic constructions of equivalent names for Point A (the dotted hatch marks were added by students).

emerging mathematical ideas are enabled and constrained by the ontogenesis of representational activity. We situate our analyses of the travel of ideas in the classroom in the interplay of genetic processes as they emerge in the collective practices of students and teachers.

Empirical techniques

Our methods for investigating the travel of ideas in classroom communities are eclectic. We collect data from a wide range of sources, and use diverse methods of data reduction and integration to reveal onto-, micro- and sociogenetic processes. Figure 13.5 is a sketch of the data sources for the equivalent fractions lesson. Student worksheets and interviews provide our primary evidence for analysis of ontogenetic change in relations between forms and functions. Video of whole-class and small-group interactions as well as students' reports of the classmates who influenced their thinking provide primary evidence of sociogenetic processes. Case analyses of microgenetic processes draw from all data sources.

Ontogenetic analyses: shifts in form–function relations in student thinking over the course of the lesson

We coded students' worksheets to document shifts in the forms of representation that students used and the functions that they used the forms to serve in solving the Problem of the Day. Figure 13.6 is a summary of group results for the first and last worksheets (left and right panel, respectively). In each panel, the bars represent the forms students used in their solutions: *geometric* (e.g., using hatch marks to repartition the number line); *arithmetic* (e.g., multiplying a number by the fractional equivalent of 1 (2/2, 3/3, etc.) to create new names for Point A); *other* (such as words like 'percent' or 'decimal' that describe the type of representation). The partitions within each bar show the four functions that the forms served. *Non-normative* solutions could not lead to equivalent fraction names for Point A; *partial* solutions were incomplete procedures to generate equivalent fraction names; *procedural* solutions were appropriate steps to generate new fraction names but without an explanation of the logic of the procedure; *principled* consisted of a correct procedure as well as an explanation of its logic. These group results provide an initial description of cohort shifts in mathematical thinking, and set the context for coordinated analyses of microgenetic shifts in individual students' understandings of relationships between forms and functions (as we explain in a subsequent section).

On the first worksheet, the forms used by most students (17 of a class of 26) were coded as 'other,' because the forms were neither arithmetic nor geometric. A minority of students (9 of 26) used geometric forms (partitioning the line further) or arithmetic forms (multiplying the fraction identified on the line by another number), but only one of these students used these forms to serve principled functions. These group results from the first worksheet indicated considerable room for progress in the classroom. By the end of the lesson (Figure 13.6, right panel), the class shifted in the ways they accomplished the equivalence problem, both in the forms used and the functions that they used those forms to

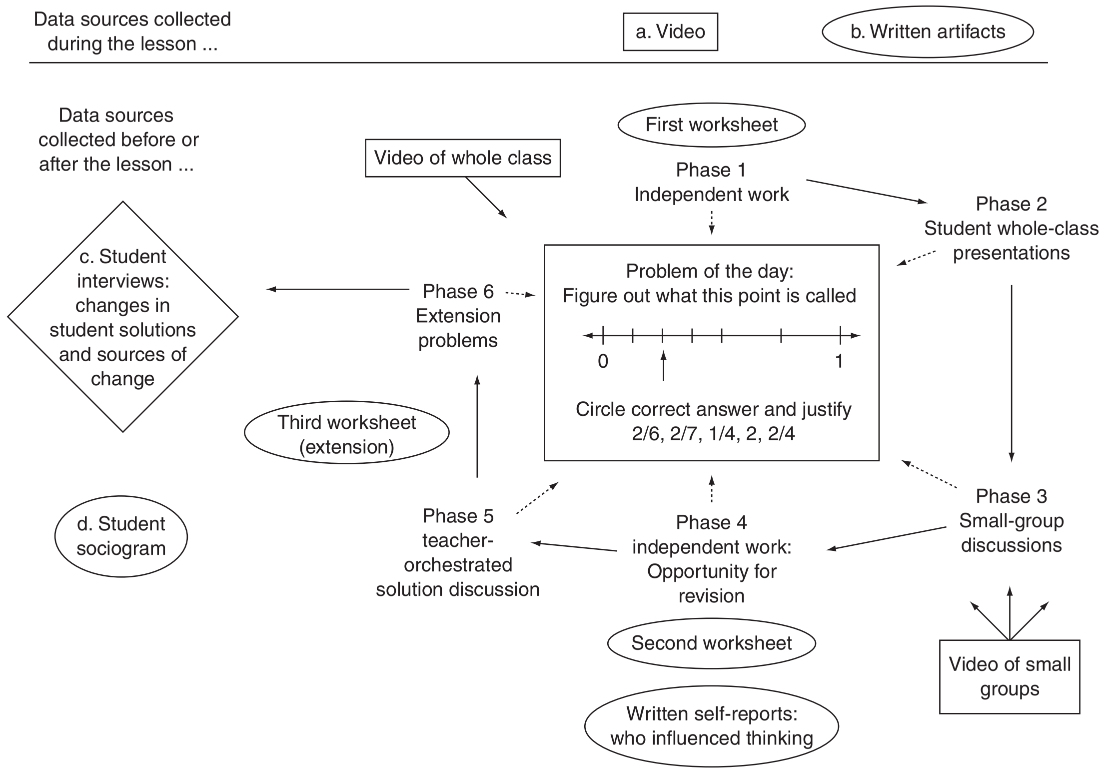

Figure 13.5 Data sources for studying the travel of ideas.

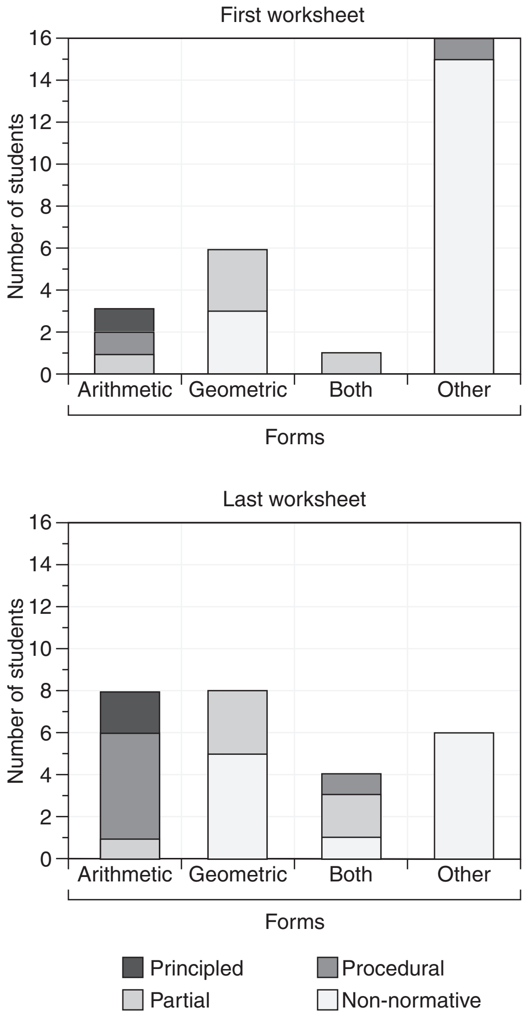

Figure 13.6 Distribution of students' uses of various forms and functions on first and last worksheets.

serve. Of 26 students, 20 used arithmetical and/or geometrical approaches, and, of those students, two used them in principled ways to serve normative functions, and six showed knowledge of procedures to accomplish the problem.

Group trends provide a summary portrait of ontogenetic change, while case analyses enable us to examine continuities and discontinuities in students' thinking over successive data points in the lesson. Each time that Damian, for example, solved an equivalence problem, the mathematical forms he used and the functions he used those forms to serve shifted.

Damian's initial conception of the problem on worksheet #1 was that there were two ways to name an equivalent fraction, 'decimal or percent.' That conception did not involve any approach to computation of equivalent fractions, and we coded the forms that he used as ‘other’ and the functions as ‘non-normative.’ After whole-class and small-group discussions, Damian shifted toward a normative definition of equivalent fractions in his second worksheet, responding that there are ‘more than 20 names for point A.’ In a debriefing interview, he explained that he added seven hatch marks between 0 and 1/3 on the number line (a geometric approach), named each of the new intervals between 0 and 1/3 ‘one-eighth,’ and then incorporated an arithmetic approach by multiplying one-eighth by one-third, yielding ‘one twenty-fourth’ as one example of an equivalent fraction. For this solution, we coded Damian as using both geometric and arithmetic forms (hatch marks and arithmetic multiplication with Hindu–Arabic numerals) for non-normative functions (using a procedure for which the function was unclear in relation to the problem). On the final worksheet and in the final debrief, Damian’s reasoning about equivalence was coded as ‘procedural,’ because he used computational procedures to generate correct equivalent fraction names without explication of principles. The task on the final worksheet was to identify how many names there were for the marked point of 2/5 (instead of 2/3). For the first time, Damian correctly identified the point and four equivalent fraction names; although he provided no explanation, his names were an arithmetic progression (2/5, 4/10, 8/20, 16/40) suggesting the use of an arithmetic procedure. In the debrief interview, Damian re-solved the initial Problem of the Day on worksheet #1 by writing on the number line from left to right, ‘1/3,’ ‘2/3,’ and ‘3/3,’ repartitioning the number line into six approximately equal intervals, and re-labeling the hatch marks as sixths.

Through pre-post comparisons of group results as well as case analyses, we document ways that students reconstruct relationships between mathematical forms and functions as they work independently and with others on equivalence problems. As we develop sequences of lessons, we will also develop techniques to follow ontogenetic progress over longer trajectories.

Microgenetic analyses: turning forms into means to serve mathematical functions

Each data point that we used in analyzing Damian’s ontogenetic progression over the course of a lesson constitutes a microgenetic construction, an occasion when Damian turned forms like number lines, fraction words, arithmetic procedures into particular meanings as he used them to cognize and accomplish goals in activity. Damian’s effort on his second worksheet is a useful illustration.

Damian’s answer on the second worksheet was that ‘there were more than 20 fractions’ for a point (2/3) on the given number line. Damian’s efforts to identify equivalent fractions for 2/3 were complex and protracted: His initial approach was through geometric partitioning, and he then applied an arithmetic multiplication to (his interpretation of) the results of his partitioning. In the geometric part, Damian began by inserting seven hatch marks between the points marked 0 and 1/3 on the number line, inadvertently creating his own unit, and then calling each of the resulting intervals ‘one-eighth.’ He appeared to count hatch marks and use the product of his count as the basis for naming the intervals ‘eighth’ and the sequence of points as ‘one-eighth,’ two-eighths,’ etc., thereby structuring lexical word forms and the intervals generated on the line into quantitative meanings in relation to one another. In the arithmetic and final part of his solution, Damian multiplied 1/3 by 1/8, and offered 'one twenty-fourth' as an example of one equivalent fraction for the labeled point (2/3). Here Damian was coordinating arithmetic procedures with lexical forms to produce a solution which was arithmetically correct, but unrelated to either the initial geometric number line representation or his transformation of it. Thus, as Damian was solving the class Problem of the Day, he was constructing goals that were his own, and his microgenetic constructions of relationships between forms and functions emerged in relation to his goals.

Students in Damian's class varied markedly in the ways that they made sense of the equivalence problem. While each student was using forms as means to construct solutions to the problem, their solutions were the products of their own microgenetic constructions. Consider the ways that three other students used lexical, geometric and/or arithmetic forms as means to solve the Problem of the Day on the first worksheet.

Figure 13.7 contains the overhead transparencies that the students created as they explained their solutions to the class. Sienna (left panel) used an arithmetic procedure, and she stated that there are infinitely many names, because 'you can keep multiplying the fraction by 2.' Daniel (middle panel) used a geometric form (hatch marks) to explain how to find one new name for the fraction 2/3; he partitioned each interval on the given line into half, determined that Point A could also be called four-sixths in addition to two-thirds, and claimed that there were no other names for Point A. Anabelle (right panel) used both arithmetic and geometric forms. She explained that one could either continue adding hatch marks or multiply both the numerator and denominator by 2, adding that doubling the number of hatch marks is the same as doubling the (numerator and denominator of the) fraction.

Sienna's arithmetic rule to multiply a fraction by 2 is – if interpreted literally – incorrect, because it cannot lead to the production of an equivalent fraction. But Sienna's actions and her verbalizations were not well-aligned, leaving us (and,

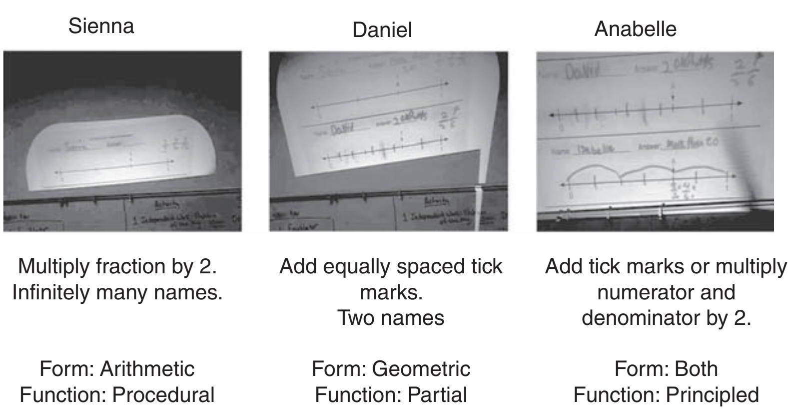

Figure 13.7 Ideas articulated by Sienna, Daniel and Anabelle in whole-class presentations (Phase 2).

most likely, Sienna) somewhat uncertain of her understanding; in action, she appropriately doubled both the numerator and denominator and indicated that the act could be repeated again and again. In Daniel's geometric construction, he showed limited understanding that the partitioning could be repeated indefinitely, appearing constrained by the geometric representation of the fraction on the line, a hurdle that did not occur with Sienna's arithmetic approach. Annabelle cited both an arithmetic multiplication and a geometric partitioning as ways to show that there is an infinite number of equivalent fractions, though she did not articulate the relationship between the two approaches.

In each of their microgenetic constructions, these three students were structuring representational forms to accomplish goals that they constructed in their efforts to solve the Problem of the Day. These students brought varied resources to their work, and they constructed varied insights. As we will show next, their public presentations of their mathematical ideas then became resources for other students when they met in small groups to negotiate consensus on the correct answer.

Sociogenetic analyses: propagation of forms and functions in the classroom community

Sienna, Daniel and Anabelle presented their work to the class, and we investigated if and how the activity of presenting ideas led them to alter how they represented their solutions. How were the forms the presenters introduced and their mathematical functions taken up by other students? The study of sociogenesis focuses on the reproduction and alteration of ideas as they propagate through communications within a community.

To investigate the propagation of ideas in the classroom, we drew upon multiple data sources. Sociograms of students' preferred tablemates for math work collected prior to the lesson provided evidence of students' initial social positions. Videotapes of social interactions in small-group and whole-class discussions were our resource for analyses of the give and take of students' microgenetic constructions from a sociogenetic perspective. Finally, students' reports of who influenced their thinking during the lesson (second worksheet and post-lesson debrief) served as evidence of the social organization and social influences within the classroom.

Figure 13.8 contains results of the initial sociogram in relation to students' reported influences on their thinking. The figure represents relationships between initial social positions and subsequent social influence on students' mathematical ideas.

In the sociogram (left panel), each student chose whom he/she wanted to sit with during math class; the left column represents the 'choosers,' the right column represents the 'chosen,' and the connecting lines students' choices. As indicated in the shaded cells, the three students who were chosen most frequently as desired tablemates for math work were Lyle, Jaquin and Pamela, suggesting that these three students had privileged status in this class as resources for mathematical ideas. The right panel connects students who reported being influenced and the students they identified as influential after the lesson. The

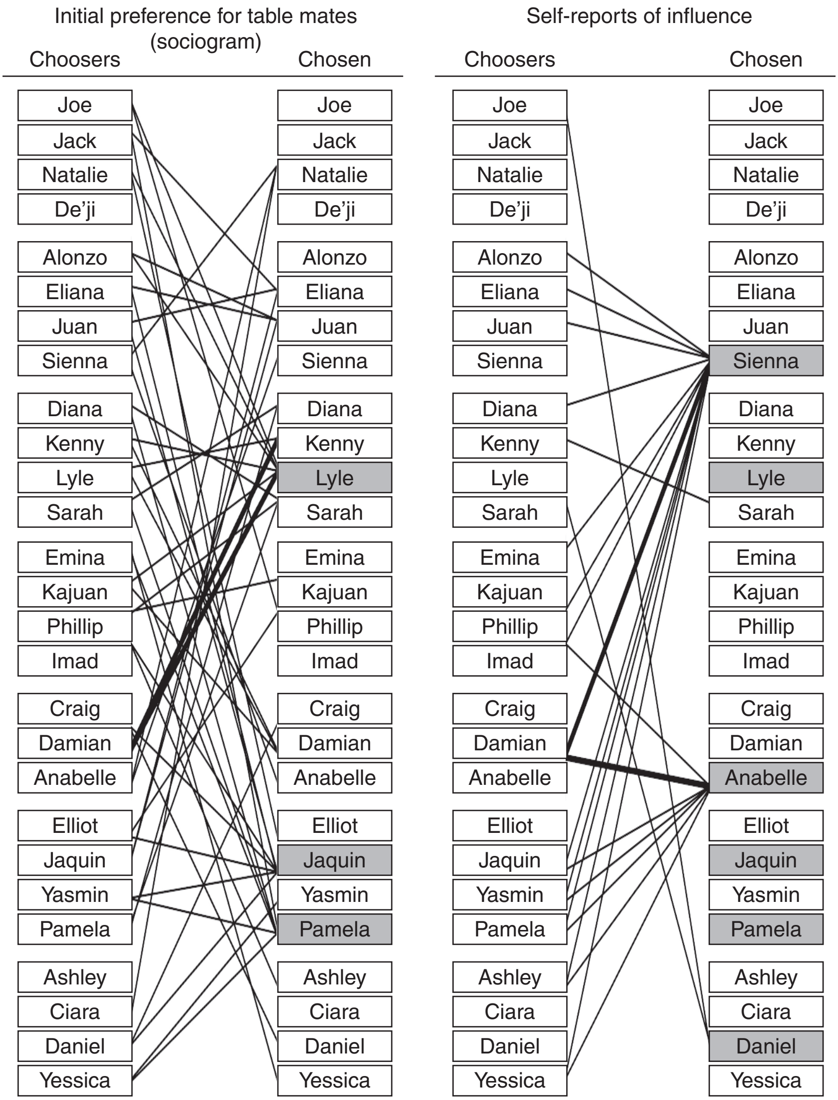

Figure 13.8 Students' preferred tablemates on the initial sociogram in relation to social influences reported on the second worksheet and final debrief.

noteworthy finding here is that the three students who were cited most frequently – Sienna, Anabelle and Daniel – were the three students who had made whole-class presentations (Figure 13.7), and none of these students was widely perceived before the lesson as valued tablemates. The discrepancy between preferred tablemates and self-reported influence suggests that students' presentations can have striking influence on the travel of mathematical ideas even when the presenters are not widely viewed as valuable members of small groups in math class. The value of these students to their classmates emerged within the context of the lesson, after they explained their mathematical thinking to the class and after students had the opportunity to continue working on the problem in collaboration with other students.

To examine the role of the presenters' ideas in the small groups after the presentations, we analyzed videotape of the group discussions. We illustrate our findings from one group shown in Figure 13.9. Damian, Anabelle and Craig were the participants, and you will see in the dialogue reproduced below how Craig rejected one of the presenter's ideas (Daniel's) and how Anabelle elaborated the ideas she had previously presented. In the excerpt, Damian is listening to Craig and Anabelle; Craig is reflecting on Daniel's solution that there are only two names, 'two thirds' and 'four sixths.'

CRAIG: [*Politely critical of Daniel's assertion*] I realized Daniel was close, but yeah…

ANABELLE: [*Interjects, explaining her own solution*] Like, see what I did was. At first I got two-thirds. And I was like, well that's … Then I got four-sixths. Well that would only be two names. And I was like, oh! If you times it by 2 you can get more.

CRAIG: [*Appearing to make an association with Anabelle's comments that there is more than one way ('times it' and partitioning), continues to reflect critically on Daniel's incomplete geometric approach, pointing out that it could be used to generate more names*] There are two ways. You either just keep doubling it or…. He [Daniel] doesn't understand. It doesn't matter if you can't fit it on your paper. It's still, like…

ANABELLE: [*Picks up on part of Craig's statement*] Yeah. I know. You can still double it.

CRAIG: [*Continues his critical reflection on Daniel's geometrical approach, commenting on making the number line 'bigger'*] Like, all you have to do is make it bigger [*gestures as if expanding the number line as he outstretches hands to either side as captured in Figure 13.9*]. [*In his debrief interview, Craig also talked about 'zooming in' on the number line, creating more space for hatch marks but conserving distance relations.*]

ANABELLE: [*Responds*] Yeah, you can either double it or times it. It works both ways. [*She is referring to 'doubling' (introduced by Craig in turn 3) as well as multiplying the numerator and denominator by the same value.*]

CRAIG: [*Still continuing his critique of Daniel's geometric approach, comments on the effects of repeated partitioning of the line*] You can actually just like make the dots, like…

ANABELLE: [*Follows up on Craig's ideas and takes up the partitioning approach*] If you have this number line, you can just keep going like, bam, bam, until you had, like marks like this small.

CRAIG: [*Attempts to interject a constraint on Anabelle's repeated partitioning expressed as 'bams'*] Yes but they all … [*Perhaps Craig is trying to add the constraint of equal spacing, but this is unclear.*]

ANABELLE: [*Interrupts Craig, arguing for the arithmetic approach over the geometric*] But then that would be more confusing than just, like, timesing it.

In this excerpt, Anabelle and Craig constructed a sequence of shifting microgenetic constructions in their abbreviated communications. It was often unclear what they were understanding at any given moment or what they intended to convey to one another. For example, what was Craig trying to convey in turn 5 when asserting that 'like all you have to do is make it bigger' (the stretching gesture captured in Figure 13.9)? Was it the idea that the number line can be stretched, preserving the ratio between hatch marks to create room for more hatch marks? Craig's talk and gesture may have been interpreted in any number of ways, and certainly the same can be said about Anabelle's abbreviated reference to arithmetic procedures. From a sociogenetic perspective, what we see here is how ideas like Daniel's may be devalued in interaction, and how ideas like Anabelle's may be brought forward. But the abbreviated nature of the exchanges, while perhaps interpretable to someone who already understands what is required for a solution to the Problem of the Day, may leave someone who is less informed uncertain about what is being asserted and why. Damian's worksheet #2, which was completed immediately following this interaction, suggests that Damian appreciated the conclusion of Anabelle and Craig (more than 20 equivalent fractions), but he took away the idea of partitioning and multiplication without rich understanding of ways they could be coordinated to solve the Problem of the Day.

To provide an index of the ways that students incorporated the presenters' mathematical ideas, we examined shifts from the first to the last worksheet for those 14 students who cited Sienna as the most influential. (Sienna was the most frequently identified as influential, and 14 students was a reasonable sample for analysis.) Figure 13.10 contains the form–function distributions for each worksheet. The results show that, even though these 14 students reported that Sienna's presentation was influential, only three students shifted to her use of arithmetic forms, and only two of these three shifted to her use of arithmetical forms for a procedural function. While the remaining 11 students reported being influenced by Sienna, they were clearly not imitating her solution on the final worksheet. We argue that Sienna's public presentation and the discussion of it were resources for students as they reconstructed their own understandings of equivalence.

Damian's shifts in thinking provide additional support for our claim that each student reconstructs relationships between mathematical forms and functions as

Figure 13.9 Small group composed of Damian (lower right), Craig (left), and Annabelle (upper right) during Phase 3 of the lesson, with Craig offering a conjecture with accompanying stretching gesture.

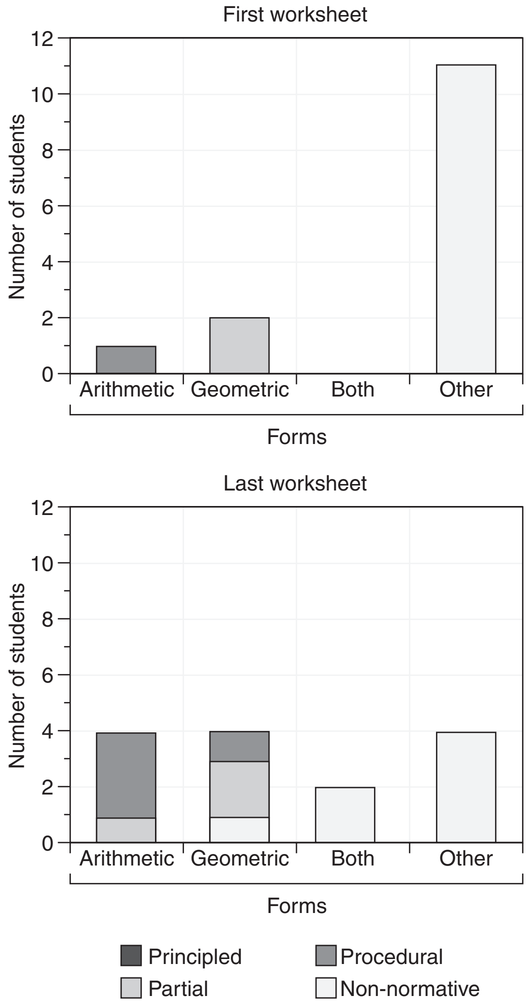

Figure 13.10 Distribution of forms and functions used on first and last worksheets by students citing Sienna.

he or she makes sense of other students' communications in relation to their own ideas. Damian cited Anabelle and Sienna as sources of influence on his thinking, and he did in fact shift to arithmetic and geometric procedures after their presentations and the small-group discussions. But Damian's use of arithmetic and geometric forms did not reflect the functions used by Anabelle or Sienna; he repartitioned only one portion of the number line (between 0 and 1/3), and he multiplied two values (1/3 and 1/8) that were different from those used by either of the presenters. We do not view Damian as misunderstanding Anabelle or Sienna (nor do we view Anabelle or Sienna as being insufficiently clear). Rather, we view Damian's solutions as microgenetic reconstruction of his own earlier ideas as he drew upon aspects of the presentations that he had just observed. Mathematical ideas travel as students make sense of them.

Concluding remarks

Developing methods to understand the travel of ideas is foundational to understanding learning in classroom communities, and we have argued that the genetic perspective on micro-, onto- and sociogenetic processes provides a useful methodological frame. But our approach is preliminary, and we hope that this chapter engenders productive conversation about methodological frameworks and empirical techniques for the study of learning in classrooms.

We are currently expanding the lesson series and conducting iterative cycles of classroom, interview and tutorial studies to guide refinement of lessons and support materials for teachers. In turn, the process of iterative refinement also serves as a laboratory for us as we refine our methodological approach and develop new empirical techniques.

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Chapter 14

A design research perspective on the identities that students are developing in mathematics classrooms

Paul Cobb, Melissa Gresalfi and Lynn Liao Hodge

Introduction

Our goal in this chapter is to present an interpretive scheme for documenting the identities that students are developing as they engage in (or resist) activities in particular mathematics classrooms. In the approach that we propose, the identities that students are developing can be made tractable for empirical analysis by documenting students' understandings and valuations of their classroom obligations. As is the case with any perspective on identity, this approach reflects a particular set of research concerns and interests. We clarify that the approach that we propose is grounded in the context of conducting design research at the classroom level. One of our primary concerns is therefore that analyses developed by using the interpretive scheme will feed back to inform the ongoing instructional design effort. An explicit focus on the identities that students are developing as doers of mathematics broadens the scope of classroom design research beyond analyses of students' mathematical reasoning by also considering the ways that students are coming to think about themselves in relation to mathematics, and the extent to which they are developing a commitment to and are coming to see value in mathematics as it is realized in the classroom. The notion of identity is pragmatically significant because it encompasses a range of issues that are typically subsumed under the heading of affective factors, including students' persistence, interest in and motivation to learn mathematics.

As a first step in clarifying what we mean by identity, we draw on the colloquial meaning of *identifying*: to associate or affiliate oneself closely with a person or group. Our concern is with both how students come to understand[1] what it means to do mathematics as it is realized in the classroom and whether and to what extent they come to identify with that activity. Analyses reported in the mathematics-education literature allow us to differentiate between three distinct cases: those in which students identify with classroom mathematical activity, merely cooperate with the teacher, or resist engaging in classroom mathematical activity, in the process developing oppositional identities (Boaler & Greeno 2000; Gutstein 2002, in press; Martin 2000). Prior investigations document that the extent to which students identify, merely cooperate, or resist can differ significantly from one classroom to another (Boaler 2000; Boaler & Greeno 2000; D'Amato 1992; Erickson 1992; Gutiérrez, Baquedano-Lopez & Tejeda 1999; Mehan, Hubbard & Villanueva 1994). These findings indicate the potential value of an interpretive scheme that enables us to analyze the relations between the microcultures established in particular classrooms and the identities that students are developing in those classrooms. To be useful, an interpretive approach should therefore attend to four interrelated issues: the nature of mathematical activity as it is realized in the classroom, what students make of that activity and both *whether* and *why* they come to identify with, merely comply with or resist engaging in that activity. As we will illustrate, the analytic scheme that we proposed in this chapter satisfies these requirements, thereby making the notion of identity as it relates to design at the classroom level both tractable and relatively concrete.

In the following paragraphs, we first outline the basic tenets of the classroom design experiment methodology and consider how the methodology might be elaborated when the intent is to support and understand students' identification with classroom mathematical activity as well as their development of significant mathematical ideas. We then present the interpretive scheme and clarify its key constructs. Finally, we place the interpretive scheme in a broader theoretical context by discussing its relation to alternative approaches that analyze the identities that students are developing across longer timescales, in the process taking account of issues of race, ethnicity and culture.

Classroom design experiments

A research team conducting a classroom experiment collaborates with a teacher, who might be a research-team member, to support a group of students' instruction for an extended period of time ranging from several weeks to an entire school year. In doing so, the research team develops instructional activities and associated resources such as computer tools that are used in the classroom to support students' learning. Given the strong pragmatic orientation of classroom design experiments, it is important to emphasize that the intent is to develop, test and refine theories, not merely to empirically tune 'what works.' These theories typically aim to account for learning processes in particular domains. For example, a research team working in the domain of geometry might aim to develop a design theory that is concerned with the students' learning of key disciplinary ideas in that domain.

One of the defining characteristics of the classroom design experiment methodology is that instructional design and research are interdependent.[2] On the one hand, the design of classroom learning environments serves as the context for research. On the other hand, ongoing and retrospective analyses are conducted in order to inform the ongoing instructional-design effort (Brown 1992; Collins, Joseph & Bielaczyc 2004; Edelson 2002; Gravemeijer 1994b). This type of research involves attempting to support students' development of particular ideas, and investigating students' actual learning as it is situated in the designed environment of the classroom. The overall intent is therefore to understand both the *process* of students' learning of specific disciplinary ideas and the means by which that learning is supported and organized.

The process of conducting a classroom design experiment consists of three broad phases: preparing for the experiment, experimenting to support students' learning and conducting retrospective analyses of the data generated in the course of the experiment. One of the challenges when preparing for an experiment is to explicate the aspects of students' learning and the classroom learning environment that will be the focus of investigation, and to distinguish them from other aspects that are either viewed as secondary or are assumed as background conditions. This specification of the investigative focus is critical because it is impossible to document everything that happens in a social setting as complex as a classroom. For the most part, design researchers in mathematics education (and other subject areas) have targeted students' development of domain-specific forms of reasoning, argumentation and tool use. In doing so, they acknowledge that it is important for students to come to identify with mathematical activity as it is realized in the design experiment classroom if the experiment to achieve its primary purpose of investigating the possibilities for students' mathematical learning. However, the process of supporting students' identification with classroom mathematical activity is typically addressed pragmatically while the experiment is in progress, and is treated as ancillary to the major research focus on students' domain-specific learning. Our agenda in developing the interpretive scheme reported in this chapter is to support the framing of students' identification with mathematical activity as an explicit focus of investigation in design research.

In the second phase of a classroom design experiment, working in a classroom to support students' learning, the objective is not to demonstrate that the instructional design formulated when preparing for the experiment works. The primary goal is not even to assess whether the initial design works, although the research team will necessarily do so. Instead, the purpose when experimenting to support learning is to improve the design by testing and revising conjectures inherent in the design about both the process of students' learning and the specific means of supporting it. This changes the question from 'Does this work?' to 'Why are things working (or not) and how does that support or challenge our initial assumptions?'. This approach of testing and revising conjectures is also central to an experiment in which the process of supporting students' identification with classroom mathematical activity is framed as an explicit focus of investigation. Testing and revising conjectures involves tightly integrated cycles of design and analysis in the course of which the research team conducts an initial analysis of each classroom session in order to plan for subsequent sessions (Brown 1992; Confrey & Lachance 2000). These initial analyses of classroom sessions involve ongoing interpretations of both students' activity and of the classroom learning environment that feed back to inform design and instructional decisions. The important point to emphasize is that these ongoing interpretations therefore profoundly influence the instructional design effort. This is the case whether the focus is on students' discipline-specific learning or on supporting their identification with classroom mathematical activity.

Given the complexity of the classroom, it is important to note that the ongoing interpretations are necessarily highly selective. This process of discrimination and selection frequently involves implicit, unarticulated assumptions about the aspects of students' activity and the classroom learning environment that are relevant to the analysis. In our view, it is critical to engage in the challenging task of explicating the key constructs used when making these interpretations so that underlying suppositions and assumptions are open to public scrutiny and critique (Cobb & Gravemeijer in press). The interpretive scheme that we report is the product of our attempt to address this challenge while conducting a design experiment in which one of the goals was to support and understand students' identification with classroom mathematical activity.

The final phase of classroom design experiment involves conducting retrospective analyses of the data generated in the course of the experiment. The ongoing analyses conducted while working with students in the classroom typically relate directly to the pragmatic goal of supporting this specific group of students' learning and their identification with classroom mathematical activity. Retrospective analyses seek to place this learning and identification in a broader theoretical context by framing them as paradigmatic cases of more encompassing phenomena. For example, an experiment that focuses on seventh-grade students' identification in a particular mathematical domain might aim to understand the process of supporting middle school students' identification with classroom mathematical activity more generally.

We will describe the types of data that might be collected to document the identities that students are developing when we discuss the key constructs of the interpretive framework. For the present, it suffices to note that the interpretive scheme that we propose was developed while conducting a retrospective analysis of a classroom design experiment conducted with a group of 11 eighth-grade students that focused on statistical data analysis. We should acknowledge that we did not frame students' identification as an explicit focus of investigation when we prepared for this experiment. However, the students' identification with classroom mathematical activity was an important pragmatic concern while the experiment was in progress and was a recurrent topic of discussion in the debriefing meetings conducted after each classroom session. There are strong indications that these efforts were successful as all 11 students had come to identify with statistical data analysis as it was realized in this classroom by the end of the 14-week experiment (Cobb, Gresalfi & Hodge in press; Hodge 2001). Data that we had generated for other purposes[3] allowed us to conduct a retrospective analysis of the identities that the students were developing in this classroom, in the course of which we developed the interpretive scheme.

Situating students' developing identities

Design researchers have expended considerable effort to develop useful ways of documenting students' mathematical learning as it is situated in the social context of the classroom (Barab & Squire 2004; Cobb & Yackel 1996; Design-Based Research Collaborative 2003; Edelson 2002; Gravemeijer 1994a; Hershkowitz & Schwarz 1999; Lehrer, Schauble & Penner 2000). The range of interpretive approaches that have been proposed are motivated by a variety of concerns and focus on differing aspects of students' mathematical activity. The various interpretive approaches share an important family resemblance in that they all delineate:

- successive patterns in the development of (aspects of) students' mathematical activity in a particular domain;
- critical aspects of the evolving classroom microculture that support the emergence of these successive patterns in activity.

Analogously, the interpretive scheme that we describe for documenting the identities that students are developing in mathematics classrooms delineates:

- critical aspects of the evolving classroom microculture with which students come to identify, merely comply, or resist;
- the personal identities that students are developing as they contribute to (or oppose) the regeneration of these aspects of the classroom microculture.

A strength of this approach is that it produces situated accounts of the personal identities that students are developing as doers of mathematics in particular classrooms. As a consequence, the resulting analyses can feed back to inform the instruction design effort because the identities that students are developing are directly related to key aspects of the social setting that support and, indeed, partially constitute their development. In presenting the interpretive scheme, we first outline the aspects of the classroom microculture that we take to be critical and then consider the personal identities that students are developing as they contribute to or oppose the regeneration of these aspects of the classroom microculture.

Delineating critical aspects of the classroom microculture

As we have clarified, the interpretive scheme was developed while conducting a retrospective analysis of a classroom experiment that focused on statistical data analysis. It proved important to focus on both the general and the specifically mathematical obligations that delineate the role of an effective student as it was constituted in the design experiment classroom. An obligation is a sociological construct and is closely related to the idea of norms. A norm can be defined as a recurrent pattern in joint activity that is regulated by the expectations that the teacher and students have for each other's actions in particular situations (Jackson 1966). Actions (including speech acts) that do not fit expectations are typically de-legitimized whereas those that fulfill expectations are, on occasion, met with explicit indications of approval (Searing 1991). This definition of norms explicitly recognizes that expectations are rarely uniform across the teacher and students (Chatman 1989; Jackson 1966). In addition, it acknowledges power differentials between the teacher and students as well as between students themselves by leaving open the likelihood that the censure and approval of the teacher and some students carry more weight than those of other students.

In this view, the classroom microculture comprises the expectations that the teacher and students have for others' actions, and the obligations that they attempt to fulfill (or resist) by acting in accord with expectations. Obligations are therefore the complement of expectations. In prior work, we have analyzed the classroom microculture by documenting both general obligations for classroom participation and obligations that are specific to action and interaction in mathematics classrooms (Yackel & Cobb 1996). As an illustration, a *general obligation* that became established in the classroom in which we conducted the statistics design experiment was that students should explain how they had completed data-analysis tasks. The teacher pressed the students to explain their reasoning during the initial lessons of the experiment and this subsequently became a taken-for-granted aspect of classroom life. A related *specifically mathematical obligation* that became established in this classroom concerned the standards that an explanation had to satisfy to be acceptable. As we document elsewhere (Cobb et al. in press), it was not sufficient for the students to present the graphical displays they had created of data sets; they also had to explain their reasons for creating particular displays when this was not readily apparent to other students. Typically, in stating these reasons, the students attempted to clarify why their data displays gave insight into the question or issue that motivated the data analysis. The evidence that this obligation had been established was relatively strong in that students increasingly pressed each other to give explanations of this type as the experiment progressed.

Taken together, an analysis of the general and specifically mathematical obligations established in a particular classroom documents the role of an effective mathematics student established in a particular classroom. This role does not therefore consist of a set of prescriptions that students are supposed to follow, but is instead reciprocally constituted in interaction and comprises the ways of acting that fulfilled others' expectations in a range of recurrent situations (cf. Jackson 1966).

Two aspects of students' general classroom obligations that proved to be important when analyzing the microculture established in the statistics design experiment classroom were the *distribution of authority*, and the ways that students were able to *exercise agency* (Engle & Conant 2002; Hull & Greeno 2002). The distribution of authority concerns the degree to which students are given opportunities to be involved in decision making about the interpretation of tasks, the reasonableness of solution methods and the legitimacy of solutions. Authority is therefore about 'who's in charge' in terms of making mathematical contributions. For example, in some classrooms authority is distributed only to the teacher, who is solely responsible for determining the legitimacy of student responses. In other classrooms, authority is distributed more broadly, with students and the teacher jointly determining the legitimacy of contributions by relying on mathematical argumentation.

The distribution of authority is inextricably linked to the ways that students are able to exercise agency in the classroom. Building on Pickering's (1995) distinction between conceptual and disciplinary practice, we differentiate between two different forms of agency that classroom practices can afford: *conceptual agency*, which involves engaging with content by choosing methods and developing meanings and relations between concepts and principles, and *disciplinary agency*, which involves using established solution methods (cf. Hull & Greeno, 2002; Gresalfi, Martin, Hand & Greeno in press). Classrooms in which authority is distributed to students and the teacher are unlikely to be effective in supporting mathematical learning unless there are opportunities for students to exercise conceptual agency (Greeno, Sommerfeld & Wiebe 2000). This is because students who have not had opportunities to exercise conceptual agency are not practiced at understanding whether or when particular kinds of disciplinary tools might be useful in solving problems. Consequently, when the teacher attempts to distribute authority by asking students to determine whether or not a solution is correct, they have had little experience in using mathematical tools to justify or refute particular claims. In contrast, classrooms in which authority rests solely with the teacher typically but not inevitably offer students opportunities to exercise only disciplinary agency.

The specifically mathematical obligations that proved to be important when documenting the role of an effective mathematics student established in the statistics design experiment classroom were the norms or standards for mathematical argumentation and normative ways of reasoning with tools and written symbols. As an illustration of the latter type of obligation, the students in the classroom in which we worked reasoned with graphical displays of data sets both when solving tasks and when explaining their solutions. Normative ways of reasoning with data displays in this classroom involved identifying trends or patterns that give insight into the phenomenon under investigation. This was indicated by the way in which analyses that involved the same data display were consistently treated as different if the students identified different patterns and developed different insights. Taken together, norms for mathematical argumentation and normative ways of reasoning with tools and written symbols delineate what counts as mathematical competence in a particular classroom (cf. Lampert 1990). Thus, the analysis of students' general classroom obligations documents both the types of agency that students are able to exercise and *to whom* students are accountable as they do so (cf. Gresalfi et al. in press). The analysis of students' specifically mathematical obligations clarifies *what* students are accountable for mathematically as they exercise those types of agency.

The interpretive approach as we have outlined it thus far assumes that the classroom discourse constituted by the teacher and students is relatively unified. However, this is not the case when a significant proportion of students are resisting and appear to be developing oppositional personal identities. In such cases, we would follow Gutierrez, Rymes and Larson (1995) in differentiating between the official classroom discourse constituted by the teacher and cooperating students and the oppositional discourse constituted primarily by resisting students. We would then document the role of an effective doer of mathematics constituted in official classroom discourse by analyzing the general and specifically mathematical obligations that students would have to fulfill to participate effectively in this discourse.

In the case both of classrooms where some students are resisting and of classrooms in which there is no oppositional discourse, an analysis of general and specifically mathematical obligations specifies the role of an effective mathematics student. We call this role the *normative identity* established in a particular classroom because students would have to identify with this role in order to develop a sense of affiliation with mathematical activity as it is realized in that classroom.[4] In the case of the classroom in which we worked, the students would have had to identify with a form of mathematical activity in which they could exercise conceptual agency as they created and interpreted data displays in order to identify trends and patterns that gave rise to insights into the phenomenon under investigation. The various facets of the normative identity as a doer of mathematics established in a particular classroom are summarized in Figure 14.1.

A precursor of the construct of normative identity can be seen in the work of Boaler and Greeno (2000) when they speak of the identity that the high school mathematics students whom they studied would have to adopt in order to become what they term 'mathematical persons.' The crucial point to note is that the term 'identity,' as Boaler and Greeno use it here, does not refer to how the students viewed or described themselves. Instead, it refers to that with which the students would have to identify to develop a sense of affiliation with classroom mathematical activity, namely the role of an effective or competent mathematics student as it was constituted in their classrooms.

To avoid possible misinterpretation, it is important to emphasize that normative identity is a collective or communal construct rather than an individualistic notion. It should therefore not be confused with consensus, agreement, or being of like mind. Consensus and agreement are individualistic notions that concern the overlap or fit in the teacher's and students' individual interpretations and valuations. However, it is quite possible that some of the students in a particular class might come to identify with their classroom obligations, whereas others might merely cooperate with the teacher, and still others actively resist engaging in classroom activities.[5] An analysis of the normative identity established in particular classrooms is useful even in such cases because it enables us to pin down with some precision *what* some students are identifying with and others are resisting. A complementary analysis of the personal identities that individual students are developing enables us to understand *why* the students are making these different valuations of their classroom obligations.

As a second, related point of clarification, it is important to note that students' general and specifically mathematical obligations are constituted in the course of the ongoing classroom interactions (Schutz 1962; Simon & Blume 1996; Voigt

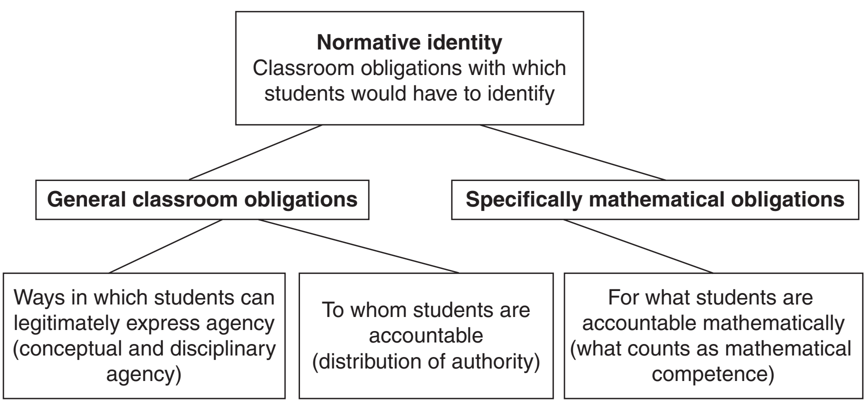

Figure 14.1 Facets of the normative identity as a doer of mathematics established in a particular classroom.

1995; Yackel & Cobb 1996). This is the case even in classrooms where the teacher is solely responsible for determining the legitimacy of students' contributions (Cobb, Jaworski & Presmeg 2000). We stress this point to ward off the interpretation that the teacher invites students to adopt a normative identity as a doer of mathematics that has been established independently of the students' participation. Instead, students who cooperate with the teacher contribute to the initial constitution and ongoing regeneration of the normative identity as a doer of mathematics established in their classroom (cf. Cooney 1985).

The data generated in the course of the design experiment reported here included video recordings of all classroom sessions. These data proved to be adequate for the purpose of documenting the normative identity as a doer of mathematics established in this classroom. As classroom video recordings are a routine aspect of most classroom design experiments, the cost of entry for documenting normative identity is relatively modest. We have described the types of evidence that we use to determine whether a particular obligation has been established in some detail elsewhere (Cobb, Stephan, McClain & Gravemeijer 2001). For our current purposes, it suffices to note that analyses of this type involve identifying patterns or regularities in the teacher's and students' ongoing interactions, and specifying students' obligations as they contribute to the enactment of these patterns (Bauersfeld 1980; Voigt 1985). The conjectures that are substantiated or refuted in the course of an analysis therefore concern whether student actions (including speech acts) are constituted as legitimate in that they fulfill the teacher's and other students' expectations.

As a final observation, it is worth noting that prior analyses of the identities that students are developing in mathematics classrooms have typically been relatively global and are not specific to action and interaction in mathematics classrooms. This is the case for even the most significant contributions to the literature. Our relatively fine-grained focus on the obligations that students' would have to fulfill to be constituted as effective doers of mathematics in a particular classroom go beyond sweeping characterizations of classrooms as traditional or reform in nature. In doing so, the interpretive approach follows Apple (1992) and Delpit (1986) by subjecting to scrutiny the claim that instruction consistent with current reform recommendations inevitably results in students coming to identify with mathematical activity as it is realized in their classroom. Studies conducted by Murrell (1999) and Lubienski (1997, in press) document the limited engagement of certain groups of students in reform classrooms, thereby demonstrating the importance of analyzing the classroom microculture at what Boaler (2002) terms an appropriate level of detail. Differences in the general and specifically mathematical obligations that we have discussed make a difference for the improvement of designs developed to support both students' identification with classroom mathematical activity and their learning of the central ideas in a particular mathematical domain.

Documenting personal identity

The construct of *personal identity* concerns the extent to which students identify with the role of an effective doer of mathematics constituted in the classroom, merely cooperate with the teacher, or resist engaging in classroom activities. Holland, Skinner, Lachicotte and Cain (1998) describe identification as a process whereby communal activities 'in which one has been acting according to the directions of others becomes a world that one uses to understand and organize aspects of one's self and at least some of one's own feelings and thoughts' (p. 121). In the case at hand, the world that students who identify with classroom mathematical activity make their own is structured in terms of general and specifically mathematical expectations and obligations. In the process of identifying, these students turn obligations-to-others into obligations-to-oneself.[6] In contrast, they remain obligations-to-others for students who merely cooperate with the teacher, and become obligations-for-others but not for oneself for students who resist participating in classroom activities.

To make the notion of personal identity a useful tool for empirical analysis, we follow Chatman (1989), Sfard (2000) and Toury (1995) in noting that normative ways of acting are not mere conventions that can be modified at will. Instead, these ways of acting are value-laden both for students who identify with classroom mathematical activity and for students who are willing to cooperate with the teacher.[7] These students' understanding of their general and specifically mathematical obligations involves a sense of 'oughtness' about what they should do (Hicks 1996; Linehan & McCarthy 2000). This is the case both when classroom obligations have become obligations-to-oneself and when they remain obligations-to-others.[8]

This moral dimension of the classroom provides us with a useful point of reference as we make the notion of personal identity tractable for empirical analysis. Specifically, it indicates the importance of documenting students' understandings of their general classroom obligations, their valuations or appraisals of these obligations, and the grounds for their valuations. Attending to the grounds for students' evaluations is important because we want to understand not merely whether but why students have come to identify with their classroom obligations, are merely cooperating with the teacher, or are developing oppositional identities. The analysis of students' views about their general classroom obligations documents their understandings and valuations of the ways in which they can legitimately exercise agency and the way in which authority is distributed in their classroom (see Figure 14.1). What gets constructed as mathematical competence in the classroom has implications for students' perceptions of themselves and their peers' relative capabilities, and thus for issues of status and power in the classroom. In documenting the personal identities that students are developing in a particular classroom it is therefore also essential to take account of their assessments of their own and others' mathematical capabilities.

Finally, in cases where a significant proportion of students resist engaging in classroom activities, it would also be important to ascertain students' obligations to and valuations of the activity of students who are contributing to oppositional discourse. In the case of the resisting students, we would not be surprised if their understanding of official classroom obligations differed from those of cooperating students. The resulting analysis would explain why resisting students are developing an affiliation with the oppositional discourse rather than with mathematical activity as it is realized in the classroom. The explanation could in turn inform the development of testable conjectures about how the classroom microculture and practices might be modified to support these students' engagement and learning.

As an illustration of an analysis of the personal identities that students are developing in a particular classroom, we again turn to the design experiment classroom. The students in this class understood their primary obligation to be identifying trends and patterns in data that give rise to insights into the phenomenon under investigation (Cobb et al. in press). There was strong evidence that the students had come to value gaining insights into issues by analyzing data, and therefore viewed the criterion for what counted as an acceptable solution as reasonable. It was also apparent that the students experienced authority as being distributed relatively widely. The pattern that emerged was that they viewed both their general and their specifically mathematical obligations as sensible and reasonable in this class. The vehemence with which they talked about the importance of fulfilling some of the obligations that defined the role of an effective mathematics student in this classroom indicates that these obligations were no longer merely directed toward others but were becoming obligations-to-oneself. In the process, the students were developing an affiliation with mathematical activity as it was realized in this classroom. In Wenger's (1998) and Nasir's (2002) terms, they were coming to view themselves as having the ability, facility and legitimacy to contribute to, take responsibility for and shape the meanings that mattered in the design experiment classroom.

We identified the various aspects of personal identities that the students were developing in the design experiment class by analyzing data generated in the course of the experiment, using open-coding with an ongoing formulation and refinement of categories (Strauss & Corbin 1990). These data consisted of audio-recorded interviews conducted with the participating 11 students. The interviews were conducted at the school while the experiment was in progress and each student was interviewed at least twice. The students were typically interviewed in pairs or groups of three because pilot work indicated that group interviews elicited richer responses than did interviews conducted with individual students (Hodge 2001). In keeping with the purpose of the interviews, the questions that were posed focused on the students' views about and appraisals of how the classroom 'worked,' rather than on interpretations of specific classroom incidents. The interviewer therefore asked students what they would tell a student who was new to the class that he or she needed to do to be successful, what they had learned in the class, whether they were a good student in this class, and so forth.

As Sfard (2002) notes, the ways that people talk about their engagement in specific settings are influenced by the immediate communicational context. This observation serves to emphasize that an interview is a social event in which the interviewer and interviewees present themselves to and recognize each other in particular ways (cf. Misher 1986). As a precaution, we therefore ensured that the interviewer was not involved in either conducting or video recording the classroom sessions during the statistics design experiment. The interviewer introduced herself to the students by explaining that she wanted to better understand their views of the lessons and the instructional activities in the design experiment classroom. This strategy appears to have been effective in that the students did make a number of negative observations about the design experiment class (Hodge 2001).

Small group interviews of this type are not a routine part of classroom design experiments. Their inclusion therefore constitutes an extension of the methodology for the purpose of documenting the personal identities that students are developing as they participate in or resist engaging in classroom activities. In our experience, the findings can be both surprising and revealing even for research-team members who observed the classroom sessions and have analyzed the video recordings of these sessions. In the case of the statistics design experiment, for example, most members of the research team focused on the significant differences in the sophistication of the students' statistical reasoning when designing instructional activities. We were therefore surprised when the students all indicated that they viewed themselves as successful in this classroom, and when most also indicated that they also viewed all the other students as 'smart.' To understand this finding, we had to take account of the students' understanding of what counted as mathematical competence in this classroom, namely to make contributions to whole-class discussions that were constituted as significant by the teacher and other students. By this criterion, all the students were indeed effective in this classroom. More generally, interviews of the type that we have outlined are critical in distinguishing between cases in which students are merely cooperating with the teacher and those in which they are coming to identify with the role of an effective doer of mathematics established in the classroom (Boaler & Greeno 2000). Furthermore, an analysis of the personal identities that individual students are developing, when combined with an analysis of the normative identity established in their classrooms, allows us to understand *why* the students are coming to identify with, merely comply with, or resist their classroom obligations at a level of detail that is adequate for the purposes of design research.

Discussion

Our purpose in this chapter has been to propose an interpretive scheme that focuses explicitly on the relation between the microcultures established in particular classrooms and personal identities that students are developing in those classrooms. An analysis of the normative identity established in a particular classroom documents the general and specifically mathematical obligations that delineate the role of an effective mathematics student constituted in that classroom. Students would have to identify with this role in order to develop a sense of affiliation with mathematical activity as it is realized in the classroom. A complementary analysis of the personal identities that individual students are developing documents both whether and why they are developing an affiliation with, merely complying with, or resist engaging in classroom mathematical activity.

As we have clarified, the notion of personal identity concerns students' views about and appraisals of how the classroom 'works' rather than their interpretations of specific classroom incidents. This formulation has the advantage of making it relatively straightforward to coordinate an analysis of the personal identities that students are developing in a particular classroom with an analysis of the normative identity as a doer of mathematics established in that classroom. As a consequence of this coordination, the resulting accounts of personal identities that students are developing can be directly related to key aspects of the classroom microculture that constitutes the immediate context of their development. This relation and the focus on specifically mathematical aspects of the classroom microculture make the resulting analyses useful for the purposes of design research.

The interpretive scheme that we have described focuses squarely on the personal identities as doers of mathematics that students are developing as they participate in or resist engaging in classroom activities. It might therefore be objected that the interpretive scheme is limited in scope because it fails to take account both of the development of students' personal identities as they move through a series of mathematics classrooms in the course of their school careers and of issues of race, ethnicity and culture. We situate the potential contributions of the interpretive scheme in a broader theoretical context by sketching its relation to alternative, largely complementary approaches that analyze the identities that students are developing across longer timescales.

The notion of personal identity as we have defined it concerns students' understandings and appraisals of their classroom obligations. However, as Horn (2006) notes, students might encounter significantly different classroom obligations as they move from one mathematics class to the next. This observation leads Horn to argue for the importance of looking at the personal identities that students are developing across as well as within mathematics classes. To this end, she analyzed students' trajectories through sequences of courses that defined the 4-year mathematics curricula in two contrasting high schools in the United States. In one school, the mathematics teachers worked collectively for all students' academic success in a rigorous de-tracked common sequence of courses, whereas in the second school the curriculum was tracked and the mathematics teachers did not believe that all students could succeed academically. Horn demonstrates convincingly that the organization of the 4-year mathematics curriculum can profoundly affect the personal identities that students are developing as doers of mathematics. As she clarifies, the official, institutional discourse constituted in a particular school associates particular positions in the curriculum with particular types of students.[9] This institutional discourse includes assumptions about both what are reasonable goals for students in particular positions in the curriculum, and what counts as mathematical competence for students in these positions. Horn's analysis is of value because she goes inside studies that document both how particular curriculum structures distribute opportunities to learn mathematics unequally in ways that correspond to race and class (Oakes & Rogers 2003), and how teachers and schools can work to minimize these inequities (cf. Gutiérrez 2000).

Horn characterizes the way in which the 4-year mathematics curriculum is organized in a particular school and the institutionalized meanings of that organization as tools or resources for identity development. For example, students' location in a curriculum (e.g., course and track) might influence their understandings of themselves as more or less mathematically competent. However, Horn also demonstrates that students' placement in a particular track can have different meanings in different mathematics classes in the same school. This implies that the institutionalized meanings of positions in the curriculum touch experience as students participate in specific activities in particular classrooms (cf. Wenger 1998). We therefore view Horn's work and the analytic scheme that we have proposed as complementary. Horn delineates an important set of resources on which students might draw as they come to view their position in the school curriculum as more or less privileged, and themselves as more or less mathematically competent. The interpretive scheme documents both the classroom social setting in which these resources come to have particular meanings for students, and the personal identities that students are developing in those settings.

Martin's (2000) work extends the timescale still further by taking account of socio-historical and community processes. Martin's goal is to understand why some African-American students in the United States succeed in mathematics whereas others fail. The socio-historical level of his analytic approach focuses on historically based discriminatory policies and practices that have prevented African-Americans from becoming substantial participants in mathematics. The community level of his approach documents 'how the historical legacy of these practices is brought to life in the narratives of African-American parents and community members – narratives characterized by repeated references to beliefs about differential treatment in mathematics related contexts' (pp. 30–31). As he clarifies, 'African-American parents and community members respond to their experiences in ways that send implicit and explicit messages – positive and negative – about the importance of mathematics learning and knowledge to their children' (p. 38). These messages in turn influence how children come to perceive activities in their mathematics classrooms. In our view, Martin convincingly demonstrates the importance of investigating how historically contingent cultural meanings about mathematics and schooling among particular groups within society affect students' learning and persistence in mathematics classrooms.

The next level of Martin's approach focuses primarily on the negotiation of general classroom norms in relatively global terms and is not specific to action and interaction in mathematics classrooms. In our view, the construct of normative identity elaborates this aspect of Martin's approach at a level of detail that is adequate for the purposes of design research. The final level of what Martin proposes is intrapersonal in focus and makes contact with the notion of personal identity as we have defined it. As we noted, the formulation of personal identity that we have outlined has the advantage of making it relatively straightforward to relate an analysis of the personal identities that students are developing in a particular classroom to an analysis of the normative identity as a doer of mathematics established in that classroom. A design research study that capitalizes on Martin's work by attending to socio-historical and communal processes would also consider how messages from parents and community members influence students' understandings and valuations of their classroom obligations.

Sfard and Prusak's (2005) narrative approach to identity is relevant to our discussion because it also focuses on 'the complex dialectic between learning and its sociocultural context' (p. 15). Sfard and Prusak argue that 'identities may be defined as collections of stories about persons' (p. 16). Their perspective on identity therefore makes contact with Martin's (2000) socio-historical and community levels of analysis. This is particularly the case given the importance that Martin attributes to narratives or stories told by parents that reflect the historical legacy of their community. A presentation of Sfard and Prusak's sophisticated analytical approach is beyond the scope of this chapter. We would, however, note that the analytical tools they describe appear to hold considerable promise for linking culture and learning. In addition to making this important contribution, Sfard and Prusak's narrative view of identity is of interest to us for two further reasons, one theoretical and the other pragmatic.

Our theoretical interest in Sfard and Prusak's work stems from a broad distinction that can be made in the general literature on identity between approaches that analyze identity in terms of position or location and those that analyze it in terms of stories or narratives (Holland et al. 1998). Sfard and Prusak's work advances narrative approaches to identity. In contrast, the interpretive scheme that we have reported is a positional approach that is specifically tailored to the requirements of design research. The use of this scheme involves documenting students' general and specifically mathematical obligations. It is with respect to the obligations constituted in the classroom that students' forms of engagement are positioned as acceptable, insightful, illegitimate and oppositional (cf. Linehan & McCarthy 2000).

We see considerable merit in Holland et al.'s (1998) and Horn's (2006) argument that positional and narrative approaches are potentially complementary. For example, Martin (2000) and Sfard and Prusak (2005) both demonstrate that students are told particular types of stories about themselves by parents and peers as a consequence of their membership in particular cultural groups and communities. Sfard and Prusak contend that students construct and reconstruct their identities by incorporating such stories about them told by significant others. For her part, Horn (2006) clarifies how students' positions within a sequence of school mathematics classes and the institutional meanings of those positions result in them being told particular types of stories about themselves as mathematics students. In our view, the illustrations we have given from Martin's and Horn's work emphasize the importance of position even to narrative treatments of identity. Taken together, these studies indicate that the types of stories that students hear about themselves are constrained by their positions at the community, school and classroom levels (cf. Willis 1977).

Our pragmatic interest in Sfard and Prusak's work stems from the distinction that they draw between what they term actual identity and designated identity:

> significant narratives about a person can be split into two subsets: *actual identity*, consisting of stories about the actual state of affairs, and *designated identity*, consisting of narratives presenting a state of affairs which, for one reason or another, is *expected* to be the case, if not now then in the future.
>
> (p. 18; italics original)

Sfard and Prusak go on to argue that a person's designated identity gives direction to actions and influences deeds. They also assert that designated identities are created, to a significant extent, by converting narratives about possible futures into the first person. Against the background of this claim, we came to realize that, in the statistics design experiment, we made few if any attempts to develop narratives with the students that indicated future opportunities would be made possible by their growing mathematical competence. In other words, we did not explicitly support the students in envisioning future trajectories that extended beyond the design experiment class. We regard this as an unfortunate oversight given Sfard and Prusak's argument that substantial learning occurs as students attempt to close a perceived gap between their actual and designated identities. To be sure, we were able to cultivate students' interest in statistical data analysis by supporting their identification with classroom mathematical activity. However, we now wonder about the extent to which the students' participation in the design experiment affected their future school mathematics careers.

In our view, Sfard and Prusak's analysis of designated identity relates directly to issues of equity in students' access to significant mathematical ideas. This becomes apparent when we follow D'Amato (1992) in distinguishing between two ways in which learning in school in general, and learning mathematics in particular, can have value to students. D'Amato refers to the first of these ways as extrinsic value or *structural significance* in that achievement in school has instrumental value as a means of attaining other ends such as entry to college and high-status careers, or acceptance and approval in the household and other social networks. D'Amato contrasts this source of value with what he terms intrinsic value or *situational significance* in which students view their engagement in classroom activities as a means of maintaining valued relationships with their teacher and peers, and of gaining access to experiences of mastery and accomplishment. The interpretive scheme we have reported is useful in guiding efforts to support students' identification with classroom mathematical activity, and thus their development of a situational rationale for learning mathematics in a particular class. However, it has little to say about giving students access to a structural rationale that can influence their learning of mathematics in the longer term. Consequently, it is, by itself, insufficient to address a significant inequity, namely that students' access to a structural rationale varies as a consequence of family history, race or ethnic history, class structure and caste structure within society (D'Amato 1992; Erickson 1992; Mehan et al. 1994).

It is in this regard that the potential contribution of Sfard and Prusak's notion of designated identity becomes apparent. This construct orients us to consider the extent to which students are told stories that support their development of structural rationale for learning mathematics in school. We should immediately clarify that we hope these stories would do more than frame mathematics achievement as a credit that can be cashed in for future educational and economic opportunities. They might instead project students' future participation in particular occupational or societal practices that would be made possible by their development of particular forms of mathematical competence.

It is also important to clarify that, in our view, mere telling of stories will not, by itself, be sufficient. Sfard and Prusak stress that stories have to be endorsable in that students would, if asked, say that they faithfully reflect the state of affairs in the world. The world in question concerns students' experience of participating in classroom mathematical activity and the sense of competence that they develop as they do so. To be endorsable, narratives that the teacher might develop with students about their possible futures have to fit with the world as students experience it. This requirement brings to the fore the central aspects of the personal identities that students are developing in the classroom as we have defined them: students' understandings and valuations of their general and specifically mathematical obligations, and their assessments of their own and their peers' mathematical competence. For this reason, we consider the interpretive scheme we have proposed and Sfard and Prusak's narrative treatment of identity to be complementary. More generally, the complementary relation between positional and narrative approaches to identity is captured by Bruner's (1987) observations that language and the world are mutually constitutive: the world that we constitute through language constrains what we can say about it.

Conclusion

As we clarified at the outset, our overall goal in this chapter has been to propose an approach to identity that is tailored to the purposes of design research. We have placed the interpretative scheme in theoretical context by discussing its relation to alternative approaches that take account of longer timescales. These relations indicate how the interpretive scheme might contribute to a program of research that addresses issues of design at the classroom level while simultaneously attending to both the structure of the school mathematics curriculum and to the practices of students' home communities. In addition, we saw that Sfard and Prusak's narrative treatment of identity has implications for the formulation of designs for supporting students' learning. In our view, one of the pressing problems for future research is to investigate how alternative analytic approaches can be coordinated and reconciled to develop empirical analyses that are comprehensive yet sufficiently detailed for our purposes as mathematics educators. The relations we have identified between different approaches indicate that classroom design experiments constitute a relevant set of contexts in which to address this problem.

Acknowledgments

The analysis reported in this chapter was supported by the National Science Foundation under grant Nos. ESI 0554535 and REC 9902982. The opinions expressed do not necessarily reflect the views of the Foundation.

Notes

1. This understanding includes implicit understanding-in-action (Schön 1986; Searle 1983) as well as explicit understanding.
2. The basic tenets of design research that serve to differentiate it from other methodologies have been discussed in some detail by Cobb, Confrey, diSessa et al. (2003) and Design-Based Research Collaborative (2003).
3. One of these other purposes was to identify and analyze situations in which individual students perceived themselves to have been silenced.
4. In colloquial terms, one might say that students would have to identify with the 'type of person' for whom the obligations have become personal values.

5. Here and elsewhere in this chapter, resistance refers to acts that are constituted as oppositional in the classroom. As Gutiérrez et al. (1999) demonstrate, the teacher might attempt to appropriate an act of resistance, in the process reframing it as a contribution to official classroom discourse.
6. This formulation of turning obligations-to-others into obligations-to-oneself directly parallels Sfard's (2006) argument that learning involves turning discourse-for-others into discourse-for-oneself. The account we have given of the process of identifying is consistent with Sfard's participationist viewpoint, the basic tenet of which is that 'patterned, collective forms of distinctly human forms of doing are developmentally prior to the activities of the individual' (p. 157).
7. The value-laden nature of classroom norms is evidenced by the challenges that teachers frequently encounter when they attempt to renegotiate classroom norms even when students do not find current forms of mathematical activity engaging.
8. This moral dimension of the classroom is apparent in the teacher's and students' sometimes vociferous responses when they perceive that a norm has been breached.
9. In Gee's (2001) terms, these institutional categories constitute the institutional identities of students who occupy particular positions in the curriculum.

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Chapter 15

Methodological considerations for the study of intersubjectivity among participants of a dialogic mathematics classroom

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The importance of intersubjectivity for the study of socially constructed knowledge

The study of the nature of social interaction and its role in the construction of knowledge is both daunting and alluring. Models of behavior and learning often stem from investigations of autonomous performance. Yet it becomes clear early in the investigations of socially mediated learning that models based on data collected at the level of the individual do not readily scale up to the group level. It is also the case that the behaviors of individuals in a social context do not behave the same as when they are acting alone (Levine, Resnick & Higgins 1993). The physical geometry of the motion of billiard balls provides a simple analogy: the behavior of a lone ball banking off the rails of a table is almost trivial to model with simple linear equations, while predicting the position of 15 balls coming off the initial break moves us to the realm of complex statistical mechanics and chaos theory.

With all these challenges, what is the allure of studying groups? It is that, despite its seeming complexities, social interaction is pervasive and fundamental to the human condition (Vygotsky 1978); it comes quite easily to us all (Garrod & Pickering 2004); and it reveals a remarkably resilient set of participation structures that can accommodate an enormous range of topics, physical situations, participants and modes of communication. In addition, the prevalence and significance of social interaction for learning and instruction in the classroom has grown (NCTM 2000; NRC 2000).

One of the central issues in the study of classroom social interaction and socially mediated learning is the establishment of common ground, or intersubjectivity, among the participants. Students and teachers must understand one another and enjoy some common referents so that the demands that come from learning and performance are not immediately thwarted by failures of comprehension.

Intersubjectivity

The traditional view defines intersubjectivity (IS) as a mutual understanding or univocality (Lotman 1988) between subjects that can either succeed or fail to occur (Cole 1991; Stahl 2006). This traditional view refers to IS as the convergence among interlocutors toward a common idea (Kapur, Voiklis, Kinzer & Black 2006) and an eventual shift from disagreement to agreement or symmetry (e.g., Wertsch 1979).

However, some scholars have proposed alternative accounts that acknowledge the important role disagreement plays in cognitive and social development and socially mediated learning (e.g., Johnson & Johnson 1989; Piaget 1975/1985; Posner, Strike, Hewson & Gertzog 1982; Vygotsky 1978). Here, disagreeing entails introducing a contested claim into the discourse (Asterhan & Schwarz this volume). Matusov (1996) presents a Participatory view of IS, where agreement and disagreement are common processes that mediate social activity within a common context. As Baker (this volume) points out, the learning that occurs within the discourse exhibits an intersubjective rationality that is predicated on the common ground that is taken as given or that is established over the course of the dialogue. Within this common context IS may show convergence on some aspects of the activity and divergence on others (Matusov & White 1996). It is this dynamic interplay between convergent and divergent processes that contribute to fostering sustained rich dialogic interactions (Bakhtin 1990).

How is enables and impacts knowledge construction in the classroom

In a recent investigation of the interactions exhibited during a whole-classroom discussion, we (Nathan, Eilam & Kim 2007) revealed several findings about the nature of intersubjectivity and its role in shaping student participation and socially mediated learning. US students in the sixth grade ($N = 24$; ages 11–12) spent over 1 hour of a double-length mathematics class collaborating on a spatial reasoning task posed by one of the students, *How do you cut a pie into eight equal-sized pieces making only three cuts?* In the weeks preceding this activity, students had spent a considerable amount of time and energy learning to listen to others, share their intellectual ideas with the group and engage each other in critical questions and responses (for more on this, see French & Nathan 2006). We observed students work out their ideas using drawings, manipulatives, constructions, hand gestures and, of course, language to publicly present and then critique one another's proposals for solving the Pie Problem.

IS was evident at several levels of analysis: the level of utterances, or single speech events (microscopic); speech-event sequences (mesoscopic); and a global perspective across the entire discourse (macroscopic). Evidence of IS+ and IS– independently coded were found to overlap in over 80% of the utterances, as one would expect from the participatory view (Matusov 1996). Thus, at the microscopic level of analysis, speech events frequently exhibited co-occurrence of both convergent (IS+) and divergent (IS–) forms of intersubjectivity. Indeed, speakers appeared to need considerable common ground to express substantive disagreement.

Intersubjectivity also appeared to play a significant role at the mesoscopic level in structuring interlocutors' interactions. As is characteristic of classroom talk, triadic dialogue – an initiation question followed by a response and then an ensuing evaluation – was the norm (Lemke 1990; Mehan 1979; Wells 1993). Often, as in the case of IRE triads, these sequences show no chaining from one discursive triad to the next. Yet in this corpus, chaining was present in over 75% of the event sequences, and covered over 80% of the time students collectively worked on the task. The recurrence of triadic sequences was preceded by IS in 88% of the occasions of chaining. In this way, IS marked, and perhaps contributed quite directly to, the dialogic nature of the class discussion.

Lastly, at the macroscopic level of analysis we saw how IS shaped both the nature of the mathematical representations used by students as they demonstrated and revised their solutions, and the way students came to talk about them and present them. This form of convergence to a common set of principles of communication to support comprehension emerged in the absence of explicit or centralized direction. It also occurred despite other aspects of the discourse community that explicitly kept members of the class from converging on a single solution.

Methodological issues with the study of intersubjectivity

Advancements in our understanding of the nature and role of IS have the potential to contribute to the theory of socially mediated knowledge construction in classrooms. However, there are considerable methodological challenges for documenting and analyzing whole-classroom discourse and socially mediated learning. We focus on two central issues: how IS in the classroom is identified and analyzed *within* each of the three levels of analysis previously reported, and how maintaining IS establishes constraints on the interaction *across* the different levels of analysis.

Contemporary models of intersubjectivity

As reviewed above, within the participatory view of IS, consensual agreement and shared use of representations, on the one hand, and differing interpretive frames and disagreement, on the other, stem from a common set of communicative and cognitive processes. This view appears to be consistent with several current models of IS that have emerged from research from the fields of cognitive neuroscience, education and psychology. Gallese (2003a, 2003b), operating from the neuroscience tradition, posits that speakers form a *shared manifold of intersubjectivity* that emerges from their empathetic responses to one another's communication and physical actions because the behaviors of specific neural mechanisms called *mirror neurons* evoke in us the bodily and affective states that we would normally occupy if we initiated those same actions ourselves. This enables interlocutors to provisionally enter into a kind of shared social space, even though the ideas and the interpretations they express may differ from one another. This account is commensurate with the Radical Constructivist view offered by Steffe and Thompson (2000), where speakers' 'conceptual structures are sufficiently compatible for successful reciprocal assimilation' (p. 193), but they need not be identical. As Rommetveit (1979) notes, this affords 'states of partial intersubjectivity' at best, as it allows speakers to temporarily bridge their 'private worlds.'

Writing from a psychological viewpoint, Garrod and Pickering (2004) argue that speakers engage in a largely unconscious process of 'interactive alignment,' whereby people align their mental structures at different linguistic levels simultaneously. In this model, alignment achieved at one level (e.g., word choice) facilitates alignment at other levels (e.g., representation use), and eventually leads to alignment at the critical level of situation-model formation (van Dijk & Kintsch 1983) that impacts one's comprehension and subsequent knowledge of the topic of investigation. Even if interlocutors argue or are insincere, they need to have substantial alignment already in place to understand each other and participate in the interaction.

Identifying and coding IS at multiple levels of analysis

In accordance with the view of communication as phenomena that are processed at multiple levels, and the notion of IS as coordinated alignment across levels of communication and behavior, we present our examination at three levels of analysis.

Microscopic level analysis

At the microscopic level of analysis the focus is on single speech acts, or utterances. In our coding of the data we were guided by the participatory view of IS, and its assertion of the mutual presence of both convergent (IS+) and divergent (IS–) aspects of a shared understanding. Specifically, in our coding system IS^+ was considered evident when students exhibited agreement, as well as when speakers operated within a shared conceptual space, as when interlocutors shared or appropriated one another's language or representations. IS^- was evident when students used the established common ground to disagree, alter one's language or representation, express confusion, or present differing interpretations. If, in addition to sharing a common representational space, students showed differing interpretations, misinterpretations or disagreements, we considered this evidence for *both* IS^- and IS^+, and coded it accordingly.

As a practical matter, we recommend that coding of IS+ and IS– be conducted independently of one another, and that each be subject to measures of inter-rater reliability on a suitable subset of the data (e.g., 10–20%). We also recommend using Cohen's *kappa* to report the level of inter-rater reliability, rather than simply reporting the percentage of agreement among coders, since *kappa* takes into account agreements that may occur by chance.

At the microscopic level of analysis the participatory view of IS focuses on 'the coordination of individual participation in *joint sociocultural activity* rather than as a relationship of correspondence of individuals' actions to each other' (Matusov 1996, p. 26; italics added). Within this view, agreement and disagreement are considered aspects of a common set of processes that mediate collective activity. In the following example (see Excerpt 1 and Figure 15.1), we see how this perspective reveals the way in which one speaker (Manisha) appropriated the representation of a peer (IS+) yet also viewed it through an alternative interpretive frame (IS–), thus demonstrating how these aspects of intersubjectivity can co-occur. See the Appendix for the transcript notation conventions.

EXCERPT 1

1 Bob: ((draws at the board)) Alright here is the to↑p ((draws a small
2 circle on the board)) and here is the bott↑om ((drawing another circle
3 under the previous circle)).
4 Bob: Just say that they're like (0.9) ((clapping vertically))
5 Bob: But, and like (.) if you cut it like (0.6) this ((draws two diagonal lines a
6 shape of the letter 'X' within the top circle))=
7 IS+ Manisha: = Then you have to go all the way through
8 ((pointing and then making a slice downward with her left hand while
9 still seated)).
10 IS+ Bob: Y↑ea↓h (0.8) then it would have to go all the way through
11 IS– ((drawing the letter 'X' within the bottom circle)). Right here i::z like,
12 ((pointing to Mary's original picture on the board)) um.
13 Manisha: Yeh. (1.6) and then (indecipherable) ((points to the board with left
14 hand)) Okay, and then, =
15 Bob: = Wait is this like the bottom?
16 IS+ ((pointing to the bottom side of Mary's drawing from earlier)) like,
17 IS+ Manisha: Y↑ea↓h.
18 IS– Boy 2: °That's the side°
19 IS– Boy 3: That's that's the [si:de.
20 IS+ Manisha: ((Manisha enters Bob's space, and gestures on his drawing))
21 IS– A]nd then you'd have to cut [it in half ((cuts the center
22 between the top and bottom of the pie with her right hand, palm flat
23 facing down))
24 IS– Boy 3: That's the [side.
25 Boy 4: I can have]
26 IS– Bob: You cut all the way down. That wouldn't make eight pieces
27 IS– Manisha: If you cut it in half ((makes a slicing gesture into the board with her
28 IS+ flat hand placed between the two layers drawn by Bob)) it would.

In lines 7–9, Manisha uses Bob's drawing to make the claim (which she simulates using her hand) that the slices shown by the Xs in Bob's drawing go all the way through the two layers shown. She accepts Bob's drawing, which is evidence of IS+ here. In line 10, Bob repeats Manisha's comment (IS+) and thus exhibits alignment at the lexical level, however, he makes a hand gesture showing the two cuts that make the 'X,' which is contrary to Manisha's horizontal cut (IS–). In lines 15–16, Bob connects his drawing of two layers to Mary's earlier drawing of a 3D perspective, and specifically links his bottom layer to the bottom portion of Mary's drawing, showing that these two representations share a common ground (IS+). In line 17, Manisha agrees (IS+) with Bob's interpretation of Mary's drawing and its relation to his. In line 20: Manisha stands and physically enters the presentation space established by Bob and appropriates his drawing for her demonstration (IS+) by talking about and gesturing along it, however, she uses it to display a horizontal cut (line 21) using a slicing motion between his two layers, contrary to the vertical cuts made by Bob (IS–). In lines 18, 19 and 24, several boys calling from their desks take the position that the area that Bob and Manisha have been calling the bottom may in fact really be the side of the pie (IS–). In line 26, Bob, contesting Manisha's claim (IS–), asserts that his drawing proves the three cuts would not yield eight pieces (rather, as we learn later, he holds a literal interpretation of the pie in this problem and believes this gives only four pieces, with the horizontal cut taken as invalid since it is slicing legitimate pieces in an uncustomary way). In lines 27–28, Manisha, who holds a geometric view of the problem that allows for the horizontal cut of any abstract entity, now challenges Bob's assertion (IS–) and states that the horizontal cut (shown being enacted in Figure 15.1) yields eight pieces from the original four. However, she does so using Bob's representation, which she accepts (IS+), but reinterprets from her frame. Thus, throughout this discussion, we see both forms of IS co-occurring in the same utterance.

Mesoscopic level analysis

At the mesoscopic level of analysis, we look at the presence and role of IS in speech-event sequences that span multiple utterances as presented at the microscopic level. The sequences we have identified (Nathan et al. 2007) are Initiation–Demonstration–Evaluation (IDE) triads, variants of the common IRE sequence. The presence of IDE triads (or some other similar sequence) provided a natural unit of analysis that stands in super-ordinate relation to the utterance level. In this data set, Evaluation events had two frequently co-present aspects: Elaboration (EL) of the demonstrated idea, and an evaluative judgment (EV).

After establishing the mesoscopic-level unit of analysis, and segmenting the transcript accordingly, we used Transana, a video-analysis tool (Fassnacht & Woods 2005) to map out the occurrences of IS codes over time in relation to mesoscopic-level unit boundaries. Using the keyword-mapping feature in Transana, we were able to show that IS codes in both the positive and negative forms frequently occurred at the ending boundaries of an IDE sequence (the E-event), and often precede the start of the next IDE sequence. In Figure 15.2, the distribution of I-, D- and E-events over time are shown in the first three rows. The shade of gray scale indicates different speech events, and their length shows the duration. The figure shows the prevalence of IDE sequences and their tendency to chain one after the other. For the third row, we specifically show the Elaboration and Evaluation (EL and EV) events that occur in the third slot of the IDE pattern. The fourth row shows the occurrence of IS (with both IS+ and IS– combined here for simplicity). The map shows that the IS codes were present during EL and EV events, especially when those events foretold the beginning of a new IDE cycle.

This role of marking the perpetuation of IDE triads suggests that the formation of IS might even serve to trigger the next initiation event, which, in turn, activates the following IDE triad. While we cannot establish causality from this analysis, we do note that IS events designate the chaining to the next IDE cycle in some 85% of the cases.

Why might IS play this role? There were many trouble spots in interpreting one another's (and even one's own) representations of the solution. For

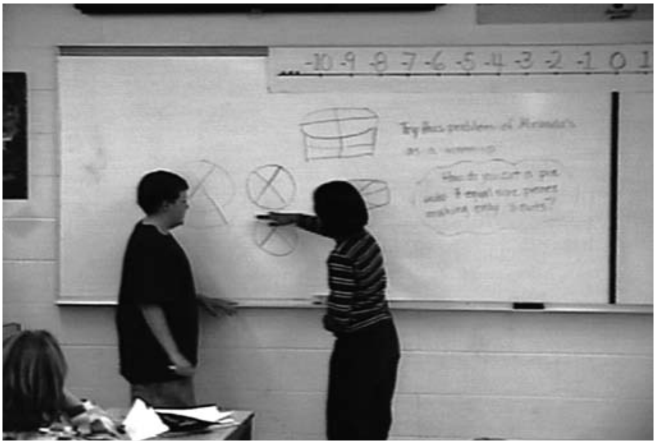

Figure 15.1 Example of the presence and co-occurrence of IS+ and IS– during student-to-student discussions of ways to represent and solve the Pie Problem.

example, several IS^- instances were the apparent result of inadequate drawing skills as students tried to convey their three-dimensional ideas using two-dimensional drawings on the white board in ways that violated principles of perspective (e.g., foreshortening) or that were too casual to interpret unambiguously. These student-led demonstrations (D-events) often triggered evaluations and elaborations (E-events) from discourse participants that often indicated a lack of comprehension, a need for clarification, or that posed an alternative interpretation (as with Excerpt 1). In this way, the placement of IS could often be interpreted as a way to invoke the next IDE sequence.

Macroscopic level analysis

At the macroscopic level of analysis the focus is on global changes across the entire discourse. Discourse-level analyses are intended to identify major features of the interactions that might otherwise go unnoticed at a finer-grained analysis. We again found that visualizing the overall pattern of the class discussion using the keyword-mapping feature of Transana was appropriate (see Figure 15.3). It allowed us to identify changes in the nature of the representations used by students over time. Solution representations were rated based on the degree to which they addressed three criteria: adherence to the principles of perspective geometry (external consistency); uniformity with which elements of the representation take on certain meaning or roles in the solution (internal consistency); and the effort and elaboration needed to interpret the drawing in an unambiguous manner (ambiguity).

Our analyses revealed a striking pattern: Out of 46 representations for the Pie Problem generated by students in about an hour, idiosyncratic representations (Levels 1 and 2, the lowest ratings) were most common during the first half of

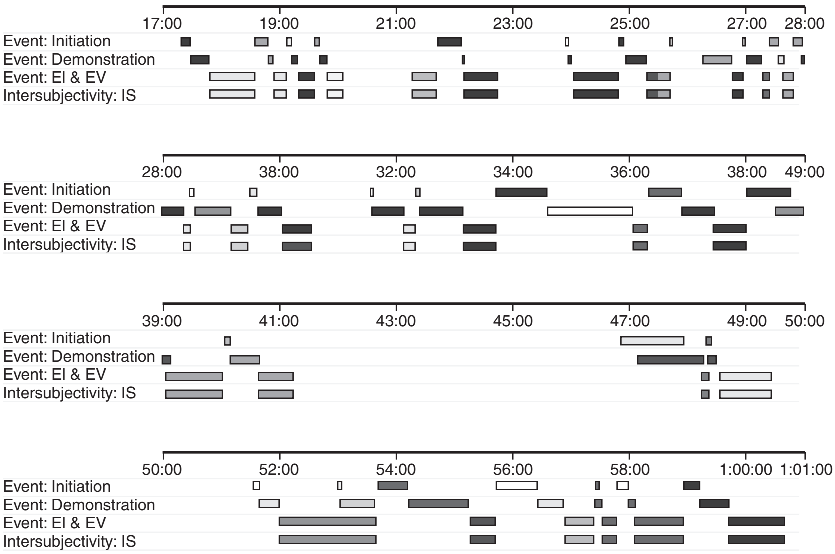

Figure 15.2 Each of the four time lines shows 11 min. of discourse. I, D and E events (the first three rows in each time line) tend to follow in sequence. The IDE sequence recurs cyclically. Occurrence of IS (bottom row of each time line) tends to mark the beginning of the subsequent IDE sequence.

the discourse, while more standardized representations that addressed ambiguity and internal and external consistency (Levels 3 and 4) were most common in the latter half of the discourse. Indeed, there were no Level 4 representations (the highest rating) made at all in the first 45 minutes of the class discussion; all (n = 5) were presented in the final 15 minutes of the discussion. This visual pattern (Figure 15.3) was statistically borne out. It was significantly more likely for students' solution representations to receive higher ratings in the second half of the discourse than the first half, $t(40) = 3.27$, MS = 0.35, $p < 0.005$. This convergence on standards for representing ideas occurred even though differences between those with a literal interpretation of the pie problem and those with more abstract interpretative frames made convergence on a single solution representation unreachable.

The influence of IS across multiple levels

One challenge facing this work is the inherent multi-layered nature of discourse (see Saxe, this volume, for a comparable methodological frame). Language can (and may even be intended to) take on multiple meanings. For example, Bateson (1972) notes in his essay, 'A theory of play and fantasy,' that communication among humans as well as among animals often serves both a denotative and metacommunicative function. For example, monkeys observed in a zoo demonstrated playful activity in which their interpersonal behaviors at the denotative

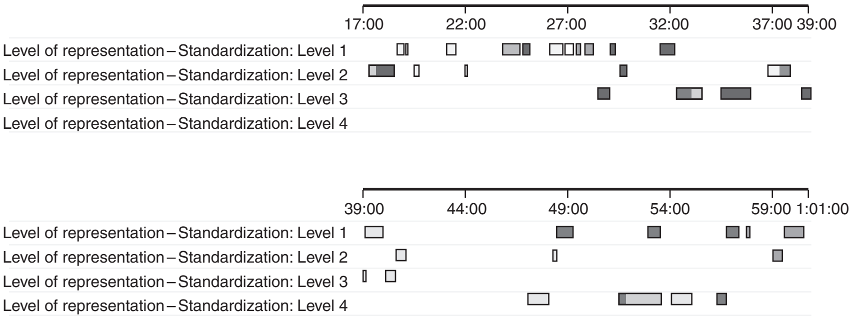

Figure 15.3 Transana keyword map showing the distribution of the four representational levels as assigned to each solution representation (n = 46) over the time course of the discourse. Each time line spans 22 min.

level could be interpreted as combat, though at a metalinguistic level they were clearly presented and interpreted as conveying a very different message, 'This is play, not fighting.'

In addition to this semiotic quality of communication, complex patterns of interaction can take place on different, although co-existing, timescales. This invites a systems-level perspective (Holland 1995; Lemke 1994), whereby structure observed at one level enables emergent behaviors or functions at another (Maturana & Varela 1980). Consider, as an example, the relation of the various levels of the discourse currently at hand. At a systems level, we can ask whether utterances operating at the microscopic level shape, or are shaped by, the global changes of representation use at the macroscopic level, and if so, we can try to explain how these mutually existing levels interact. One way that phenomena at different levels of a system interact is by providing constraints. We can imagine that comprehension is a highly constraint-driven process; too much information is difficult to process because of limits on attention and cognitive processing (e.g., Miller 1957); while too little, or information that is low fidelity, does not enable the cognitive systems to settle on a proper interpretation. Here we also see the need to share background knowledge so that relevant information, presented in an accessible way, can be meaningfully interpreted. The need for comprehension at the utterance level is actually something that was imposed earlier on the students through their training and norms for proper group interaction. This created the climate in which students were inclined and able to question, critique and elaborate on one another's comments, and provide reasonable responses in return. It is here that IS at the utterance level may be motivated.

There were also clearly considerable challenges to overcome when making one's ideas comprehensible to others. One of these challenges, which we mentioned briefly, was one's skill at using representations such as drawings or object constructions to convey one's thoughts. There are, of course, inherent limitations with representing three-dimensional (3D) objects in two-dimensional (2D) media, such as drawings. The added need to explain the cutting process as it unfolds over time adds yet a fourth dimension (4D) that is not easily supported by drawings. Conventions have developed to facilitate this all-important goal. Yet these were clearly not universally understood or applied by the students in this sixth-grade class. Consequently there were many attempts to pass off inadequate representations that simply could not stand up to the critical process that had come to be expected in this class.

At a gross level, the representations offered can be roughly grouped into 2D drawings, 2D, 3D and 4D gestural enactments, and 3D and 4D materialized enactments. If we take drawing and object use as examples, we can consider the nature of constraints that need to be met for each of these two forms of media to facilitate comprehension and support a shared understanding among interlocutors.

For drawings, we can consider their adherence to the principles of perspective drawing, including the attempt to convey 'vanishing point' (often referred to as linear perspective), to hide parts of the object that would, in 3D, be visually occluded, and to include proper size and shape variation (e.g., circles receding may look like ellipses). Alternatively, students may avoid perspective altogether and provide separate 2D views from different viewpoints that must then be explicitly knitted together. In these ways, we can have a reasonable expectation that 2D representations will support comprehension by adopting conventions that convey three and four dimensions in increasingly standardized ways, as discussed earlier.

In fact, this is what we observed. Drawings intended to capture depth, occlusion and action tended to emerge over time. Initially, these features were absent from the drawings. Over time, students came to apply elements like foreshortening to convey depth and color to distinguish the outlines from the cuts that were being applied to the referent object. We also saw late in the discussion the use of multiple viewpoints (like that of Figure 15.1), and even later, the incorporation of labels to denote the relationships intended between them.

Materialized demonstrations inherently captured 3D through their objectivity. Still, these were not actual pies, but mere proxies. Furthermore, they still needed to be perceived as having been processed convincingly (i.e., sliced) to serve as mediators of students' proposed solutions. Thus, even *objects* had to meet constraints on their mappings to the intended realm (the proverbial pie) by being authentic to viewers and yielding the expected outcome of producing eight equivalent pieces. So it is interesting to briefly deconstruct one ambitious demonstration by a student that used material but that fell short of meeting the constraint of authenticity.

Figure 15.4 shows a girl using a paper construction to suggest the pie. Her general approach seems valid, that there should be a way to fold a single sheet of paper to make a 3D form that will yield the requisite number of pieces of equal size and shape. Yet, as we see from Excerpt 2, this approach fails to meet the intended goal.

Excerpt 2

1 T: Mary's got a model idea that might help straighten this out.
2 Mary: Okay, this is a pie. ((Mary holds a folded piece of paper in her left hand

3 and scissors in her right hand.))
4 Mary: Okay what you do is you cut it in half. ((Mary uses scissors to cut along
5 a fold that leaves half of the paper construction dangling down))
6 T: So, in terms of cutting in half you're going down straight through the pie?
7 Mary: Yeah like this, and now there are two pieces. ((Mary holds two flat
8 pieces of paper parallel to one another)
9 Mary: And then you … then you put them together again,
10 T: You put them, you put em on top of each other?
11 Mary: yeah and then you cut it this way
12 S: That's wrong
13 Mary: or you can cut it, you'd make two cut
14 S: There's four pieces.
15 S: Yeah that's four pieces.
16 Mary: No then you make two cuts ((Mary drops the paper and
17 is now kneeling with the paper pieces in hand)).
18 T: Stand up please, Mary
19 Mary: So you make two cuts and now that it's like this you can cut it, you can cut it like
20 T: You can put … ((Moves over to use the table in front of her)) Go ahead, use the table
21 and then put the scissors down and hold up to say what you're talking
22 about okay … hold them up, they can't see it when it's on the table.
23 Mary: And then you have these, and then you cut it again like this, ((Mary is now using
24 the table to support the pieces as she cuts)) across the whole thing as one
25 cut and then you have .. I accidentally … You have eight.
26 S: Looks like twenty
27 Mary: I know cuz I folded it before
28 T: Oh it's a folded piece of paper.
29 Mary: And then you have eight.
30 T: Did that help?
31 Ss: No. (many students mumble.)
32 S: You have ((Points to pieces of paper)) one, two, three, four, five, six,
33 seven.
34 T: Who has another modeling idea? Draper?

The demonstration falls short in some important ways. First, the cutting process she is enacting is not visible to the members of the class, because her construction is too thin, and her attempts to hold the paper object and cut it all in her two hands without the use of a surface prove to be too cumbersome for her (lines 16–17). Second, prior to her public demonstration Mary made some hidden folds in the initial paper construction (as revealed in lines 27–28) that led her to produce far more than the expected eight pieces, though the exact number was never actually established (line 26). Finally, neither the process nor the pieces produced were made available for inspection, so they lacked validation process. Thus, in the end, this non-authentic use of material enactment produced utterances indicating disbelief and confusion (lines 31–33) as to its relevance (all coded as IS–) and no statements indicating agreement or common ground. Consistent with this, this demonstration did not get picked up by subsequent speakers, it was never again referred to, and so failed to become part of the shared understanding that the students were co-constructing.

These accounts provide examples that help us see how phenomena at the microscopic level (focused on contributions to IS of individual utterances) provide constraints on the global behavior of the discourse that influenced the use of representations, even though this was never centrally directed. Drawings, for example, became more standardized over time to address concerns and minimize questions and elaborations. Material enactments had to become more authentic and more explicitly linked to the pie scenario. When they lacked this authenticity, as in Excerpt 2, they contributed little to the maintenance of common ground.

The interaction between levels may be reciprocal, as well. That is, at the macroscopic level, we can view the entire discourse as oriented around the common problem brought to the class by Manisha. Once the teacher deemed this the focus of the class, students organized in a different manner. And that meant that the new contest shaped the manner of evaluating one another's comments and contributing to the construction and maintenance of the intersubjectivity manifold.

There appear to be multiple phenomena that emerged at the macroscopic level in response to the utterance-level role of IS. First, rules for presenting and representing demonstrations emerged that enhanced their comprehension and relevance. Second, interlocutors formed a common basis for communication, even though differing interpretive frames prevented convergence toward a common solution or depiction. Students came to standardize their representations in a way that contributed to the development of an intersubjective manifold. This occurred even though, throughout the discourse, students maintained strong differences of opinion about the interpretation of the problem and the criteria for evaluating an appropriate solution (cf. Baker, this volume). Consistent with the participatory view, the discussion did not lead to many conversions of opinion about the Pie Problem; members of the literal group insisted that horizontal cuts to pies were not appropriate; while those in the geometric group maintained that all cuts are possible in the abstract. However, disagreements and a desire to establish and maintain common ground drove students to express these views in more conventional and comprehensible ways. Third, an engaged and highly interactive classroom environment was created that exercised many of the principles of *productive discourse*, whereby 'forms of social exchange [were able to] provide participants with an avenue to construct and build upon mathematically correct conceptions through their interactions with other class members' (Nathan & Knuth 2003, p. 204). Specifically, students listened to each another and were genuinely interested in one another's ideas and opinions (Rommetveit 1989), they addressed and often built upon one another's representations (Latour 1996), and they evaluated and reflected on the problem-solving activity itself.

Figure 15.4 A materialized enactment of a solution that fails to contribute to common ground.

Discussion and conclusions

This work has implications for the study of learning and discourse and for instructional practices. On a theoretical level, identification and explication of theoretical constructs such as dialogic interaction and intersubjectivity help us to advance our knowledge of learning and participation in collaborative settings. Documentation of alternative forms of discourse structure (i.e., I–D–E vs. I–R–E) may provide valuable markers for and potential causes for engaging and productive discourse. On a practical level, the research may also inform teaching and teacher educators by providing a rich example of how norms and practices that foster engaged and sustained discourse contribute to the engineering of beneficial collaborations in the classroom.

In contemporary classrooms in the United States and elsewhere, learners are encouraged to share their ideas and collaboratively construct knowledge through observing and interacting with others and the environment. To engage in these settings, participants have to understand each other's ideas and be able to communicate them. Three main factors may contribute to establishing common ground: context, the activity and multi-modal communication.

Context

The achievement of IS calls for an empathetic classroom climate that supports the group's shared understanding by being responsive to the intellectual and social needs of the individuals as learners. The classroom must certainly support their reasoning processes. In addition, the classroom culture must encourage free expression of ideas and engender trust that those ideas will be respected. Establishing a *community of learners* (Bereiter & Scardamalia 1989; Brown & Campione 1994) such as this requires a substantial commitment among students and their teacher of attention to such matters and time for experimentation, reflection and revision (Pintrich, Marx & Boyle 1993). The teacher's role, in particular, must not be underestimated. Even in her decentralized function, she helped keep the discussion organized and focused, clarified the tacit rules of the problem (e.g., what constitutes a cut), and reframed comments from students. And while she established important norms early in the school year, the range of knowledge and practices is ever-changing as new tasks arise.

A common, goal-oriented activity

The task itself provided students with a common agenda that focused their attention and creative energies. This is often recognized as a critical element to effective collaboration (Cognition and Technology Group at Vanderbilt 1997; Cohen 1992; Johnson & Johnson 1989). Still, one can ask about the relative importance of this particular task in shaping the ensuing dialogue. While it is clear that task selection is significant, and can draw in or repel students, we tend to favor the impact of the classroom environment for germinating this productive discussion, and we would generally predict that emphasis on a climate supportive of dialogic interaction will bring about socially mediated knowledge construction more reliably than merely attending to the task that comes before the group.

Multi-modal communication

This third factor, which directly influences learners' chances of achieving IS, reflects the different modes of communicational means used by students for presenting their ideas and for promoting others' ability to understand them. A better understanding of these processes of building intersubjectivity was achieved in the present chapter by a close examination of the communicational means that generated them. These included ephemeral means such as speech and gestures, inscriptions that exist in material form and can be shared by learners (Roth 1998), and various objects.

We may naturally perceive language as the essential 'cultural tool' for carrying out processes of forming and expressing ideas and experiences (Mercer 1995). However, as these data make clear, speech was insufficient, and speakers regularly relied on gestures, objects, drawings and other inscriptions to publicly convey their ideas. This raises new issues for future curricula, since students' successes at communication and persuasion greatly depended on their abilities to use the various media effectively. The convergence toward standardized forms of representation (though by no means did they 'arrive' at this ideal) underscores the important role that intersubjectivity played in shaping the forms that solutions acquired over time.

Context, activity and forms of communication simultaneously contributed to the formation of a manifold of IS that came to encompass and influence the nature of the participants' interactions. These influences invited a multi-level analysis that helped identify the nature of participants' interactions as well as document the social accumulation of knowledge as students' strove to analyze and express their ideas to their peers.

Acknowledgments

This research was funded in part by a grant entitled ‘Understanding and Cultivating the Transition from Arithmetic to Algebraic Reasoning,’ awarded to the first author by the Interagency Educational Research Initiative, an alliance of the National Science Foundation, the Department of Education’s Institute of Educational Sciences, and the National Institute of Child Health and Human Development within the National Institutes of Health. This work was made possible in part by the support of the University of Haifa to the third author.

Appendix

Transcription excerpts with Jeffersonian notation transcription conventions:

[	Point of overlap onset
]	Point of overlap termination
=	No interval between adjacent two turns
(2.3)	Interval between utterances (in seconds)
(.)	Very short untimed pause
word	Speaker emphasis
the:::	Lengthening of the preceding sound
?	Rising intonation, not necessarily a question
,	Low-rising intonation, suggesting continuation
.	Falling (final) intonation
CAPS	Especially loud sounds relative to surrounding talk
° °	Utterances between degree signs are noticeably quieter than surrounding talk
↑ ↓	Marked shifts into higher or lower pitch in the utterance following the arrow
()	A stretch of unclear or unintelligible speech
(())	Nonverbal actions

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Chapter 16

Comparing and contrasting methodologies

A commentary

Angelika Bikner-Ahsbahs and Gaye Williams

A short overview

The term 'methodology' is discussed before we consider the methodological contributions of each team of chapter authors (Cobb, Gresalfi & Hodge; Nathan, Eilam & Kim; and Saxe, Gearhart, Shaughnessy, Earnest, Cremer, Itabkhan, Platas & Young) and examine links between them. We generate questions arising from our analyses of the three chapters in this section and formulate views on classroom learning in mathematics that could be researched through complementary analyses. The subsequent discussion of data-collection instruments appropriate to further analyses is informed by our own research perspectives. This commentary concludes with a summary of what we have learnt through comparing the three methodologies and how simultaneously focusing on data from different theoretical perspectives might help to show the way forward in researching the richness of learning in classrooms.

'Research methodology': what does it mean?

Background theories determine the kind of research questions that can be investigated and the kind of methods that are suitable. They provide the frame of how research objects are shaped and what kind of aims are followed (Mason & Waywood 1996; Bikner-Ahsbahs & Prediger 2006). Seiffert & Radnitzky (1989, p. 465) describe a methodology as a technology for epistemological progress that should not merely be regarded as a set of algorithms; rather it comprises quite vague rules which allow a researcher to handle methods in a creative way. Such creative ways of handling methods are described in the three methodology chapters in this section.

The methodology of a research study cannot simply be reduced to the set of methods. Methods control the way of coming to know and a methodology controls the choice of methods. The methodology's normative rules tell us how research results can be valued and applied and how they are related to the empirical world. Therefore it has to fit the background theory and paradigm in which research is conducted. Research results might not be compatible due to incompatible assumptions. That is why researchers should feel an obligation to make explicit their normative rules for their choice of methods. Reflecting on the topics of the three studies and their connections to their views on learning, research objects, observation tools, situations of investigation and their aims, we will illustrate the notion of 'methodology' in more detail.

All three research teams use a study on learning mathematics to inform their methodologies. Cobb and colleagues describe what design research means methodologically using a concept of a pair of situational subject-based identities developed in a statistics classroom. Saxe and colleagues outline a methodological frame for investigation of the travel of ideas which they illustrate by a fraction lesson shaped as inquiry learning. Nathan and colleagues present the analysis of a dialogical learning situation with a focus on developing intersubjectivity and consider methodological issues.

Learning mathematics, which is at the centre of all the three chapters, has different meanings for different authors. Nathan and colleagues speak of 'socially mediated learning' which implicitly indicates an *individual concept of learning*. Saxe and colleagues focus on the travel of ideas that shape *learning as a social process* that is interrelated to individual learning mediated by participation in the travel of ideas. Cobb's more dialectical view considers the co-development of two identities, one on the social level and the other on the individual level; learning mathematics takes place as *co-constructions* of both of them.

Concerning the concepts of learning, the three authors require different kinds of *observation tools* in their empirical research. Dialogical learning environments (Nathan et al.) are especially effective for investigating learning as part of a discursive process with tools that grasp the specificity of the dialogue and its impact. For inquiry learning the core idea is the production of mathematical ideas which Saxe and colleagues investigate through the change of forms and functions in the ideas that travel.

Classroom learning takes place within a class as a result of teaching and the teacher's philosophy of teaching and learning. Therefore research on learning mathematics in the classroom must also take into account the *situation under investigation* and this can be regarded *at different levels*, e.g. at the level of microgenesis, sociogenesis and ontogenesis of inquiry-learning processes in the class (Saxe and colleagues) or at the micro-, meso- and macro-level of a dialogical process within one lesson (Nathan and colleagues).

Methodological considerations taking into account all these aspects show how choices of methods and techniques are to some extent predetermined. However, the choice of methods is also influenced by the nature of the results that are of interest; for example, Saxe and colleagues and Cobb and colleagues use a *recursive method* of investigation because research is done for the purpose of curriculum development or design.

We conclude that a good methodology relates all the important research aspects to each other and to the theoretical background leading the researcher to well-grounded choices about methods, tools and techniques that capture just what the researcher wants to observe before and during the research conducted. An illustration that deepens our understanding of such connections between analysis tools and theory is presented by Hershkowitz in this volume.

Some questions arising from these methodology chapters

These chapters stimulated the following reflections and questions for us:

1. Saxe and colleagues have investigated the travel of ideas. Understanding the kinds of participation in the travel of ideas seems to be crucial to examining how mathematical understanding develops. We want to know more about this, including: What kinds of participation were observed? What kinds were found productive? If changes of forms and functions indicate learning, do some kinds of forms and functions occur before others? When and why are students satisfied to know only how (described as procedural)? What kind of conditions support asking why (described as normative)? The authors stated: 'Mathematical ideas travel as students make sense of them'. To what extent is sense making captured through the research design? For example, Annabelle realized that adding further intervals (geometric representation) gave the same answer as 'doubling (the numerator and denominator of) the fraction'. Did she only see a pattern linking these representations or did she know why the pattern existed?
2. Is Saxe and colleagues' focus on the change of ideas sufficient to examine the learning that occurred? Change of ideas need not indicate making progress, and change of forms and functions might not be enough to describe learning. For example, adding an idea to one's own ideas is a learning step but need not result in a change of form and function. Transforming an idea into one's own view could be a change of form and function but need not be a step in progress. Linking to Cobb and colleagues' chapter, how can student participation in the travel of ideas be improved?
3. Saxe and colleagues collect data on 'influences' on learning by asking students whose ideas influenced them most. Students tended to select one of the three students who presented their work to the class rather than a member of their own groups. Given previous research on how much students can learn from each other in groups, a closer examination of how students interpreted the term 'influence' is required. In making their decisions, did these students use a criterion like the 'amount of time spent listening'? Or did they make their selection on the 'significance of the change in ideas'?
4. Mathematical ideas can be considered at the level of content, and at a meta-level. Dreyfus, Hershkowitz and Schwarz (2001) describe a meta-level change in ideas that occurred when a student pair realized algebra could be used as a tool to develop an argument. Meta-level change in ideas might not be easily observable. Do these types of meta-ideas 'travel'? And if so, how? Can the data-collection instruments used capture this?
5. Cobb and colleagues investigated two mutually dependent concepts of identity. We would have liked to see illustrative examples of the enactment of normative and personal identity, and students' responses to questions about their perceptions of competence in these classrooms. This would have provided opportunities to consider the nature of the methodology in more detail and to retrace results from data. We wonder what such illustrations would have 'looked like'?

6. Have Cobb and colleagues found that normative and personal identity depend on the mathematical topic and the aims of the lesson or do they reflect a special kind of engagement no matter what kind of mathematical topic the students work on? For example, do they consider that preparing for a test requires another kind of normative identity than discovering a pattern, or developing a concept?
7. In what ways could simultaneous study of the work on identity by Cobb and colleagues and Sfard and Prusak (as cited in Cobb and colleagues) inform our understanding of mathematics learning? Complementary analyses of a situational and a stable concept of pairs of identity might shed light on the question of how a learner creates a specific relationship to a mathematical content, as well as the conditions that support and hinder this process.
8. Nathan and colleagues have found that a dialogical process leads to more standardized practices and have employed intersubjectivity as a way to study this process. What differences were there in the nature of the developing intersubjectivity displayed towards the beginnings and the ends of exploratory intervals? Answers to this question could lead to further insights into the quality of learning mathematics in dialogical settings and into how the nature of the dialogue can influence the process of knowledge development.

Interest: origins and fluctuations

A theme that permeates all three research studies is student interest and fluctuations in interest over time.

Fluctuations in interest

The episodes described by Saxe and colleagues and Nathan and colleagues were not uniform with regard to the degree of interest displayed by the students. Fluctuations in interest can be considered in terms of what Bikner-Ahsbahs (2005, 2003) has called 'interest-dense situations'.

In interest-dense situations, the students experience what it is like to learn mathematics with interest, and participate in an epistemic process that requires students to take responsibility for their own learning processes. In a socially supportive learning environment, students begin to value mathematics and experience positive affect in class. Without being forced to, they become involved in the activity of constructing mathematical meanings which often appear as an evolution of mathematical ideas within a dialogue. This way, they develop a feeling of competence, and experience autonomy and social relatedness. Such interactions are evident in the research described by Saxe and colleagues and Nathan and colleagues. Repeated experience of interest-dense situations increases the chance to develop a fruitful and interest-based relationship to mathematics (Krapp 2003).

Taking Cobb's normative view of identity, interest-dense situations build a normative identity that describes a mathematically interested student experiencing competence, autonomy and social relatedness (Deci 1998). Such an interest-based identity can lead to seeing mathematical structures and valuing mathematics highly by creating mathematical ideas, building concepts and inventing, discovering and checking mathematical patterns. Interest-dense situations offer increased opportunities to affiliate with mathematics and to participate in doing mathematics in a creative way. The extent to which a student identifies with such a normative identity or with parts of it provides evidence of situational interest (Mitchell 1993). According to self-determination theory (Deci 1998), this kind of interest is not far away from a more stable kind of interest which is called personal interest (Mitchell 1993).

Personally interested students are already affiliated with mathematics before the lesson begins. They bring their interest with them into the class and influence the building of a special kind of normative identity through their involvement in the activity. How do Cobb and colleagues capture this kind of phenomenon since personal identity is a situational concept which does not seem adequate to grasp stable features? Sfard and Prusak's construct of identity (in Cobb et al.) might provide this complementary view.

Within interest-dense situations the students participate in a 'travel of ideas' that leads to seeing mathematical structures. The teacher may support this process by abstaining from her/his own content-specific expectations. At the same time the students take over the responsibility for their learning process following their own and their classmates' process of producing ideas while considering them to be mathematically valuable. Saxe and colleagues' concept of learning is restricted to the change of ideas according to their forms and functions. To what extent do they include the quality of learning mathematics as it is part of interest-dense situations? Some progress of learning mathematics is described in their results in distinguishing between normative, procedural, partial and non-normative functions. Do the authors want to find out what kind of progress is observed in the travel of ideas? If so, why do they only distinguish between arithmetic and geometric forms and gather all the other forms into the category 'other'? Changing real-world forms, such as pizza cuts, into arithmetic forms may not occur in this way. Former experiences such as decimals, percentages or pizza cuts and how they link with fractions might influence the way the students 'catch ideas'. New ideas can have the function of structuring a group of ideas to build a rule, for example expanding a fraction with the natural numbers 2, 3, 4 or 'any natural number' (Bikner-Ahsbahs 2005). In addition, interest can have a significant effect on the travel of ideas. How do the authors grasp these kinds of phenomena?

The last example in Nathan and colleagues' chapter about pie cuts shows that the interest-dense situations can break down when the flow of ideas begins to dry up. This happens because complicated representations are difficult for the students to understand and the teacher does not intervene; therefore, ideas cannot be grasped and situational interest decreases. Do Saxe and colleagues investigate what conditions prevent students from grasping ideas? This would inform research on fluctuations in interest.

Nathan and colleagues show that, in a dialogical environment, there is a 'convergence on standards for representing ideas'. This convergence seems to be a natural phenomenon if learning through dialogue is experienced because understanding the other is essential for this kind of lesson. However, does this lead to deepened construction of knowledge as it can be observed within interest-dense situations? Do Saxe and colleagues observe the travel of ideas leading to some standardized forms as well? Is this enough to make progress in learning? Bikner-Ahsbahs suggests not. Imagine that a false idea is standardized and accepted.

Quest for the origins of interest

Nathan and colleagues showed that the process of developing intersubjectivity can stimulate deepening interest. At the start of an interval, a common direction of focus is identified; a common mathematical background and a common language are required to explore. Towards the end of an interval of exploration, students have developed new ideas: they need to crystallize their findings and develop a common language to communicate them. The conditions they develop during this spontaneous activity fit the conditions for flow (Csikszentmihalyi & Csikszentmihalyi 1992) during mathematical problem solving (Williams 2001, 2002). Flow is a state of high positive affect during creative mathematical activity that occurs when people spontaneously set themselves intellectual challenges that require them working above their present conceptual understandings to overcome them (Williams 2006).

But, why do such spontaneous explorations begin? Saxe and colleagues' task with unequal intervals on a number line led to spontaneous exploration when students became aware that there were mathematical implications to these unequal intervals that they had not thought about previously. In other words, they discovered a mathematical complexity they had not previously been aware of and decided they needed to explore it. The origin of interest was the students' idiosyncratic attention to a feature embedded in the task that had not been evident to them earlier. Exploring this feature to make sense of its implications involved students working outside their present mathematical understandings. They had created the conditions for flow during mathematical problem solving. The students' intense interest accompanied their creative development of new knowledge. Such flow activity is also identifiable in student responses in the research of others (e.g. Kieran & Guzmàn 2003; Dreyfus & Tsamir 2004).

Why do some students become intensely interested while others do not? Considered from the perspective of Cobb and colleagues, why do some students possess personal identities consistent with the normative identity of a classroom that includes activity associated with spontaneous exploration, while others do not? Or in terms of Williams' research (2008), why are some students inclined to explore new mathematical ideas while others want to remain within the boundaries of what they already know? How are the constructs of personal identity and inclination to explore linked and can research in one area assist in illuminating the other?

Williams (2008) has linked the inclination to explore with the personal characteristic of 'optimism' (Seligman 1995) which is associated with the ways in which people respond to successes and failures. 'Failure' during mathematical problem solving can include encountering unsuccessful pathways when trying to find out more about an unfamiliar complexity (Williams 2008). An optimistic student possesses psychological characteristics that provide them with the capacity to change a state of failure into a success. They perceive failure as temporary and able to be overcome through applying personal effort, and have the capacity to look into the situation of failure and find those factors that they can vary to increase the likelihood of success. Pessimistic students see failure as permanent and resulting from characteristics of themselves: 'I failed, I am stupid.' Optimistic students are inclined to explore new ideas (see, for example, Williams 2006).

Optimism can be built through students experiencing successes during flow situations (Seligman 1995). Will psychological changes along the dimensions of optimism affect a student's personal identity? Such a question could assist with finding out more about links between the situational and stable identity constructs discussed earlier. Optimism building may have occurred in the research settings studied by Nathan and colleagues and by Saxe and colleagues because the students focused their own challenges and worked outside their present mathematical understanding to develop new mathematical ideas.

Cobb and colleagues were surprised that all of the students in the class they studied perceived themselves as competent in that classroom. Did these students all perceive themselves as competent because they were in optimism-building classrooms (e.g., see Williams 2008)? Answering this question could help to illuminate what can bring about changes in personal identity over time.

Data-collection instruments used by Williams (2008) included multiple cameras in the classroom to capture student activity and provide mixed-image video that was used to stimulate student reconstruction of their thinking in class to generate data to study: (a) mathematical structures students developed; (b) the process of development of new ideas; (c) classroom activity that influenced student thinking; and (d) student optimism or lack thereof and student enactment of this. The assembly of such data-collection instruments could assist in answering some of the questions posed in this commentary.

Capturing different views on learning mathematics

On the basis of considering the three chapters and our own methodologies, we have experienced the value of complementary analyses: complementary application of compatible methodologies deepened our understanding of learning as considered from a variety of perspectives. Recently published papers use the idea of networking different theoretical approaches to link views, to gain deepened insights into learning mathematics from different perspectives and to develop research strategies for this complex purpose (Kidron, Lenfant, Bikner-Ahsbahs, Artigues & Dreyfus 2008; Prediger, Bikner-Ahsbahs & Arzarello 2008). We have extracted three complementary perspectives on learning from these research studies and present some considerations on methods for multi-perspective research referring to them.

1. Learning as a way to develop a specific relationship between the individual or the class and the mathematical content, regarded as identity, interest relation or optimistic enactment (Cobb et al., this volume; Bikner-Ahsbahs 2005; Williams 2008).
2. Learning as participation in a specific learning environment. This involves influences of the design of the lesson and its philosophy; like, for example,

dialogical learning or inquiry learning. Dialogical learning involves establishing common ground, and inquiry learning requires students to produce mathematically valuable ideas (see Nathan et al., this volume; Saxe et al., this volume; see also Williams 2002; Bikner-Ahsbahs 2005).

3. Learning as making progress, within an epistemic process either individual or social. Here, investigating the conditions that foster or hinder the process of learning makes sense (Dreyfus et al. 2001; Williams 2007, 2008; Bikner-Ahsbahs 2005).

Complementary analyses of learning mathematics from these views could provide insight into the kind of 'growing' knowledge that co-evolves with the students' developing relationships with mathematics in a specific learning environment. They could also clarify how these three views on learning can be mutually informative.

Methods to combine different perspectives

The authors of the methodology chapter all videoed lessons and triangulated this with data from a variety of sources including student work, questionnaires and interviews. The nature of activities in classrooms was studied from the perspectives of: (a) exploring participation in normative activity; (b) developing common ground; (c) developing mathematical ideas with a focus on representations; and (d) to some extent developing mathematical structures. The students' relations to mathematics could be studied in the light of interrelationships between individual learning and learning situations. Video-stimulated student interviews would contribute additional data to enhance these complementary perspectives and deepen insight.

Some methodological reflections

Comparing and contrasting different methodologies, we have looked at some critical points. However, there is still much to be learned about methodologies and how they provide the choice for methods and techniques. Where data showed how results were obtained there was opportunity for us to evaluate those results. Where critical data were presented and variation in data was shown, there was opportunity for us to learn from the research. We have learned that the use of paradigmatic examples can be an effective methodological tool to communicate theoretical concepts. We have also learned two methods to bridge studies of social and individual learning. Saxe and colleagues and Nathan and colleagues have empirically bridged the gap between the individual and the social by multi-level analyses, whereas Cobb and colleagues worked this out theoretically by constructing a pair of identity concepts that interact with each other.

Engaging in networking among the three papers, their methodologies and our research backgrounds, we gained the impression that three views on learning could be linked. We have presented some tentative ideas about this and we know that in doing so, we have sometimes gone outside the theoretical perspectives of the research teams adding complementary views. We both felt that a great deal could be gained from considering a classroom informed by different theoretical perspectives and a lot could be learned by comparing and contrasting the methodologies of the three chapters. Networking (Prediger et al. 2008) theoretical perspectives may detect incompatible assumptions of the theoretical approaches. However, we regard this to be a useful conflict that promotes overcoming empirical single-perspective research. It may lead to creating methodologies for multi-perspective research which could better grasp the complexity of 'guided construction of knowledge in classrooms'.

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Part 5

General reflections on transformation of knowledge in classroom interaction

Chapter 17

Contour lines between a model as a theoretical framework and the same model as methodological tool

Rina Hershkowitz

Opening

The flow of theoretical and methodological paradigms which determine the frames for research work in science and mathematics learning has become rich and more and more sophisticated. However, it seems that more than in the past, researchers today do not feel obliged to and/or satisfied with sticking to one methodological paradigm. Research trends in our area are nowadays characterized by flexibility and creativity in combining research methods and methodological tools, which fit the researchers' theoretical framework and meet their goals and needs to explain and answer some 'big questions' emerging from their explorations (e.g. see the chapter by Saxe et al. in this volume).

In this chapter I first discuss issues concerning the contour lines between the theoretical framework and the methods and methodological tools within the same research work. I argue that in more and more research work these boundaries are flexible and even a bit vague in the sense that the same scheme or model may serve as a theoretical framework in one piece of research, as a methodological tool in a second one, and as both of them in a third piece of research. I will discuss these issues via two examples, which illustrate dynamic relationships between theory and methodology in two different research domains of mathematics learning. In each example, the questions will emerge from analyses of the above relationships in a few research papers concerning a particular topic, looking at differences and similarities between theoretical and methodological frameworks. The first example is taken from research on argumentation in mathematics learning, and the second example is taken from research using the RBC+C model for abstraction in context.

Example 1: classroom research in the role of argumentation in mathematics learning

The book edited by Cobb and Bauersfeld (1995) is a meaningful contribution and a trigger to research in mathematics education, which investigates mathematics learning in classrooms via the psychological and the socio-cultural lenses in parallel. The book represents a collective effort to create a theoretical frame for classroom research in mathematics. The authors use a common corpus of data to elaborate their different perspectives.

Krummheuer's chapter (1995) in the above book deals with what he calls 'the ethnography of argumentation', whereby his theoretical interest is in the social genesis of argumentation. His research is targeted to two crucial goals: To develop a theoretical framework of argumentation in mathematics, and to incorporate it into a social context. He is interested in 'collective argumentation' as a classroom practice. In this sense Krummheuer considers argumentation as a 'social phenomenon; when cooperating individuals tried to adjust their intentions and interpretations by verbally presenting the rationale of their actions' (Krummheuer 1995, p. 229). He bases his study of argumentation mainly on Toulmin's (1969) argumentation model (scheme) and shows that this theoretical model is valid and applicable to identify argumentative processes in mathematics classrooms. In a sequence of episodes with various students from mathematics classrooms he shows step by step that the elements of Toulmin's model: *claims – conclusions*, *data*, *warrants* and *backings*, which support, justify and explain the claims, are appropriate elements for tracing a collective argumentation in mathematics classrooms. In addition these elements have internal relationships which together present a process of argumentation and *collective argumentation* in mathematics. The above model elements, in a certain collective argumentation process, might be given by different students in the classroom as well as by the teacher. Does this model serve as a theoretical framework for Krummheuer's work? As a methodological tool? As both?

It seems that for Krummheuer, Toulmin's model is a core for both theoretical framework and methodological tool. His intention is to create an appropriate theoretical frame for studying the emergence of argumentative processes in mathematics classrooms. But, in order to build it step by step, through analysing the classroom episodes and to show how various elements of the model are taking place in this context, he does not have any choice but to use the model itself first, in order to interpret and explain the argumentation that emerges in the episodes, and doing this gives evidence that this model is appropriate as a theoretical framework for mathematical argumentation.

Yackel (2002) used Krummheuer's work, while emphasizing the aspect of the model as a methodological tool, in exploring the role of the teacher in the classroom. She wrote: 'this approach to argumentation is useful as a methodological tool for documenting the collective learning of a class because it provides a way to demonstrate changes that take place over time' (p. 424). She used it as a methodological tool for the purpose of investigating the dynamic role of the teacher in encouraging mathematical argumentation in the classroom. Because her research aim was not the investigation of the argumentation process itself, nor the validation of the model, she needed and could use a less refined version of the Toulmin/Krummheuer scheme. (She mostly did not differentiate between *warrant* and *backing*.) Does this support the hypothesis that when the 'model' serves mostly as a methodological tool for classroom research, where composite interactive processes are taking place, and where the research aim is not any more to construct, or elaborate, or confirm the model, then a simpler dynamic tool is preferable to a very detailed one, and the research tool in use becomes less refined?

A complementary finding arises from the work of Whitenack and Knipping (2002). They used Toulmin's model and Krummheuer's work to learn more deeply how the emerging of collective argumentation is *socially accomplished*. In contrast to Yackel's research, the (collective) argumentation and the arguments themselves became again the focus of research and therefore their evolution has to be presented and analysed in a very refined way. In their micro-analysis of classroom work they showed how different students, one after the other, add an additional *warrant* to support the same conclusion and even the teacher contributes her *warrant* to the same collective construct.

Whether the Toulmin/Krummheuer model is used as a theoretical framework or as a methodological tool, some global methodologies were seen to be used in the research work in addition to the model (e.g. protocol documentation and analyses). The model, at any level of refinement, is used as the lens through which the data are described and analysed. This analysis suggests that when it is mostly a theoretical framework, a process of refinement and confirmation of the theory is taking place. When the research goal is beyond the model itself, the model is used in a more global, unrefined way.

Example 2: the RBC+C model for abstraction in context

In this example I will elaborate further on the issue of the flexible nature of the contour lines between a model as a theoretical framework and the same model as methodological framework and/or methodological tool. I will again focus on a few research papers in which a certain model, this time the RBC+C model, serves either as theoretical framework or as a methodological tool, or both.

Seven years of research and more than 30 research publications, contributed by more than a few people, separate the 'birth' of the RBC model, as an empirically based theoretical framework, from recent publications that use this model as one of two or more 'conceptual frameworks' (Kidron 2008; Wood, Williams & McNeal 2006).

The researchers, at the beginning, came up with a first hypothesis for the model using both theoretical considerations and the analysis of considerable amounts of data. In this undertaking, they were led by the need to give theoretical expression to the specific characteristics of their data which pointed to constructing of knowledge by means of mathematical thinking. In the process, they took into account and incorporated elements of exiting theories. Abstracting was taken as human activity of mathematization, specifically 'vertical mathematization' (Treffers & Goffree 1985). Vertical mathematization represents the process of constructing a new construct in mathematics by learner(s) within the mathematics itself and by mathematical means; like reorganization of previous mathematical constructs, interweaving them into one process of mathematical thinking with the purpose of constructing a new mathematical construct. Abstracting in context has emerged and was described at first by means of illustrative examples in different contexts. For a detailed description and theoretical analysis of this model, see Schwarz, Dreyfus and Hershkowitz in this volume.

At that stage, the researchers found themselves in a situation where theory stemmed from the analysis of data, and the analysed data served as evidence for validating the theory. They were quite aware of this situation and explained: 'This definition (of abstraction) is a result of the dialectical bottom-up approach … a product of our oscillations between theoretical perspective on abstraction and experimental observations of students' actions, actions we judged to be evidence of abstraction' (Hershkowitz, Schwarz & Dreyfus 2001, p. 202). It is clear that for analysing the above actions, the researchers had to use some basic methodologies which fit protocol analyses of an individual and the more complicated analysis of cognitive and interactive work within dyads. The three epistemic actions, recognizing, building-with and constructing, and the dynamically nested relationships between them were hypothesized as the main building blocks of the model, and at the same time were validated as well, and also used as the lens and compass to describe and interpret the data analyses themselves. Such a situation held for the first steps towards the validation of the model as a theoretical framework.

Since then, the RBC+C model has been validated, both as a theoretical framework and as a methodological tool, in various social settings and learning environments. The settings considered include (teacher-led) classroom discussion, small-group problem-solving processes, tutoring situations and individual activities (e.g. introspective self-reports of single learners). The age range of the learners extends from elementary school to adult experts and the longitudinal dimension varied. And, indeed, research made it clear that the RBC+C model is an appropriate tool/theory/methodological tool/methodology to describe abstraction and provide insight into processes of abstraction in a wide range of situations of abstraction and consolidation on a medium-term timescale, where consolidation is a process by which the construct becomes progressively more self-evident, the student's awareness of the construct increases and the use of the construct becomes more flexible (Dreyfus, Hadas, Hershkowitz & Schwarz 2006; Hershkowitz, Hadas, Dreyfus & Schwarz 2007).

It is interesting to follow the role of the RBC+C model in different pieces of research and how it is changing with the changes of the research goals and the researchers' interests: In Dreyfus and Kidron (2006), the model is not the focus any more but becomes a tool to examine whether the learner gains insight into the mathematical situation she is investigating. The researchers used the RBC model as a methodological tool for the analysis of constructing knowledge with two 'new' features: very advanced mathematics and a solitary learner. But it appeared that the model turned out to be only partially sufficient and the analysis of the data turned into theory building, namely the extension of the model to interacting parallel constructions, the refinement of the epistemic actions and the connection between these two. Note that there was another 'surprise', an even more positive one, namely that the interactions among the parallel constructions led to a deep insight into processes of justification as part of abstraction (Kidron & Dreyfus 2007).

The paper by Tabach, Hershkowitz and Schwarz (2006) presents an example of knowledge constructing within the context of peer learning in a working classroom. It shows how the design of the tasks and the computerized tools available to the students afford the constructing of conceptual knowledge (the phenomenon of exponential growth and variation, as it is expressed in its numerical and graphical representations). The researchers trace the constructing of knowledge through a series of dyadic sessions for a few months in a classroom environment, and analysed three of them with intervals of a few months between them. The analysis shows that knowledge is constructed cumulatively, each activity allowing for consolidating previous constructs. This pattern indicates the nature of the processes involved in creating a new abstract entity: knowledge constructing and consolidating are dialectical processes, developing over time, where new constructs stem from old ones already consolidated, and old constructs are further consolidated through the new constructing. As the aim of this research was beyond the model in itself, to trace the global constructing and consolidating of specific conceptual knowledge along successive dyadic interactions, with a few months' interval in between, the main function of the RBC+C model in this research was to serve as a methodological tool to illustrate construction and consolidation processes. The data did not serve, in turn, to polish and refine the model theoretically.

In research published recently, Wood et al. (2006) examined 'the relationship between the patterns of interaction that exist in the classroom and children's expressed mathematical thinking' (p. 228) in classes from different cultures, and for this aim the RBC model served as the 'conceptual framework employed to examine the quality of students' expressed thinking' (p. 225). I think that the term conceptual framework, when applied to a certain model, expresses the flexibility with which this model may be applied as a framework for both theory and methodology. And indeed in Wood et al.'s research, it seems that on the one hand the authors believe theoretically that the model with its three epistemic actions expresses quite accurately the level of mathematical thinking that children have; on the other hand they use the model for analysing the protocols of the class members and identifying the levels of thinking expressed by class members. They accumulate these data for the purpose of quantitative analyses of the levels of thinking expressed by the class members in discussions in the different classrooms. This research shows some maturation of the model as a theoretical framework and as a methodology. The authors needed two conceptual frameworks in their study and the model is one of them. The model is not any more the focus of the study. It allows the researchers to determine levels of thinking that are available for inspection in the classroom.

The last research example in this section is Dooley's classroom research (2007) concerning what I would like to call *collective abstraction processes*, which emerged in one lesson, mostly during the last phase of the lesson, where a whole-class interaction took place. The researcher's aim was to show how the class community, as one entity, reaches 'sophisticated constructions'. She explained: 'One pupil's "recognizing" led to "building with" by another and to "constructing" of new ideas and strategies by others' (p. 1658). Again, this is a situation where the researcher uses the RBC model as a methodological tool to explore the existence and nature of *collective abstraction*, and by doing this she confirms the RBC model as a conceptual framework for *collective abstraction* as well. The above research process is in analogy to the *collective argumentation* research process of Krummheuer (1995), which was also investigated by Whitenack and Knipping (2002). In both cases the process is distributed among quite a few students in the same classroom.

Concluding remarks

The two research models by which I tried to exemplify the flexible contour lines between the model as theory and the model as methodological tool have some common features: Both of them deal with a model that is aimed to serve as a framework for describing, analysing and interpreting a human mental activity. Both of them are appropriate for exploring individual student mental activity as well as for exploring collective mental activity which is distributed in the classroom among different individuals.

The elements of Toulmin's model for argumentation, as well as the three epistemic actions of the RBC+C model, have a very general nature (general in a sense that they can be used in many and varied contexts). The relationships between the elements of Toulmin's model on one hand, and the nested relationships among the epistemic actions of the RBC+C model on the other, are global as well. The elements of both models are observable and can be identified. Therefore the models lend themselves easily to be adapted and to contribute to research in many different contexts of argumentation or abstracting.

However, the notion of *collective abstracting*, and in a similar way the notion of *collective argumentation*, raises many questions, such as: What can we learn from this kind of research about abstracting/arguing, or more generally about learning processes and knowledge constructing in classrooms? What can we say about the individual students in the classroom and the classroom community not only as one entity, but as a community that consists of all the individuals who belong to this community? Do we have a methodology/methodological tool by which we will be able to conduct the kind of research that gives some answers to such questions?

Similar questions are at the heart of classroom research in general. See, for example, the research that focuses on 'the travel of ideas' in the classroom, by Saxe et al. in this volume, which claims that 'developing methods to understand the travel of ideas is foundational to understanding learning in classroom communities', where the individuals in the classroom have a main role in the travel of ideas. Researchers who plan to observe and analyse in detail micro-processes of constructing knowledge, or argumentation, or any other learning processes which are related to the classroom activity in a given context, along a time segment that may range from minutes to weeks, and do not want to ignore the researched phenomena as it is expressed by the individuals, face great difficulties: The observation and documentation processes are complicated, data are messy and massive and there are no systematic clear-cut methodologies for analysing them (Schoenfeld 1992).

However, research interested in relationships between learning processes of individuals and communities in classrooms faces further questions in the same direction: How far will one be able to expand the investigation of communities' constructing of knowledge/learning versus the individual's constructing of knowledge/learning? How far will one be able to 'keep an eye' on both, the community on one hand, and each individual who belongs to this community on the other? There is no doubt that it is very difficult to absorb such a situation.

It seems that there is no theoretical problem to conduct such classroom research, which analyses in parallel the knowledge-constructing processes in the different social settings, which are formed in a natural way in the classroom, including the paths of such processes within individuals and among them, and then to interweave the analyses together. This seems the optimal way for gaining insight into processes of constructing knowledge in the everyday classroom. But will the 'heavy battery' needed for documenting all of the above not affect the natural learning environment in the classroom? And what about creating the methodologies for analysing the huge volume of the accumulated data needed for interweaving the findings together?

It seems logical to assume that observing different kinds of interactions/ learning settings in parallel in the classroom on one hand, and overcoming the methodological problems of analysing, interpreting and interweaving the findings of the different research settings on the other, may give researchers coherent and meaningful insight into natural processes of learning activities (in the widest sense) in the classroom. The research of Saxe et al. (this volume) seems to come close to this optimal way. In the researchers' words: 'We collect data from a wide range of sources, and use diverse methods of data reduction and integration to reveal onto-, micro- and socio-genetic processes.' By means of these three kinds of processes the researchers describe and analyse the travel of ideas among a classroom's individuals, which are, as they believe, 'foundational to understanding learning in classroom communities'.

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Chapter 18

Learning in schools

A dialectical materialistic, cultural-historical activity-theoretic perspective

Wolff-Michael Roth

> For a long time education has been not so much *culture* as *structure* – a *structure* of institutional subordination.
>
> (Mikhailov 2006, p. 55)

The individual chapter contributions to this book specifically – and therefore this book as a whole – deal with knowing, learning and teaching in science and mathematics classrooms. The major threads include: (a) the construction of knowledge; (b) guidance in classrooms; (c) dialogue and argumentation; and (d) methods for studying key components of knowing and learning in (science, mathematics) classrooms. Some of the additional specific theoretical concepts used include *meaning*, *activity*, *intersubjectivity*, *actions*, and the temporal scales of development (*microgenetic*, *ontogenetic* and *sociogenetic*). This rapid overview already provides an indication of the complexity and breadth of educational research on knowing and learning in schools in general and in science and mathematics more specifically. The studies assembled in the foregoing chapters cover considerable terrain, often from competing perspectives, which makes it somewhat impossible to integrate them into one consistent statement. The purpose of this chapter is to survey – from a dialectical-materialist, cultural-historical activity-theoretical perspective and in a necessarily sketchy way – some of the terrain covered by the contributors and some of the blind spots that have been left uncovered. In so doing, I am aware that I myself am subject to taking a position, which, coming with its *dis*positions, *sup*positions, and *pre*suppositions, comes with its particular perspective, blind spots, and *pro*positions.

Doing a critical, integrative review of a diverse set of chapters is no small feat indeed. Because all observation is theory-laden, any honest review articulates the theoretical underpinnings or describes (by way of example) its way(s) of viewing the world. For it is one of the great dangers in, of and for educational research to use unarticulated knowledge and familiarity with the school situation in the interpretation of data sources. This unarticulated knowledge, as all knowledge, is ideological and cannot be separated from its societal-hierarchical relations (Bakhtine 1977). The opening quotation is especially relevant: (a) All human activity is mediated by its place and function *in society*, and (b) heretofore education and educational research has focused on *structural* rather than *cultural* (agential) aspects. Omitting *societal* mediation and focusing on structural aspects of learning comes with pitfalls: it leaves out essential moments – i.e. constitutive and irreducible components – of cognition. Thus, to address the danger of using unarticulated knowledge in the interpretation of data, I like to make explicit what allows me to hear and see events that I have no other information about than the videotape or transcript. I focus on conversations because independent of their particular topic, the chapters either analyse language or presuppose that language is used to make available such things as concepts, ideas or positions. As analyst, everything that I can support claiming therefore has to be available in the data sources themselves, or I do not allow myself to use it in *explaining* what is happening. Taking nothing for granted is an approach that leads us to *first principles*, for example, of the very possibility to 'have' conceptions (Tiberghien, Hoppe, Asterhan, Saxe), misconceptions (e.g., Michaels, Asterhan), preconceptions (Howe), or 'mathematically correct conception' (Nathan).

Specifying the position for this review

To set us up for the particular course and content of my framing of the chapters presented here, consider the following episode, which I analyse in my preferred way both in reading and writing research. No background is provided so that readers may want to imagine themselves overhearing the conversation somewhere or coming into a room where there is a recorder playing a tape that they do not know where it has been recorded. Let us see what we can find out by looking at a transcript that also includes indications of the way in which the speech was delivered, i.e. its prosody.

1 D: its um its sort of, its sort of an (0.24) <<dim>um a different way of thinking [abou::t graphs].>
2 A: [it certainly is] it certainly [is]
3 D: ['um]
4 (1.75)
5 ^um=hmmm
6 (0.69)
7 ↑`OKAY:: WHAT `s[o (0.50)] [i=ll ↑`give you]
8 A: [maybe just] [a little hint] more a
9 D: i=ll give you more [a: hint]
10 A: [a ^hint]
11 (0.20)
12 D: hint so=f if 'your in region three.
13 (0.61)
14 A: yeah.
15 (0.15)
16 D: an your an your death rate=s (.) higher than your birthrate;
17 A: right;
18 D: <<dim>your population=s going to start dwindling.>

The key questions for analysts such as myself are these: What kind of situation do the two participants recognizably produce and reproduce in and through their talk? What are their institutional relations? How do they achieve the collaborative production of the situation that we come to be of a specific type? As we listen to the two individuals, the lower pitch register (as measured by the absolute pitch [F0]) of the first voice (D) and the much higher register of the second voice (A) allow us to hear the conversation as one that involves a male and a female participant. I therefore designate them as David and Annemarie, respectively. In the first turn, David appears to be completing something that has happened before as a different way of thinking about graphs, a commentary that Annemarie affirms in and with the next turn. Together, turns 1 and 2 thereby realize a speech act that makes a statement about something that has happened before. After some hesitation on the part of David (turns 3, 5) and after some pauses that they co-produce (turns 4, 6), David appears to be giving something to his interlocutor, which she names to be 'just a little hint' (turn 8). The utterance therefore can be heard as an offer; and what is offered is a hint. Whether it is understood as an offer *in that situation*, however, can *only* be taken from the next action on the part of Annemarie. She confirms the offer in and through the subsequent two turns, in which David reiterates providing a hint, where Annemarie overlaps him uttering the same words. That is, the speech act itself, the offer as offer, is available only across two or more turns and cannot be reduced to a single speaker (Bakhtin 1986). The meaning of an utterance – which in fact is a cultural-historical speech act – as it is pertinent *in and for this conversation*, is spread across two turns and two participants!

David then continues uttering something about a 'region three' with a decreasing pitch (as indicated by period at end of utterance) as if he were coming to an endpoint in his utterance. But grammatically, what has been uttered so far is only a first part of a conditional, the premise; we may therefore expect him to continue. But this expectation is not warranted as the two co-produce a longer pause; and it is Annemarie who, by uttering 'right' (turn 14), brings the pause to an end. With this brief utterance, she in fact signals having attended to and having been following her counterpart's unfolding talk. The speaker therefore may continue. Advancing the tape, we would find out that this is the case, as David goes on almost immediately. His utterance 'an[d] your an[d] your' can be heard as an indication that he is offering the elaboration of the premise. Annemarie utters another affirmation that signals attention and understanding ('right') before David articulates what may turn out to be (if we were to listen further) the consequence of an implicature, 'your population is going to start dwindling' (turn 18).

As culturally competent members, we can hear what is happening here as a conversation in which one person offers and another person requests the provision of a hint about a way of thinking about graphs. But this recognition is itself a societal-hierarchically mediated achievement through and through, as they participate in a recognizable collective activity that they themselves produce. There may be a particular graph present in which there are at least three regions, in the third of which the death rate is higher than the birth rate, so that the graph implies a decreasing population. When we are now asked to make some inference about the situation, we might, given our experiences in educational systems, intimate there to be some sort of didactic exchange (tutoring, lesson) in which

Daniel assists (guides, teaches) Annemarie to read a graph that contains at least one birth-rate and one death-rate curve. In this hypothesis, Annemarie would then be identified as a student (tutee) and Daniel as the teacher (tutor). Graphs are topics that are used seldom in everyday life, and being taught how to read a graph may be most typical of educational settings. If we were provided then with the institutional positions of the two and a name of the locale where the tape was recorded, we would find out that we were entirely wrong: Annemarie is a professor in a science department with more than 30 years of teaching experience (including teaching awards) and David is a third-year undergraduate student majoring in the same department. This immediately raises the question, 'Why would David teach Annemarie how to read a graph that is part of the introductory material of a second-year course?'. If we were now told that Annemarie is a physics professor and that David conducts interviews about graphs for a professor in applied cognitive science interested in how professionals read graphs out of field and that the particular graph at hand is from the field of ecology, it might dawn on us that David has recruited Annemarie to participate in a particular form of *activity*, research in the social sciences conducted by means of interview/think-aloud protocols. But why would David tutor Annemarie if his assignment had been to interview her/make her think-aloud about graphs?

The mystery might begin to unravel if we were then provided with further information that during the interview, Annemarie, based on all signs available, could not provide the requested expert reading of the graph, indicated this to be the case, and then began to ask the reluctant David repeatedly for hints until he offered providing them to her.

What is interesting about the episode is that without this context that I subsequently provided, we are not likely to understand what the conversation really is about – something to be kept in mind when we read the chapters presenting transcripts and interpretations thereof. It is the collaborative production of an interview/think-aloud protocol that orients and mediates what the two participants do. More so, the participants themselves would not understand what the utterances that they concretely produce one for the other are about. What they say makes sense *because* they are in *this* situation that has as a larger, collectively mediated motive the production of data for a research project conducted by a professor of applied cognitive science who is interested in graphing-related expertise and workplace knowledge. We also need to understand that David does a work term required by his co-op programme; and the condition for his employment was the production of research data involving physics professors as research participants. But the situation is not a pre-existing box in which people follow prescribed behaviours, for the whole episode *only exists in and through* these speech actions. These actions thereby realize the activity in a very concrete manner, they both presuppose and constitute the activity, which both presupposes and gives sense to the speech actions. We cannot even understand each utterance in and for itself, as if produced by an individual independent of the interlocutor, because each utterance inherently must take into account the respective listener for which it has been produced and a more general, culturally competent audience that constitutes the ideal (but non-present) listener to whom both are accountable (Bakhtin 1986). Each participant says what he/she says oriented towards the (unstated) motive for the utterances; and this motive is the production of an interview/think-aloud text. That is, the talk not only realizes the interview as an activity in a concrete way, but also presupposes the latter. The talk therefore both produces *this* interview as a singular event and reproduces the interview as a culturally recognizable phenomenon (activity). They do so even though neither has a schema that *determines* how *this* interview is to turn out; it is only under the condition of uncertainty and the absence of a *determinant*, mechanically realized script that an interview/think-aloud session can turn into a tutoring event. That is, the participants have been able to move from doing an interview to doing a tutoring session without any apparent break, smoothly moving from one into the other.

There is at least one additional dimension of everyday conversations available in the transcript, one that most science and mathematics educators do not yet attend to, but which plays an extremely important role in everyday life: prosody. This dimension is important because it provides participants in a situation to make inferences or simply to understand the emotional engagement of their mutual others. There are pitch contours (‘‘‘, ‘`’, ‘^’); rising (‘,’), level (no punctuation) and falling pitch towards what become ends of utterances (‘;’, ‘.’); drawn out sounds (‘:’); sounds run together (‘=’); sudden shifts in the pitch level (‘↑’); changes in speech volume (CAPS=much louder than normal); and decreasing loudness (‘<<dim>’). Together, these features are expressions of emotional states, agreement and disagreement, harmonious and discordant forms of collaboration in producing the situation at hand (e.g. Roth, Tobin, Carambo & Dalland 2005). Thus, for example, listening to the tape we can clearly hear (experience) Daniel’s resistance (3–6), and his ‘resolution’ to provide Annemarie with a hint (7, 9). Each activity is characterized through and through by its emotional-evaluative dimensions that participants make available to each other, among others, in and through their voices.

Much of the understanding that hearers/readers develop during a hearing/reading of the episode for a first time is based on our cultural competence, including the emotional states of the participants. What we hear are not just sound patterns with clear pitch modulations, pauses and variations in intensity, but words. (In cases of poor signal-to-noise ratios, we may not hear words but indeed mere sound.) We hear that what has been phonetically transcribed as ‘your’ (in IPA [ju:r] or [juɘ]) would in written English be transcribed as ‘you’re’, constituting the shortened form of ‘you are’. This is important, for we know that members of some Asian cultures do not hear the difference between l and r so that the sound might be transcribed by them as ‘yu:l’. We also understand the repeated sound ‘an’ (turn 16) as a shortening of ‘and’. To fully understand utterances and other communicative productions in the way they were meant requires us to know – according to a line of psychological and philosophical reasoning that falls under the term of cultural-historical activity theory – the *activity*, that is, one identifiable part or dimension of society that produces outcomes relevant to its maintenance, sustenance and development. If we do hear the recorded speech in the way articulated here, it is because of our own cultural competences that generally remain un-thematized and therefore constitute our analytic blind spots, including our blind spots with respect to the societal-hierarchical relations that mediate any activity, as well as the actions and unconscious operations that realize them.

What can we learn from this analysis and where should learning researchers head next? Lev Vygotsky's (1986) method has considerable implications, which he had not fully developed so that it was up to others to formulate unit analysis. The function of speech is communication. This sets up a way of understanding the word, meaning and speech in a much broader sense: as collective phenomena. For example, words cannot be reduced to the individual, because the basic characteristic of words is the generalized reflection of reality. Words specifically and language more generally 'is a practical consciousness-for-others and, consequently, consciousness-for-myself' (p. 256). 'The word', notes Vygotsky, following the materialist philosopher Ludwig Feuerbach, 'is a think in our consciousness … that is absolutely impossible for one person, but that becomes a reality for two' (p. 256). Clearly, the word is not individual, as it 'is a direct expression of the historical consciousness of human consciousness' (p. 256). The unresolved question is how the word and language, clearly changing at historical timescales, can change unless there is a mechanism of change *explicitly built in* to the theory of speaking.

In this exemplifying case analysis, we can see how human beings concretely realize a collectively motivated activity that orients them but only exists in and through their concrete work (actions, operations). More so, there are many aspects in their concrete actions that the participants are not aware of, such as prosody, or what the next word will be that comes to their mouths; and these non-conscious aspects of the production of actions are operations. In this commentary, I use this way of looking at situation and discourse to identify commonalities and differences that exist with a special interest in identifying inner contradictions within and across the research methods and research results presented by the different contributors. Consistent with my dialectical materialist approach, I follow Evald Il'enkov (1982) in considering contradictions

> not as a mere *subjective* phenomenon which regrettably occurs in thought due to the imperfections of the latter, while dialectics considers it as the *necessary logical* form of the development of thought, of the transition from ignorance to knowledge, from an abstract reflection of the object in thought to an ever more concrete reflection of it.
>
> (p. 234)

It is out of such contradictions that development and growth of scientific fields generally, and of the learning sciences particularly, arise.

From units of analysis to the analysis of units

In any research, the question of the unit of analysis is primordial, because it is impossible to integrate when the different moments – structural parts that cannot be analysed independently of other parts because they are mutually constitutive – to be considered deal with different entities and phenomena. The analytic units chosen by the contributors to this volume differ, sometimes focusing on thinking, speaking and learning by individuals (e.g. Schwarz et al., Yerushalmy, Howe), sometimes on groups and their language (e.g. Tiberghien, Asterhan). In any event, the largest unit considered in the chapters is the classroom, where students are studied engaging in some task, which may be building an artefact (e.g. Hakkarainen), arguing and seeking consensus (e.g. Howe, Baker), using technology (e.g. Yerushalmy, Hoppe), or producing identity (e.g. Cobb). These units generally are below the level in which I am personally interested at this moment in my career: How do societal-hierarchical structures and associated cultural practices mediate student learning? Answering requires me to study what students are engaged in, what and where they go when they leave home in the morning: 'schooling'. Thus, I am both interested in a more overarching constitution of each situation and in a more microgenetic analysis of how participants produce it. It is only when I take this aspect into my research, theoretically and methodologically, that I can begin to understand why African-American and Aboriginal students do worse on standardized examinations than middle-class students. I understand my unit of analysis to include those of the chapter authors and the real question is under which assumptions I can abstract their situations from the broader societal, ruling relations in which their research participants find themselves.

The chapters provide interesting accounts of how students talk and argue. However, I need to know more to understand not only *their* students' successes but also the failure of working-class students around the world. Can I generalize the findings about cognition in the present contributors' classrooms to classrooms generally? The chapters provide me with descriptions of what might happen to discourse and argument when students buy into the activity of schooling and align their own goals accordingly; I know what happens when I look into these classrooms as bubbles disconnected from everything else, including the school, its neighbourhood, its community, its urban context and so on. But without the connection to the larger context of each situation, I do not get a general description of learning. I do not know, for example, about those students who are not present, which, in some of the schools where I taught and conducted research, might account for 25% of the population, many of whom were excluded for 'inappropriate behaviour', 'lateness', and other 'truancies'. Those who are present contribute to the reproduction of schooling by not challenging its status quo. Thus, to properly understand the actions of any subject, I personally take into account the relevant activity that they realize in so doing. Staying at the task level – which many chapters describe explicitly (e.g. Schwarz, Michaels, Asterhan) – I do not know why individuals do what they do. More so, I personally take into account the motive of the activity and the extent to which the individual has taken it up because it will mediate whether students engage in expansive learning, which is motivated by the increasing control over students' life conditions, or whether they engage in defensive learning, which is motivated by the avoidance of punishment, suspension, low grades and other repercussions (Holzkamp 1993). For me, students participate in and are agential subjects of the activity of *schooling*. In everything they do, they produce and reproduce schooling as a form of activity, in all its variations, realizing all its possibilities, desired or undesired by school administrators, including 'being off-task', 'screwing around', 'hanging out', 'smoking in the toilet', and other forms of (undesired) behaviour.

The question of the appropriate unit of analysis for me is the most fundamental question for conducting research, for its choice determines what lies inside and outside the field being considered and what is relevant in the theory of the phenomenon of interest. The focus on tasks and actions reduces knowing to 'cold cognition' and eliminates concerns for emotions, motives, motivations, ethical relations and so forth. I ask myself, 'Why would Schwarz's students produce epistemic actions, why would Tiberghien's students construct physics knowledge, or why would Howe's students seek consensus?'. So, whereas these studies show us in great detail how some students participate and learn under certain (to-be-specified) conditions, I would, in my research, seek more to provide accounts of and theorize learning *generally*, including emotion. For cultural-historical activity theorists, it is only through the motive-giving activity that emotions come to be the driver of thinking. It is therefore surprising that we do not find emotion more centrally in the literature. Writing in the early 20th century, Vygotsky (1986) suggested that research in which psychological wholes are analysed in terms of elements thought to constitute the former 'provides no adequate basis for the study of the multiform concrete relations between thought and language that arise in the course of the development and functioning of verbal thought in its various aspects' (p. 5). Accordingly, he proposes substituting psychologists' traditional methods of elemental analysis with *unit analysis*. To him, the proper unit for studying the 'internal aspect of the word' and the 'internal aspect of speech' is *word meaning* (p. 5). He argues that in distinction to sensation, thought is 'a *generalised* reflection of reality, which is also the essence of word meaning; and consequently that meaning is an act of thought in the full sense of the term' (p. 6). For Bakhtine (1977), too, all speech is characterized by its place in the hierarchical relations of society: 'We always have to take into account of the social-hierarchical situation of speech in the course of transmission' (p. 171). Thus, I hear Sarah Michael's participants Ms Davies and Paulo as speaking in ways that not only produce the topic at hand but also reproduce *schooling* in both structure and content. In so doing they collude, thereby making what happens recognizably a moment of schooling. I attempt to take into account the specific discourses in the linguistic consciousness of a social group in a given epoch because 'the conditions of verbal communication, its forms, means of differentiations, are determined by the socioeconomic conditions of this epoch' (p. 172).

For me, activities are entirely marked by their *societal* character, which mediates what the participating subjects produce, learn and (can) do. It is easier to study actions and operations so that psychological research generally focuses on these units (Zinchenko 2004), leaving aside any concern about motives, motivations and emotions that are treated as external to cognition. If I do not take into account the level of the activity, schooling, then I cut myself off from understanding how inequities in society, how, for example, the very structure of schooling mitigates against the participation and therefore learning of students from under-represented classes of society, gender or culture/language. Without further research, I do not know whether the conversations Ms Davies has with her students are consistent or inconsistent with their normal ways of being and therefore whether they enact symbolic violence. I do not know whether to accept the authors' abstraction from the context as legitimate or whether I should have qualms about the analytic move. I am certain that the First Nations students in the Pacific North West would not participate in the forms of argumentation that Asterhan champions, because in this culture, function and structure of argument goes against the culture. That is, separating cognition from the cultural-historical instant, separating form and content, and, therefore, dissociating cognition from societal-hierarchical contexts leads me to the implicit or explicit identification of those identified as 'being outside reason', that is, 'colonial people, women, proletariat, the mad, children' (Walkerdine 1997, p. 59). Unless I know the role of the emic (insider) concept of *respect* in the streets of Philadelphia, I cannot understand that 'dissing' (i.e. showing disrespect) a teacher in a science classroom actually may lead to increase the amount and levels of respect a student gains within his peer community. Unless I know about the forms of ruling relations among urban and middle-class youths and how schooling is consonant with the cultural practices of the latter and dissonant with the cultural practices of the former, I do not know why students do what they do, the extent to which they take up the goals in accomplishing school tasks. Failing to take into account the forms of telling stories in the homes and culture makes me miss the very reason why Hawaiian children do not do well in schools that produce and reproduce the culture of the white, Western people (Au 1980). Thus, it would certainly be astonishing to some educational researchers with cultural-historical sensitivities why, for example, Baker and Asterhan advocate argumentation and why Howe supports consensus building – typical middle-class behaviours – without also investigating the level to which such school tasks are in/consistent with the students' everyday culture and therefore how the latter mediates students' engagement in the task.

In thinking about unit analysis, a problem derives from the choice of the concept 'activity', which, when it occurs, pertains to the tasks or activeness rather than to what activity theorists mean by the term. Linguistically more sensitive translations use the terms 'activeness' and 'activity' to produce equivalent expressions to the original (German/Russian) *Aktivität/aktivnost'* and *Tätigkeit/deiatel'nost'*, the latter of which clearly is societal in nature. The theoretical distinction allows me to better understand the Schwarz chapter, where activity denotes a process at the task level that should be 'driven by the goals of understanding and convincing'. In part, the disattention to the societal mediation arises from the fact that the German/Russian adjectives and adverbs pertaining to society ('*gesellschaftlich*', '*obshchestvo*') are translated as 'social' rather than 'societal'. This makes it difficult to understand how schooling practices lead students from some social classes preferentially into blue-collar jobs (e.g. Willis 1977) or unemployment and poverty and students from other social classes (middle, upper) back into the same social classes. Schooling viewed as societal-hierarchical activity, with its own ideologies, discourses and ruling relations brings into my focus this reproduction of inequities through the reproduction of differences in cognition and achievement. If there is a consistent positive correlation between socioeconomic status and school achievement, which has increased over the past two to three decades (Sirin 2005), then I take this as an indication for the mediational role of societal structure on what and how students learn and on their future choices. In this way, 'our society may be failing in one of the greatest commitments of every modern society, that is, the responsibility to provide educational opportunities for each student regardless of social and economic background' (p. 45).

Timescales, developmental scales

Related to the question concerning the unit that retains the phenomenon in its entirety is the question about the scales of time at which phenomena occur. Thus, activities, actions and operations, though part of the same unit, nevertheless occur and unfold at different scales of time. There is then a question about change and how quickly or slowly these changes occur at the different levels within a unit. In most of the chapters, the scales of time on which school-related phenomena are dealt with are implicit (e.g. Howe, Yerushalmy, Hoppe, Baker). In other cases, timescales are explicit in the focus on – to use Saxe's concepts – microgenetic, ontogenetic ('mesoscopic') and sociogenetic ('macroscopic') processes in the classroom. In one chapter (Hakkarainen), the learning processes occur and are studied at even longer timescales extending over several semesters. I would be interested in seeing how the authors as a collective would study the interrelation between culture more generally, which embeds current schooling practices, the timescales at which it develops, and the ways these mediate individual developments (ontogenesis). I suspect that what and how students learn in a mathematics classroom today is likely to differ from what and how other students learned a decade ago so that for me, a *general* theory of cognition *must* include such changes or it misses its central phenomenon much as linguistics misses its central phenomenon by studying language as static. This is a direct consequence of the assumption that culture and language undergo change each time a word is said, the only way that we can think of change as inherent in and driving a system. More so, the relations between processes occurring at the different scales are, at best, external and, at worst, completely unrelated. Collectively, the chapters do not allow me to understand *how* change can occur at all or *why* it occurs. Why and in which directions does classroom discourse change? Are changes this year the same as they were last year? Why would students want to change? How does mathematical learning change because of changes in language, curriculum, setting, culture and so on? Unless I seek a dynamic understanding of cognition, the movements of the systems I study (mind, culture and activity) are due to external rather than internal forces.

In many ways, these conceptualizations of time that I find in the chapters are consistent with my own previous view of three orthogonal temporal axes (Roth 2001). The first axis allows researchers to engage in microgenetic studies of the moment-to-moment production of actions and discourse in a classroom; the second axis focus concerns the ontogenetic development and transformation of individual actions and knowledge; and the third axis concerns the transformation of classroom-level cultural practices. But theorizing and thinking in terms of three orthogonal timescales, processes and forms of development (micro-, onto- and sociogenesis) keeps time external and the linkages between them an external relation. Over time I have come to understand that to properly conceptualize activity, I need something else that would allow me to arrive at the kind of analyses that I provided in the introductory section: a way in which neighbouring temporal scales – or rather, the corresponding processes – can be thought as constitutive of one another, that is, I need to think them in the same dialectical fashion in which I think the relation between activity, actions and operations (which do, in fact, occur at different timescales). Unless learning – detectable in students' changing language – is *inherent* in every act, I cannot explain changes in action across mesoscopic (ontogenetic) and macroscopic (sociogenetic) timescales as system-internal.

To theorize a system (students within classrooms within schools within society) dynamically, change for me has to be inherent and has to be produced at every instant, which is the case when thought and speech are theorized as changing processes related by means of a changing relation. Every speech act then not only constitutes a reproduction of a sign but also a creative production that changes spoken language as such in the very speaking – not only at the individual or classroom level, but cumulatively and in both synchronous and diachronic ways, in society as a whole. Even if the same word were repeated within the same utterance, it would no longer be the same *precisely because* of this repetition and the associated change. Thus, I understand each of Saxe's, Cobb's or Nathan's students not only as agents who produce and reproduce classroom life but also as constitutive moments of their culture more generally and the continuous changes it undergoes. If I think these students as reproducing (mathematical, scientific) culture, language, discourse, identities and the like, I merely get a static theory of learning. But it is a fact that we all continually witness changes in culture generally and school culture specifically: what was possible yesterday no longer is so today, and what could not even be thought yesterday is common practice today. To build change into a theory of culture (language), it has to occur each and every time someone acts because, 'language is not transmitted, it endures and perdures in the form of an uninterrupted process of evolution' (Bakhtine 1977, p. 117).

Central to all chapters in this book are the foci of thinking or speaking. If I theorize thinking and speaking as mutually constitutive (see next section) and dynamic processes, as Vygotsky proposed, then I arrive at a model that is inherently dynamic, one in which the changes at all levels are directly linked. If each speech act, word or utterance is a creative act that changes not only what is available to the individual (see speaking and its relation to thinking) but also what is available to the group, classroom and culture, then the microgenesis comes to be directly and, importantly, *internally* linked to ontogenesis, sociogenesis and cultural evolution (Roth 2007). Cultures and languages change because each speech act changes not only the local situation but also culture as a whole. The particular trajectory of the conversation in the class that Michaels and her colleagues analyse then is understood as coming out of the question–response pair that Ms Davies and Paulo enact. In its unfolding, their question–answer pair *is* constitutive of a changing culture. I might expect very different trajectory of the conversation, ontogenesis and sociogenesis if Paulo had uttered, 'twenty-four is even because divisible by two'. That is, his incorrect answer sets up new and (likely) unanticipated opportunities for engagement and learning. I understand each subsequent turn in this way, as the contingent production of resources for further development of the conversation (system) specifically and culture more generally.

Thinking (inside) and speaking (outside)

Speaking, and its reduction/abstraction into written language, is the dominant mode of human–human transactions. Without speaking, there would not be anything like consciousness, culture and thinking. The two participants in the interview/think-aloud protocol at the beginning of this chapter (David, Annemarie) use language as their primary mode of participating in the event. They not only find themselves *in* the event where they talk, they also *make* the event for what it is. Without their talk about graphs, the interview would not exist; but without the interview they would not talk or talk in this way. They do not produce an interview as an entirely new thing, as something utterly singular, but they produce the event in a way that they and others recognize *as* an interview/think-aloud protocol. What they say cannot be understood other than out of their participation in this kind of situation. Scientists like Annemarie talk very differently about graphs when they are at work and again differently when they write about them in a scientific publication. But in talking, both presuppose the intelligibility of what they are saying, which means that both content and form already have to be within the range of possibilities of the specific and generalized intended audience. That is, although David and Annemarie produce the sounds that we hear in the form of the words captured in the transcript, these words are irreducibly societal-hierarchical and ideological: the talk clearly is not intended for a car mechanic or palliative nurse.

Language allows us to appreciate in a different way the nature of intersubjectivity, the topic of Nathan et al.'s chapter, which is taken to be the result of an active production very similar to consensus production (Howe). For Bakhtin and Vygotsky, speaking *presupposes* consensus and intersubjectivity: Language is practical consciousness-*for-others* as much as it is consciousness-*for-myself* (Vygotsky 1986, p. 256) and every 'utterance is social in nature' (Bakhtine 1977, p. 119). Bakhtin and Vygotsky consider every discourse, every utterance, as reflecting the societal-hierarchical relations in which the present moment is embedded. But if every utterance is societal-hierarchical and ideological, then consciousness not only is for the other but also has come from the other to which it returns. Consciousness is collective knowing, that is, knowing (sciere) together (con-) in the pursuit of controlling the environment to secure it for the species and all the individuals that constitute it.

Some contributors to this volume make language an explicit topic (Michaels), others leave it more implicit (Tiberghien, Saxe), and again others do not make it thematic (Yerushalmy, Hakkarainen). To be able to integrate these different studies, I require some theory that relates thinking (concepts) on the one hand and speaking on the other. There are different implicit theories about this relation at work which renders it difficult for me to understand how the contributors think about what occurs between individuals (interpersonal, -subjective, -individual) and other (corresponding?), and within individuals (intrapersonal, -subjective, -individual). Thus, in the Hoppe chapter, I read about the 'externalization of design ideas' and about 'external constraints on the learner'. In the Asterhan chapter, I have descriptions of 'socially constructed scripts' that individuals appropriate so that they become 'individual internal scripts'. Baker theorizes personal opinions as intrasubjective phenomena and publicly accepted matters as intersubjective phenomena. In focusing on intersubjectivity, Nathan problematizes the relationship *between* people. For Schwarz there is a difference between internal representations and epistemic actions that clearly have an external orientation and place. Tiberghien writes about knowledge as something that is shifted from the outside, knowledge to be taught and known by the teacher, to a place inside the student, something to be learned and appropriated as a function of the didactical contract between the parties involved. I ask myself, 'Why is this separation between the internal and external so pervasive when cultural-historical researchers attempt to overcome and abandon it?' and 'Is this separation part of an ideology that allows testing students independent of context, making assessments, and attributing achievement and knowledge to individuals according to a Gaussian distribution along a single dimension?'

To integrate across chapters, I need to know how the authors theorize thinking and speaking. Baker, for example, writes about the student Chloé as expressing a neutral opinion and thereby fails to account for the relation between a personal thought/opinion and the context in which language is a medium from, for and returning to the other. For me, what Chloé says cannot be singular, for no one would understand her. What she can say is constrained by the culturally and historically possible and situated ways of speaking; and because what has been said mediates the development of thinking, 'her' opinion cannot be hers because it is inherently a function of language and therefore ideological (societal-hierarchical). The idea of language as a system of stable forms and thought as a singular, subjective experience are abstractions and fictions that do not adequately render the concrete reality of language, for 'language constitutes a process of uninterrupted evolution that realises itself through the social verbal interaction of speakers' (Bakhtine 1977, p. 141). The laws of this evolution essentially are sociological laws and the *structure of the utterance* is entirely societal-hierarchical in nature. But language and thought are not related in a linear way; nor do they develop along the same microgenetic, ontogenetic and cultural-historical trajectories. Bakhtin and Vygotsky theorize speech and thought as obliquely related so that speech cannot be an external representation of an inner thought that has been expressed like toothpaste from a tube. These constitute two processes that overlap, intersect, but undergo their own trajectories. In fact, in thinking (process and content), speaking changes because in the articulatory process it is forced to appropriate external matter, which comes with its own rules that are external to internal thought. It is only for idealist subjectivism (Kant, constructivism) that the expression is formed on the inside and externalized via a process of translation.

To overcome the theoretical problems that arise from the inside/outside dichotomy, we might want to abandon the qualitative distinction between the inside and outside (Bakhtine 1977). Others in the cultural-historical tradition – including Leont'ev, Mikhailov and Vygotsky – fully agree with this position, theorizing the relationship between consciousness and material reality as a dialectical one. Thus, when Chloé (in Baker's chapter) says, 'I don't yet have a fixed idea, I think that there are as many arguments for as against GMOs', she must presuppose the intelligibility of this utterance, which therefore realizes in a concrete and singular manner the collective possibility of describing a state of affairs. What she says not only describes a general possibility but also allows her to understand herself at the moment in cultural-historically appropriate ways. This is so because 'it is not mental activity that organises the expression but the expression that organises mental activity, which models it and determines its orientation' (pp. 122–123). Others concur. Thus, a 'language can live only when all the means of people's objective activity, all the historically evolved objects of culture become in it and through it means of living intercourse between people and the individual's internal communion with himself' (Mikhailov 1980, p. 230).

The question of meaning

One of the hard questions, if not the hardest of all in our field, is that of meaning. Confusion easily arises because educational scholars tend to use the term in different ways all the while assuming they are talking about the same thing. For me, David and Annemarie in the introductory episode do not have to *construct* (understood as conscious action) meaning. Knowing that they participate in an interview/think-aloud protocol mediates what they say, which in turn reproduces the event in its particular form. The event is experienced as an immediately meaningful one, even in those moments when the two deviate from the protocol and produce a tutoring session. They do deviate without, as the transcript shows, expressing doubt. There is a great consistency across the chapters assembled here in using certain theoretical terms, but I suspect that they are used differently. To integrate, I would first have to ascertain whether authors use the same terms similarly or differently. There are both possibilities and constraints in using the same terms differently within the same volume. On the positive side, different uses make salient the range of semantic variations a concept has within a community of practice (here, our own); and, on the negative side, different uses lead to Babylonian misunderstandings.

In my reading, the contributors use 'meaning' differently – and therefore, in Wittgenstein's (1958) way of thinking, produce different meanings of 'meaning'. What I would need to successfully integrate across the chapters is a conversation among the authors about how they see themselves differing from or aligning with each other. Thus, the term and its associated adjective (meaningful) and adverb (meaningfully) appear in a variety of contexts and guises, and sometimes it does not appear at all (Howe, Asterhan). For many authors, it is a product of individual or social action ('making meaning'), may exist in 'connections' to other meanings (Hakkarainen), and becomes contextualized (Michaels). It may be 'broad', 'everyday', 'conventional', 'anthropological' and a function of context (Tiberghien). Meaning may be 'symbolic' (therefore inherently materially embodied), so that it can be 'signalled' and 'interpreted' and owned to become 'students' meaning' (Michaels) or it may be 'mathematical' (Schwarz, Cobb). Meaning somehow comes to be attached to words such as in talk about the 'meaning of indicators' (Hoppe) or in the meaning of 'function representation' and 'equal sign and rewriting equations' (Yerushalmy). Meaning may be 'institutionalized' and 'institutional' (Cobb), 'quantitative', 'different' (Cobb), 'multiple' (Nathan), 'twofold' (Hoppe), 'colloquial' (Cobb), 'particular' (Saxe, Nathan, Cobb) and 'cultural' (Cobb). Ordinary processes may become modified when they are full of meaning, marked by the adjective or adverb 'meaningful'. Thus, not only 'learning' (Hakkarainen, Saxe) and 'engagement' (Hakkarainen) may be meaningful but also 'interpretations' (Nathan), 'questions' (Baker), 'situations' (Schwarz) and 'language' (Yerushalmy).

For me, the difficulties with the term arise from its unspecific relation to another, cognate one: sense. Thus, students may 'make sense of results' (Schwarz, Michaels, Saxe) and something – model, word, concepts, equivalence problem, communications, mathematical ideas – may be used in one (some) rather than another sense ('in the sense') (Schwarz, Tiberghien, Yerushalmy, Hakkarainen, Howe, Hoppe, Baker, Asterhan), which may be 'traditional', 'concrete' (Hakkarainen), 'shared' (Michaels), 'shallow' (Hoppe), 'cultural' (Asterhan) or 'purely literal' (Baker). We also may have a sense of something, such as a sense of 'efficacy in school', 'running [one's own experiment]' (Michaels), or 'affiliation', 'oughtness' and 'competence' (Cobb). Language is the tool for making this sense, which may be that 'of the world' (Michaels). Furthermore, the expression 'in the sense of' implies multiple senses of which one is stated following the expression. To make sense, the sense of a word has to be contingent: there are as many senses as there are contexts and possible significations. Contrasted with this plurivocity (multi-voicedness) of the word across situations, dialectics, intonations and so on is its unicity: there is one word (i.e. the one has, is used in, the different senses). To me, this polar opposition between plurivocity and unicity can be overcome only in a materialist-dialectical approach that studies the living and dynamic comprehension on the part of the speaking subject engaged in verbal communicative processes.

To confuse the matters even further, the terms sense and meaning are related to a third term: reference. A diagram may be used to referentially link utterances to previous utterances (Schwarz) and teachers may use a single reference where they ought to use multiple references to something like 'the inertial principle' thereby establishing a link between 'taught knowledge' and 'disciplinary knowledge' (Tiberghien). Theories and predications may be explicated 'with reference to shared observable physical events' or we may use language to present ourselves in 'reference to the collective endeavour' (Michaels), talk produces 'abbreviated reference to arithmetic procedures' (Saxe) or makes 'repeated references to beliefs' (Cobb). More indirectly, reference is articulated in talk *about* a variety of topics: 'solutions', 'demonstrations' (Nathan), 'views' (Michaels, Cobb), 'persons' and 'possible futures' (Cobb), 'different ways of repartitioning the number line', 'equivalence', and other '[mathematical, scientific] topics' (Saxe, Howe, Yerushalmy, Schwarz) or '[scientific, terribly important] issues' (Howe, Hakkarainen, Schwarz) such as 'survival and selection' (Asterhan), 'what should or should not be done' (Baker), 'ideas' (Michaels, Howe), or 'views' (Michaels, Tiberghien).

Vygotsky (1986) does not always help us in better understanding the meaning of 'meaning', at least not in the English translations of his work. Thus, 'meaning is only one of the zones of sense, the most stable and precise zone … and remains stable throughout the changes of sense' (p. 245); and any change occurs as a function of context. But elsewhere in *Thought and Language* he suggests that word meaning develops, together with the continually developing processes of thinking and speaking, where he understands these developmental processes to occur on and be constitutive of microgenetic, ontogenetic and phylogenetic (cultural-historical) timescales. To him, word meaning itself is a *process* that yokes together – in a dialectical fashion – two independent but criss-crossing, mutually constitutive processes of thinking and speaking. Word meaning simultaneously belongs to 'two different spheres of psychic life' (p. 212). In fact, meaning itself is not some *thing* but a continuously unfolding process whereby 'subjective mental activity dissolves itself in the objective fact of the utterance having taken form, whereas the articulated speech subjectivises itself in the act of decoding that sooner or later the encoding of a response' (Bakhtine 1977, p. 67).

More confusion concerning the issue of meaning is created when we look at two different translations of Bakhtin into English and French. What comes to be consistently denoted in French by referent ('*référent*') and '*sens*' is translated into English into the same word 'meaning' ('everything ideological possess meaning it represents, depicts, or stands for something lying outside of itself'); similarly, what is rendered by means of the French word '*sens*' appears in English most frequently as 'meaning' but sometimes also as 'sense'. In the translations of the works of Leont'ev, the German '*Bedeutung*' has in English sometimes the correspondence in 'signification' and at other times as 'meaning'. In the sense of signification, '*Bedeutung*' (etymologically related to '*deuten*', to point) is related to the French *référent*, especially given the pairings with the German '*Sinn*' and French '*sens*'.

Discussing the ideas of Bertrand Russell, Mikhailov (1980) suggests that for the former, 'social (universal) significance' and meaning, were the same and characterized by 'social "depersonalised" language'. He uses the example of the word 'rain', for which the meaning is 'only that which is repeated in an autumn drizzle and a tropical downpour' (p. 184). For Mikhailov, on the other hand, the 'meaning of things [is] objectified in practical activity' (p. 197). If anything is meaningful at all then it is purposeful, motive-oriented, practical activity with all of its contextual particulars. It is in and to this intuitively understood practical activity that new words and things accrue, and it is precisely because words and things can denote in a metonymic fashion the situation, activity or context as a whole that human beings come to experience something as having meaning or being meaningful. It is precisely (a) when the object or word comes to denote 'the aim of collective action' that the 'individual will experience the image' (word, idea) as an external object that it (b) comes to have 'meaning in itself' (p. 197).

For Leontjew (1982), sense expresses the relationship of the motive of activity and immediate goal of action, where motive constitutes the objective states and relations that orient the activity towards a determinant outcome. Meaning, on the other hand,

> is the reflection of reality independent from the individual relations of individual humans have to it; the human being finds an already completed, historically developed system of meaning and appropriates it in the same manner that it appropriates the tool, the carrier of meaning.
>
> (Leontjew 1982, pp. 259–260)

Reality presents itself in the form of meaning, which thereby mediates the coming into consciousness of the word. People are conscious of the world as a societal phenomenon because the reflection of the world is grounded in and accounts for societal praxis. Meaning only exists in and is realized by concrete human beings in concrete relation with each other and the material world. It is that form in which individual human beings appropriate generalized and reflected human experience.

Emotion, affect

If there is one aspect that I would like to have seen addressed more in the collection of chapters then it is this: affect. I am convinced that the study of language, generally recorded by electronic means, would give the contributors to the present volume the data sources that they need to study at least some aspect of this phenomenon. Cultural-theorists from Vygotsky and Bakhtin to Leont'ev and Mikhailov insist that without emotion we cannot understand a single thing about cognition, language, human behaviour, personality and learning. Without emotion, cognition is cold and easily reduced to information processing. More so, 'scientific activity, with a purely cognitive motivation, disappears and there occurs – this has been clinically demonstrated – a decline in the substance of cognitive activity' (Leontiev 2005, p. 49). To me it is a bit surprising that the contributors to the present volume, all more or less squarely situating themselves in socio-cultural perspective, pay little attention to this central moment of consciousness, thinking and cognition. Cobb mentions once that identity issues are frequently subsumed to affective factors; and Michaels mentions emotions in passing as the distinguishing feature between the physical and social world. The chapters suggest to me that thinking seems to be for itself, pushing and developing itself, finding 'reason' only in itself. The difference, however, between mere cognitive agents – e.g. computers, AI, disembodied thinkers – and living human beings is precisely that the former *process information*, react to and build representations of recurrences, whereas real human beings actually *think* and are conscious.

On the other hand, the emotive quality of everyday interactions is easily shown in the tone and intonation of continuing everyday speech. Thus, considering the introductory episode involving David and Annemarie, especially watching the associated video clip, it becomes immediately evident that their communication does not only involve words. The two individuals are conscious beings, attending to the business at hand whatever it might be, and relating to the social and material setting. In their talking, I hear changes in pitch (jumps, contours), changes in speech intensity (loudness), and changes in speech rate. Not indicated in my transcript but visible in the speech-analysis program I use, there are changes in the timbres of the voices, changes in speech energy across the spectrum, and changes in the contours of the higher frequency that constitute their voices. I can literally hear pleas for hints and resistance to give them; both are objectively available to anyone listening to the speech produced in the give and take of the conversation. All of these aspects of the human voice that constitute the accent and intonation of an utterance communicate appreciation, stance and emotion;

and this expressive intonation is the fundamental way in which societal-hierarchical relations are produced and reproduced (Bakhtine 1977).

Consciousness cannot be understood if we add up and relate thought, perception, memory, skills and emotions. An integrative framework requires me to theorize emotion and affect as a moment of the same analytic unit as cognition. Thus, Bakhtin, Vygotsky and Leont'ev think of emotion and affect not as an important *factor* in cognition and consciousness but as the very heart of collective activity, actions and consciousness. Emotion and associated motivation move out of sight (focus) when we look at goal-directed actions and conditioned operations (Leontiev 2005), the moments of human activity at the centre of much of learning-science research. It is only when I take into account the motive-driven activity that the central nature of emotion and motivation comes to the fore. Vygotsky (1986) noted that the separation of intellect and affect is one of the great shortcomings of psychology. The scant attention to emotion in the learning-sciences literature is evidence that not much has changed since he wrote *Thinking and Speaking* nearly 80 years ago. The result of omitting affect is that 'it makes the thought process appear as an autonomous flow of "thoughts thinking themselves", segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses, of the thinker' (p. 10). He suggests that

> such segregated thought must be viewed either as a meaningless epiphenomenon incapable of changing anything in the life or conduct of a person or else as some kind of primeval force exerting an influence on personal life in an inexplicable, mysterious way.
>
> (p. 10)

The important aspect of Vygotsky's approach is that emotion and affect constitute a core moment of human activity; they are not just *external* factors that mediate the practical consciousness that goes with the activity. Affect is not a *factor* but a *constitutional moment* of thinking. This is important, for if affect is not internal, then its constitutive relation in thinking cannot be understood, for example, when thinking produces changes in emotions or emotional states and task-related emotional valences change thinking. Vygotsky notes, 'Thought is not the superior authority in [verbal thought]. Thought is not begotten by thought; it is engendered by motivation, i.e., by our desires and needs, our interests and emotions' (p. 252). In his analysis, the unit now is understood as activity, which, therefore, cannot be understood independent of emotion. Emotions reflect in a refracted way the relationships between collective motives and (the possibility of) success (Leontjew 1982). For Leont'ev, emotions are relevant at the level of activity. Even when motives remain unrecognized, when the relation between action and activity remains unaccounted for, they are reflected in the human psyche in the form of emotional colouring of the action. Without affect at the very heart of cognition and consciousness, human beings would not be able to cognize something as dangerous, threatening, interesting or rewarding. Even talk can be properly understood only when I take into account emotions, not as mere psychological phenomena but as central to human activity generally and therefore subject to sociological, anthropological and social-psychological study.

The present contributors have in their recorded speech all that is needed to track emotions on a microgenetic scale, for each utterance communicates affective valences and appreciative orientations and values, mainly by means of expressive intonation (Bakhtine 1977). These orientations, valences and values are not external to expressions but are their very ground. In the same way as language, affect also is social through and through. Affect is one of the moments of the activity – understood in a cultural-historical activity-theoretic sense – as a whole. It is precisely because it is at the core of activity, individually and collectively, that emotions and speech can mutually mediate one another.

Coda: toward integration

After reading the different chapters, I wondered about the conditions that would make it possible to have sets of chapters about learning and teaching that are relatively diverse. In thinking about the source of this diversity, which is one across the field of learning sciences and educational research, my first thought was that it might be the result of thinking about cognition in terms of its attributes. But a combination of factors cannot constitute cognition or law (Hegel 1806/1977). 'Being externalities, [factors] are indifferent towards each other, and lack the *necessity* for one another that ought to lie in the relation of an outer and inner' (p. 188 [¶314]; emphasis added). Throughout this chapter I take a perspective that allows me to think of the various moments of knowing and learning in a coherent way, coherent because these moments are internal to the phenomenon of interest. But this coherent way itself embodies inner contradiction, which constitutes the *necessary* condition for any system of knowledge, including epistemology (Schelling 1977).

The chapters in this book articulate the state of the art of research on knowing and learning in mathematics and science classrooms and a number of important moments that characterize schooling processes (consensus building, making arguments, building artefacts) and their outcomes (knowledge, understanding, practices). At the same time, the reading of these chapters left me with the sense that the different research agendas occur independently and have little influence on one another. But most people do experience life as a single entity. A student goes to school, realizing motives and goals, producing and reproducing emotive stances, participates in conversations in which emotive stances are produced and reproduced, and, in the process, learns a variety of things often unrelated to school subjects and develops aspects of an identity. A common framework is required to allow me to understand how all these different research pursuits hang together such that they reproduce and represent the unicity of a student life for truly understanding what it means to learn in the here and now of the situation, under these circumstances in my life, and at this point in history.

Both explicitly and implicitly I make the case for the use of a more integrating framework that encompasses not only all of the issues addressed in the various chapters but also shows how these mediate one another in mutually constitutive ways. For me personally, cultural-historical activity theory with its grounding in dialectical materialism constitutes a suitable solution to the problem. More so, the various dimensions that are only externally related or that occur at different levels and scales of time mutually influence one another, as they are thought in a dialectical and therefore irreducibly connected way. Thus, I propose that we *not* look at:

- students' conceptions and misconceptions independent of the setting in which they are produced;
- misconceptions independent of the linguistic competencies that allow both the educator/psychologist and student to participate in interviews about misconceptions – the very fact that the interviewer/analyst understands the relevant talk, even though it comes to be denoted by the interpretive-analytic theme of misconception;
- classroom situations independent of the motives and motivation of schooling and the goals of students and teacher;
- classroom events outside their cultural-historical trajectories and the biographical trajectories of all of their students;
- classroom and interview situations independent of the emotion, important in the assessment of whether motives and goals have been achieved;
- language independent of the societal-hierarchical structures (ideologies) that it produces and reproduces; or
- identities independent of the culture, language and biographies that embed student life outside the classroom, both in the school (e.g. hallways, lunch rooms, school yards) and at home, in the street, in the pursuit of hobbies and sports.

Important in the form of research that I advocate and exemplify in my introductory analysis is the dialectical approach that recognizes and theorizes the mutual dependencies on the inside of human activities considered as the minimal unit of analysis. All the items in the bulleted list need to be considered. At a minimum, I encourage researchers to indicate under which conditions this or that moment of activity may be disregarded for the moment and still achieve (generalizable, transportable) knowledge that can be used in other settings to make sense there as well.

In closing, I reiterate my (sup-, presup-, pro-) position that it is legitimate to abstract learning situations from the context as long as we provide empirical evidence that this abstraction does not limit the generality of claims. In other words, when I do abstract learning situations from their societal context, then I articulate the extent to which what I have learned in and through a study generalizes to situations other than the one I have been looking at. One of the unresolved issues in qualitatively oriented learning-science research is the accumulation of studies the results of which are non-comparable because of the lack of context, which thereby interferes with any attempt to learn how anything learned in one situation can be transported to inform other situations.

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Chapter 19

Sociogenesis and cognition

The struggle between social and cognitive activities

Sten Ludvigsen

Introduction

The chapters in this book give us very rich descriptions and ways of understanding how learning occurs in classroom settings and learning environments more generally. An important question then becomes: in which ways do these descriptions contribute and add to our understanding of learning activities? More specifically, how do different types of interactions contribute to students' participation and understanding of concepts and conceptual systems in domains like mathematics and science? The chapters also provide us with rich frames of interpretation of empirical data.

We have been through a period of 20 years where different perspectives and metaphors for learning and knowledge construction have been discussed (e.g. Anderson, Reder & Simon 1996; Greeno, Smith & Moore 1993; Hutchins 1995; Lave 1988; Packer & Goicoechea 2000; Roth 2008; Vera & Simon 1993), where differences and incommensurability have played a major role in the discussion. We must take a step forward and explore what the different perspectives and metaphors can explain and in which ways these explanations could be connected and combined (e.g. Arnseth & Ludvigsen 2006).

In this integrative chapter, I will discuss how the different chapters in this volume can contribute to a richer understanding of how students learn to participate in learning activities, as well as of students' use and understanding of specific concepts in knowledge domains such as mathematics and science. In order to explore the relationship between the different chapters, both with regard to their frames of interpretation and empirical analysis, I need to establish a framework of interpretation that allows the possibility of moving between different levels of social and cognitive activities.

In the socio-cultural stance toward learning, development and cognition, we can differentiate between different levels of descriptions and explanation (e.g. Saxe, this volume; Valsiner & van der Veer 2000; Wells 1999). In all the chapters, social interaction constitutes the empirical focus; this refers to how teachers and students engage with material artefacts and language as tools. The goal of this endeavour is to understand how students make use of specific concepts in a knowledge domain. These processes can be categorized as the microgenesis level of knowledge construction. This level of description represents the empirical level, where the phenomena we want to understand unfold and are played out.

The question then becomes how to establish frames for interpreting what is going on at the level of microgenesis. In several chapters (which will be specified below), the authors turn their analysis to what we call ontogenesis, which refers to how individual students use and understand the concepts with which they work. Here, the activities are interpreted as the mental activities (mental models, cognitive structures, etc.) of the students, which become accessible to the researchers through different sets of methods such as thinking-aloud protocols or interactional data. In these chapters, the authors give very detailed and precise descriptions and analyses of how the mental activity is played out in the interaction. In such studies, it is important to know what the students bring to situations where their activities are analysed in order to understand what kind of comprehension emerges and whether the students make any progress.

In other chapters, authors make interpretative turns towards social norms and values, and the role of the institutional aspects of learning activities. Institutional aspects here would be, for example, the curriculum, the construction of tasks, the tools that students can use and how learning activities are socially organized (Lund & Rasmussen 2008; Rasmussen 2005). When the social organization of learning is the analytic focus, we can categorize this as the level of sociogenesis. At this level, there is an explicit focus on how knowledge is constructed as part of the intersection between the historical development of the institution and the teachers and students (Ludvigsen in press). The use of specific artefacts and language plays a major role in the researchers' analytic endeavours. In addition, we can understand the conditions for knowledge construction over longer historical periods of time (e.g. Donald 1991; Engeström 1987; Säljö 2000). This type of analysis is called phylogenesis.

I will analyse the different chapters in this book from the framework of interpretation described above, and explore how rich descriptions of learning events can give us a more robust understanding of knowledge construction as a multi-level phenomenon, where the different levels are understood as dependent on each other.

Framing assumptions: social and cognitive aspects

If we start to understand human cognition from the perspective of the social self, it means that we see cognition as constituted by historical, cultural and social aspects. Cognition is not reducible to innate or mental structures. Cognitive and social activities are interwoven, and can only be separated analytically. Human cognition is built up through experience and knowledge by participation in different settings. This means that human cognition is historically constituted, and that this development always unfolds and is played out in situ. The social self is a historically anchored subject. This level of description in terms of cognition is what I have called ontogenesis. All humans have their own unique development, which is of course connected with development on other layers, but can analytically be described as a layer in its own right. This perspective on human cognition also takes social interaction as the empirical starting point; however, as I mentioned in the introduction, human cognition is constituted as a multi-layered phenomenon.

Artefacts play a significant role in human history and for human cognitive development. An almost classical example is the graphical calculator. Knowledge over generations is built into an artefact like the calculator, and students can use it to perform mathematical activities in school and in the workplace. The artefact becomes a tool that mediates human cognition. It is the connection between the historical artefacts, and the use of the artefact as a tool, that creates the basic unit of analysis in a socio-cultural perspective; this basic unit is called the 'activity'. Activities are constituted by individual actions over time, but cannot be reduced to single actions. Actions represent the methodological starting point when we want to understand human cognition as sense-making in socially organized activities.

The level of ontogenesis describes individual knowledge construction anchored in social activities; microgenesis describes knowledge constructions through interaction and conversation between two or more humans; and socio-genesis takes into account historically developed knowledge in terms of artefacts and the social organization of activities within and across institutions. What unites these perspectives is that activities include social and cognitive elements, which are non-reducible entities. This means that giving privilege to either of these levels is not an adequate reduction. The different layers can of course be highlighted and emphasized, but they cannot be reduced to only one layer. The socio-cultural perspective takes the sense-making of the individual in collective activities as the premise for understanding human cognition.

I will make a brief remark about the style of this integrative chapter. It is written based on the premise that the reader has already read the other chapters in the book. I will restrict my comments to highlighting a few aspects from the chapters that are necessary for understanding the discussion in this chapter. I have taken up the chapters thematically, not in the order in which they appear in the book.

Transformation of knowledge in classroom interaction

The authors of this book all 'struggle' with the difficulty of understanding the relation between social and cognitive activities. This 'struggle' is dealt with at different levels: the theoretical, the methodological and the empirical. In some chapters, all of these levels are included, while in others, one or two are more prominent.

Abstraction in context

The chapter by Schwarz, Dreyfus and Hershkowitz ('The nested epistemic actions model for abstraction in context') is very rich, giving a historical overview and discussing different approaches to abstraction in the domain of mathematics, or to use their own term, abstraction in context. The chapter also includes extracts that give the discussion a concrete empirical layer. The idea of 'abstraction in context' already includes the layers discussed in the introduction; abstraction is perceived as part of an activity that is related to context. I will discuss their model, 'Recognizing (R), Building-with (B) and Constructing + Consolidating' (RBC+C). RBC+C are perceived as nested epistemic actions. Epistemic action implies a specific type of agency that involves cognitive operation with mathematical concepts, or to put it another way, that involves a high level of consciousness. The authors define 'abstraction as an activity of vertically reorganizing previous mathematical concepts within mathematics and by mathematical means so as to lead to a construct that is new to the learner'. These four types of actions are, of course, all important for the student's conceptual development. However, I will dwell here on the action of consolidation. My interpretation is that consolidation implies a type of generalization that creates a disposition for further abstraction in context.

Schwarz et al. point to three mechanisms for consolidation. The first is consolidation that occurs during a new construction that capitalizes on the previous one; the second is where a construct becomes the most frequently used, which means that the students can use it in a flexible way; and the third is when the construct becomes an object for reflection. I see consolidation as the key to transfer, because the mechanism involved implies different types of cognitive activities, which means that the students develop new concepts that are part of the activity of doing mathematics. The transfer between the situations is not dependent on the mental models of the individual, but is rather linked to the relation between situations and activities. The idea of developmental transfer developed by Gröhn and Engeström (2003) is based on the idea that the transfer of knowledge is located in activities, and in the students' sense making. For learning new concepts in school, the consolidation phase allows students to firm up and fix their understanding of a set of concepts. These concepts become relevant in new and related activities.

In order to conceptualize students' abstraction in context, the authors make use of Davydov's dialectical view on abstraction. The process of abstraction in this view is not detached from the learning processes, but rather is a process that involves analysis, synthesis and a return to the concrete use, which means that the abstraction is part of the actions performed, and is anchored in the activities. Here, it is easy to see the link between an idea of abstraction that ends as a new action through analysis and synthesis, which means that generalization involves the construction of new concepts and the use of these concepts in new activities.

In the chapter, the different mechanisms for consolidating are mainly described in relation to the two layers of the interpretative framework that I use as a point of departure: microgenesis and ontogenesis. The authors argue that they do not try to address the more difficult questions about what fuels and sustains abstraction in context. I can understand this position, but I think that by expanding their framework, these questions could be addressed as well. The weaker part of their argument for abstraction in context has to do with the 'struggle' of the relationship between social and cognitive activities, especially at the level of sociogenesis. This layer deals with questions about how learning is socially organized, where knowledge originates, how it is sustained over time, how knowledge is structured and organized in different settings and how institutions like schools organize the production of knowledge in their everyday activities. When Schwarz et al. come to the social context in the process of abstraction, they provide less convincing evidence for their RBC+C model. I think this could be explained in that their main interest lies in the interaction that takes place, which includes the tools used, and its relation to the individual student's conceptual change. When following this interest, their line of interpretation limits the possibility for moving to the question that is central for including institutional aspects in a systematic way. If Schwarz et al. want to examine what the institutional layer means for students' conceptual change, they need to develop their model so as to include an additional layer in their analysis. This could be accomplished in studies where this focus on the institutional level becomes one of the main lines of interpretation. Here, the role of the teachers and the role of artefacts and computer tools would also become more visible in the analysis.

The role of communication for conceptual development

In their chapter ('Transformation of robust misconceptions through peer argumentation'), Asterhan and Schwarz go to the core of the aims and ambitions of educational systems: conceptual development and the education of rational and well-informed citizens that can take part in dialogues to improve society and its institutions. The strategy for developing students and citizens is argumentation. When I say that the authors go to the heart of these systems, it is also because they design for both critical (dialectical) and consensus-oriented interactions. These are often seen as end points on a continuum. The ambition in this chapter is to discuss and create insight concerning factors responsible for improvement of conceptual learning, which cannot be interpreted based only on descriptive data.

In a series of experimental studies, they investigated the conditions under which conceptual gain could be achieved. They claim that the students who took part in scripted dialectical argumentation showed more conceptual gains than the control group. In one of the studies, they created a design that isolated argumentation from the interactional aspect of the collaboration. The results give clear indications that the conceptual gain could only be explained by the interaction between the students. Given that the study was designed to demonstrate the effects of peer-argumentation, the conclusion supports the idea that peer interaction is the most important explanation for the differences between the experimental group and the control group. This finding can be explained through socio-cognitive theories which assert that students will go through the processes of assimilation and accommodation, and ultimately attain a state of shared knowledge (equilibrium). However, it is important to point out that not in all dyads students gained knowledge under experimental conditions. The authors analyse these differences both at a micro-level and at a macro-level to capture the more socio-cognitive features of the talk. The difference between the gaining and non-gaining dyads is explained by noting that the non-gaining dyads engaged in one-sided argumentation, while the gaining dyads were characterized by interpersonal repartition and a dialectical argumentative structure. This finding demonstrates that neither a critical nor a consensus-oriented argumentative structure is sufficient to support conceptual gains. In a critical-argumentative structure, the competition risks provoking argumentation in which ideas are not shared; on the other hand, consensus-oriented argumentation creates dialogues which do not problematize the content in the domain. This finding also shows the interwoven character of the social and cognitive aspects of knowledge construction, which includes norms and rules for how to engage in peer talk and talk with teachers. This interpretation is also supported by a number of studies in the chapter.

The promising finding in this chapter has been produced by a very careful task design as well as by the instruction of the participants. Given the experimental design, the authors provide us with clear evidence of how the effects were produced by the participants in their interaction. From a learning-design point of view, these findings are not easy to replicate in natural classroom settings, as we know that aspects other than the task design and 'short-term' instruction often play a significant role (Furberg & Ludvigsen 2008). I think we can argue, however, that curriculum development for schools could create spaces where students would be exposed to such a task design. We could consider such designs as 'germ cells' (Engeström 1987) that could be the first steps towards institutional change.

From a theoretical perspective, the findings also call for reflection, since they clearly demonstrate the interwovenness of different types of social and cognitive actions. In experimental designs, the relationship between the ontogenetic and microgenetic levels can be investigated to a certain degree. The experimental design is much weaker in terms of the possibility of addressing questions related to cultural and social norms, as well as rules and conventions, since these types of phenomena have a history that overrides the short-term experiment. Social practices must be seen as a set of interrelated activities that always come with a multi-voiced history. If we interpret the situated experiments as part of the social practices of research, it becomes a richer resource for the interpretation of data, which can be taken beyond the experiment itself. In the discussion of all the chapters, I will come back to the problem of generalization.

In Howe's chapter ('Expert support for group work in elementary science: the role of consensus'), the problem of expert support for group work is addressed. Howe's main claim is that it is 'logically' impossible for students in groups to learn advanced concepts and theories without expert support. I think that, as a premise, this is reasonable and sometimes overlooked in studies of peers, dyads, etc., working on problems in science and mathematics. This is the case in many experimental studies in the computer-supported collaborative learning (CSCL) field, which often 'zero out' the role of teachers or experts in their design. The other key assumption in Howe's chapter is the role of consensus, rather than any global agreement. Consensus is here understood as a positional consensus that is related to what the students share as an effect of their talk.

I think that analysing the role of the teacher in the experimental set up is an interesting turn in order to understand how conceptual change can take place. Given that overall consensus is seen as positive, the question becomes: how can we explain this? I think this is related to the issue of the relationship between critical (dialectical) and consensus-oriented interactions, raised by Asterhan and Schwarz. One crucial issue becomes the agency of the students. Is the teacher's intervention perceived as institutional support, or as input to a given answer defined by the institutions and the teachers? If the students do not 'own' the concepts, they are less likely to be able to reconstruct and generalize the concepts in a new setting in which they are relevant. It seems to me that a detailed analysis of who is doing the demanding cognitive work in the consolidating phase is one way to go if one wants to explain the role of the teachers and the students in more detail in these experiments. Howe's approach also moves from microgenesis to ontogenesis in order to understand how contextual issues are part of cognitive activities over time. The sociogenetic level must also be seen as crucial in order to understand how students can engage in conceptual development in schools as social institutions; this layer is not included in Howe's analysis.

Baker ('Intersubjective and intrasubjective rationalities in pedagogical debates: realizing what one thinks') addresses a theoretical problem, between dialectical and dialogical approaches. The chapter also includes an empirical analysis. More generally, he claims that the relationship between theories of learning and theories about communication is not worked out at a sufficient level of detail. Baker differentiates between two types of rationalities: first, the intersubjective rationality that has its source in a dialectical analysis based on publicly accepted or recognized knowledge; and second, intrasubjective rationality, which is concerned with changes in views, values and opinions by the participants involved. The intersubjective dimension obviously must involve meaning making that is related to two or more participants, since it cannot be invoked by the participants individually.

In the framework I use here, I would categorize the intersubjective process as microgenesis, which is connected to sociogenesis through artefacts. The intrasubjective process is concerned with the ontogenetic level. The connection between the two rationalities proposed by Baker, or the three geneses I use in this chapter, cannot be straightforward. This is because both the rationalities and the geneses operate at different timescales, and the connection will be negotiated in particular spatial-temporal settings like schools or other types of institutions. This means that the degree of tension and the potential breakdowns between these rationalities and geneses are part of the development of the participants and how knowledge is organized beyond each participant in a concrete situation and activity.

The students move between the two rationalities; this involves arguments related to the knowledge domain and questions that go beyond the local argumentation. From an empirical point of view, this is important, since the students seldom move only along one rationality; rather, they move on different levels in order to align and frame the problems with which they work (Lund & Rasmussen 2008). When Baker raises the question of a bridging or overarching theory that integrates the dialectical and dialogical approaches, the question becomes: what does it mean to create a coherent theory? I will argue that the idea of the socio-micro-ontogenesis serves as an explicit starting point.

Conceptual development and new learning environments

Yerushalmy's chapter ('Technology-based algebra learning: epistemological discontinuities and curricular implications') focuses on how students can learn algebra using the computer as a tool, given the presence of epistemological discontinuities. This idea has a strong family resemblance with central aspects in the RBC+C model. The resemblance is related to the breakdown of concepts that could be used to understand the problem at hand, and the idea that new concepts must replace old ones and must be consolidated in order to become robust enough to form part of students' broader repertoire in problem-solving activities in mathematics. There are a number of interesting elements in this chapter. In particular, I will discuss what the author calls opportunities for learning algebra in the new design. The first opportunity is related to the design of the task, which pushed the students into a metacognitive analysis of their understanding of the previous concepts in order to understand an equation in one variable. The second opportunity was to design a solution for a procedure that was yet unknown, but built directly on what they had learned about equations of one variable. The students had to change their view on the representation and meanings of the symbols in the computer environment. The third opportunity was related to the idea that the symbol system of algebra provides a rich resource for understanding what an equation is and what it represents.

My interpretation of these three opportunities is that we need to look at the genesis of mathematics, computer tools and students as the units of analysis. Computer tools and environments provide new and very rich ways of representing the knowledge domain of mathematics, but also other domains such as science. The rhetoric used in discussing the possibilities of the new 'learning' environment is often very strong, but the empirical results demonstrate what I will call a reasonable ambiguity (Krange & Ludvigsen 2008). The problem is not the computer tools themselves, but the hype and rhetoric around the possibilities that computers create for learning. Computational representation has obviously changed the conditions for knowledge construction. However, as we see in Yerushalmy's chapter, there are other aspects that are more important than a careful design.

These aspects are related to the cognitive and social demands that are built into the design to promote learning mathematical concepts. If the design presupposes cognitive activities that are too advanced, we cannot take for granted that the students will pick this up, as the analysis shows. The question becomes: what kind of technological and social scaffolds can be brought in so that the students can bridge and build up new concepts? It seems to me that these processes would need to be consolidated in order to make use of the new opportunities. One could infer that as deep a change in conceptual orientation as using analogies and changing viewpoints on representations takes longer than the time period available to the students in this experiment. The same argument is valid for the development of more flexible metaphors of functions and equations. It seems to be the case that the design created the discontinuities, but that the solutions to the epistemological discontinuities are less obvious. I will argue that the solutions should be seen as threefold. A careful redesign with a set of scaffolds as part of the computer tool is step one. A possible second step consists of a different design of the teachers' role as instructors and supervisors in problem-solving activities. The third step, which bridges the first two, is an integrated analysis of the genesis of algebra and students' conceptual orientations. The tools must then be designed in the zone of what the students can possibly learn. This is what I mean by stressing that the unit of analysis must include premises from mathematics, as well as cognitive and social demands in the design and the students' conceptual orientations.

In Hoppe, De Groot and Hever's chapter ('Implementing technology-facilitated collaboration and awareness in the classroom: roles for teachers, educational researchers and technology experts'), two important problems are addressed in terms of CSCL community. First, the authors discuss how we should conceptualize the idea of a 'learning environment' and, second, they ask how it is possible to design for a smooth flow between different media and representations. One of their assumptions is that the teachers as professionals are the gatekeepers in relation to pedagogical and technical innovation in schools.

The meaning of the term 'learning environment' is controversial in the CSCL community. Some define it as the computational system or even as tools (microworlds, simulation, etc.). I agree with Hoppe, De-Groot and Hever that this perception and conceptualization is very problematic. The environment should include where learning takes place, that is, the pedagogical, organizational and technical contexts. The students move in and out of these environments through their activities. Learning activities cannot be defined by the boundaries of an environment, but rather must be understood in relation to the participants' movements across settings and types of media. The students' experience with technologies can help to fill gaps in these movements, so that the educational workflow is enhanced and sustained over time. The focus on the teachers' workflow in relation to the students, and how this can be supported by new computational systems and tools, takes what I call multiplicity as a starting point in designing for educational innovation (Ludvigsen in press). Here, multiplicity points to the history of the institutions through teachers' work, as well as the kinds of new tools that can enhance their practice; in other words, the sociogenetic aspects of practice become an explicit part of the design.

The design problem is addressed from different viewpoints. Design always involves a reduction of a problem; the question is whether this is an adequate reduction. The way Hoppe, De-Groot and Hever involve different experts in the design process optimizes the potential for a transformation not only for technical innovation, but for educational innovation that could be sustained over time and transform school practices.

Multi-level analysis of learning activities and accountable talk

Tiberghien and Malkoun ('The construction of physics knowledge from different perspectives: the classroom as a community and the students as individuals') present an argument for how students can learn concepts in domains like physics, as well as a framework for how teaching and learning can be analysed. I will here deal with the problem of the 'scale of the analysis'. Tiberghien and Malkoun are inspired by Lemke (2000) to argue that one needs to trace classroom activities using different timescales in order to understand teaching and learning. The first scale is related to the macroscopic level; the scale of analysis concerns the whole teaching sequence. This could be part of the curriculum for a longer period of time, which could mean days and weeks. Here, the norms, values and conventions are central for teachers and students to develop an understanding of the domain and its concepts. The next scale is the mesoscopic level. Here, the premises are more detailed and the analyst looks for a thematic analysis within the broader macroscopic scale. The didactical contract between the teacher and students, as well as how the students and teachers work with the knowledge, can be investigated in detail. A theme is not given any specific duration, but can last from 1 minute up to 45 minutes. The focus for the analysis on this scale is thematic coherence and the discourse between students and teachers. The last scale is microscopic. Here, the focus is on the small elements of knowledge (facets), which could also be understood as concepts. The key point here is that these scales of analysis complement one another. The phenomenon under investigation needs to incorporate all three scales in order to generate a robust understanding of the students' uptake of knowledge presented by the teachers. The genesis of the knowledge domain is part of the history of the teaching–learning sequence, and cuts across the scales.

These scales of analysis are quite similar to the socio-micro-ontogenesis framework used in the present integrative chapter. The framework presented by Tiberghien and Malkoun is more directed to the scales of the activities in classroom settings; however, it is less clear how tools, artefacts and social conventions are included in the analysis at the empirical level. The chapter deals with the meso- and micro-levels, which means that the knowledge dimension is privileged, and social factors are not integrated in an explicit way. In the way I have used the term here, sociogenesis is not included in the empirical analysis; however, it is part of the interpretive framework.

The issue of intersubjectivity is also the focus in Nathan, Kim and Eilam's chapter ('Methodological considerations for the study of intersubjectivity among participants of a dialogic mathematics classroom'). This chapter takes up the methodological challenges inherent in studying the phenomenon of intersubjectivity. The authors suggest a multi-layer analysis that shares many of the same aspects as Tiberghien's framework, but is different in some important respects. The most important difference is that Tiberghien focuses explicitly on knowledge construction and the teachers' role; in Nathan, Kim and Eilam's model, teaching and learning are conceptualized, based on the overall ideas in the curriculum. The micro-level is understood as single speech acts or utterances. The authors recommend a coding schema so that the utterances can be seen as convergent or divergent when it comes to the achievement of intersubjectivity. However, it is not clear how the coding schema deals with the relationship between interactional moves and the knowledge produced.

At the meso-level, the structure of the interaction is analysed in well-known structures of talk like Initiative–Response–Evaluation or, in the example given, an Initiation–Demonstration–Evaluation triad (IDE). In their mapping analysis of features in the dialogues' intersubjectivity, one of the empirical findings seems especially interesting: Evaluation seems to have a crucial role, both when ending a sequence and when starting a new IDE sequence. I strongly relate this to the significance of the consolidating phase in Schwarz et al.'s work: The students need to confirm their understanding, whether they agree or not, at some point in the dialogue.

The macro-analysis is defined as global changes in the entire discourse (or corpus of data). In this analysis, the content of knowledge is emphasized. The analysis is presented in graphical format. However, even if I have great sympathy with the three-layer analysis, it is less clear to me how these layers are related in order to allow us to understand more about communicative and learning activities in the classroom. It seems to me that instead of developing a multi-level model, the authors move from the microgenesis to individual learning. The level of sociogenesis is not really explored.

The chapter by Sohmer, Michaels, O'Connor and Resnick ('Guided construction of knowledge in the classroom: The troika of talk, tasks and tools') focuses on what the authors call 'accountable talk'. Under this label, they have synthesized different approaches concerning the guided construction of knowledge in the classroom. Their idea is close to the multi-layer or multi-scale approach that many of the contributions in this book promote. Accountable talk, in their approach, connects three different aspects: the community level, knowledge and the accepted standard for reasoning. The way I interpret these three aspects of accountability is that they combine a high level of structure, given by the knowledge domain, with a high level of relevance for the students. The community level involves explicit social norms, values and conventions. The authors argue that this is the most straightforward aspect of accountable talk. I do not find much evidence of this rather broad claim in the chapter, nor would I say that there is much evidence in the research literature. Norms in talk involve cultural issues, and students' beliefs seem to be a rather difficult issue to structure or 'engineer'.

Accountability, when it comes to standards of reasoning, involves what we can call a discourse genre, which has to do with ways of talking and conventions for what is relevant evidence for claims put forward. Of course, we do this all the time in everyday life, but when it comes to a specific knowledge domain, particular claims must be justified by very precise evidence that is involved in problem-solving activities. A knowledge domain in schools puts quite different cognitive and social demands on the students. In schools, not only is gaining experience about a phenomenon the goal; concepts and a conceptual system are also objectives. Sohmer et al. claim that the accountability for knowledge is most difficult to achieve in schools. The literature from conceptual-change research amply justifies this last claim. In making the claim that implementing social conventions is easy while the cognitive achievement is hard, however, the authors run the risk of promulgating the very dichotomy against which they themselves argue.

At the practical level, the authors argue for a model that involves the following activities: recitation, stop-and-talk, student presentations and group critiques, and whole-group 'position-driven' discussions. If we compare these activities with the RBC+C model proposed by Schwarz et al., there are many similar features. The main difference is that the idea of accountable talk is based on both cognitive and social validation of the talk. Sohmer et al's model has been developed and implemented to improve classroom practices.

The interesting issue related to accountable talk in the context of this book is the relationship between dialectical and dialogical activities. As shown by Asterhan and Schwarz, it is not trivial to balance the critical and problematizing accounts with a social climate that is inclusive and supports all the students. Learning in schools is basically about involving students in activities in which they do not have the knowledge to solve a problem or perform a task. This means that students must challenge each other, and the teachers must challenge the students by direct instruction and by intervening in the activities performed. In addition, more guided processes of inquiry use a combination of knowledge resources from books and the Internet, and here the solution to the problem must be based on criteria based on the knowledge and relevance for the community. This involves understanding how the sociogenesis of schooling plays out as an intrinsic part of the learning activities that take place in the school as a social institution in a highly differentiated society.

Normative models of identity and a new learning metaphor

The two chapters I discuss in this section address more theoretical issues: The chapter by Cobb, Gresalfi and Hodge ('A design research perspective on the identities that students are developing in mathematics classrooms'), and the chapter by Hakkarainen and Paavola ('Toward a trialogical approach on learning'). I will first look at Cobb et al.'s chapter. The key idea in this chapter has to do with the construction of identity and how a conceptualization of this construct can improve design research. The focus on identity as an important aspect of learning in different knowledge domains has been developed in recent years. In the turns towards the linguistic and social issues in learning theory, concepts that can connect the individual with the culture become central.

Cobb et al. discuss two constructs: normative identity and personal identity. Normative identity is based on studies that document how effective mathematics students perform in classroom settings. This normative identity involves several aspects that concern what kind of agency is seen as adequate, and to whom and to what students are accountable. This involves the teacher and co-students, and mathematics as the knowledge domain. Cobb et al. emphasize that normative identity is a collective construct.

Personal identity, on the other hand, concerns the individual and is seen as an empirical construct that is used to analyse and understand how students choose to orient themselves in mathematical activities. The authors point out that these two aspects of students' identity produce situated accounts of students' personal identities. This situated account can give feedback and inform the instructional design in different areas of mathematics. The central question becomes: how can the analyst move from an empirical account of situated identities to a normative account that informs design? One possible solution to this problem is that this relationship could be conceptualized as dialectical movements between the activities, abstractions and generalizations that develop into concrete design guidelines.

Cobb et al. also build on studies of identity that have followed students in mathematical activities over a longer period of time. What emerges from these studies is that the empirical analysis does not generate unified results that can create a coherent instructional design. Instead, it creates a set of analytical concepts that can be used to understand the situated identities that the students produce in their institutional setting. I think that longitudinal studies show that the broader institutional context influences microgenesis. The implication is that design research cannot only be based on the idea of refinements that gradually improve theory and practice in school, but rather, it must be based on a dialectical and dialogical understanding of the learning activities.

What can be seen as the next step in the discussion about the normative construction of identity and an empirical account of identity is how we understand the intersection between them. A normative account can demonstrate the analyst's biases if the normative construct is seen as a template for how students should behave. As analysts, we must be sensitive to how the students choose to orient themselves within the institutional arrangement of which they are a part. As a theoretical reflection, it seems to me that there is a tension between the sociogenesis of schools and learning activities and the design for micro- and ontogenesis in Cobb et al.'s account of the two types of identities.

Over the past years, Hakkarainen and Paavola, together with colleagues at the University of Helsinki, have developed an idea that they call the trialogical approach to learning. The theoretical work that is the foundation for this idea is based on two lines of reasoning. First, it is based on a critique of the knowledge-acquisition metaphor, which takes the individual as the unit of analysis and of the participant metaphor. The second line builds on three different approaches to collective knowledge creation.

The knowledge-acquisition metaphor is criticized because it privileges the formation of mental structures. The participant metaphor grew out of, among others, Lave and Wenger's (1991) work in the late 1980s and early 1990s. Hakkarainen and Paavola criticize the participant metaphor for not differentiating between knowledge as part of a social practice and as an outcome or product of this practice. One can say that they argue for a more differentiated idea related to how knowledge is produced and how it is represented across situations, activities and settings.

Based on the criticism of these two metaphors, the authors argue for a third metaphor, the trialogical approach. The new idea that is introduced is based on the collaborative development of mediating objects. These objects are produced by the participants in activities that are mediated by different types of artefacts over longer periods of time. The foundation for this idea comes from three different sources: Bereiter's theory of knowledge building, Nonaka and Takeuchi's theory of knowledge creation and Engeström's theory of expansive learning (Hakkarainen and Paavola, this volume). When the trialogical approach builds on the idea of the collaboratively developed object, we can trace this idea to Bereiter's work, inspired by Karl Popper, about world-three objects, which refers to objects that are not located in any person, but outside as an object with which we could choose to engage. A theory is often used as an example of a world-three object.

In Engeström's theory about expansive learning, objects could be seen as one of the main concepts in his work. According to Engeström (1987), it is through the change and expansion of an object that we can identify learning at both the individual and collective level. To discuss the differences between the object as a notion in Popper's as opposed to Engeström's work is beyond the scope of this chapter. However, the meaning of object in the trialogical approach is conceptually weak, since it draws on different intellectual traditions. A concept comes with a history, which has implications for how it can be integrated into a conceptual framework. Hence, we need to be concerned not only with the concept of object itself, but also with which concepts will be needed to analyse object-oriented activities. In the three case studies presented in the chapter, the empirical layer is conveyed through examples of the concepts used in the trialogical approach. The implication is that the approach should be seen as a theoretical idea which would need to develop analytic concepts and a methodology in order to be seen as a new and robust perspective on learning. We would also need a high number of empirical studies in order to validate the approach. The approach must be able to demonstrate its superiority to provide us with more advanced understanding of how knowledge is created by individuals and collectives over short and long periods of time under different institutional and technological conditions. This is the claim put forward by the authors of the idea of trialogical learning.

The travel of ideas: discussion and concluding remarks

The task of writing an integrative chapter involves a large number of decisions about what to stress. One of the chapters that I will discuss in this last part is the one by Saxe and colleagues ('A methodological framework and empirical techniques for studying the travel of ideas in classroom communities'), which explicitly discusses the layers of human genesis, as I have done throughout this chapter. Their focus is on the methodological implications of the multi-layered model of knowledge construction in classrooms. Their approach concerning the 'travel of ideas in classrooms' gives us a powerful metaphor when we want to understand the construction of knowledge in these settings. Ideas need to be made concrete, relevant, connected to experience and other concepts, and play a role in the problem-solving activities in the classroom.

In their chapter, the authors specify how they have designed their lessons based on three main principles: lessons should target core mathematical ideas, lessons should target all students and lessons should be organized as mathematically coherent series. These design principles are broken down into six different phases. The phases are: independent work and pre-assessment, students' whole-class presentations, small-group discussion, opportunity for revision, teacher-orchestrated discussion, and independent work and pre-assessment. If we compare these phases with the RBC+C model, they are directed towards many of the same social and cognitive activities.

The RBC+C model has a more cognitive orientation, while the design principles from Saxe et al. incorporate social processes and social validation in an explicit way. Together, these models would optimize the opportunity for the students to recognize the relevance of the problem, develop strategies to revise and build up new concepts, and solve the task by constructing new concepts. Consolidation adds a concrete layer and meta-layer, so that the students can monitor and reflect on their own development in the context of mathematical activities, and develop a deeper sense of the concepts. Together, these principles specify how students can engage with the travel of the idea in the classroom through social interaction with adequate resources, as participants with individual agency and as members of a community that is accountable to different rationalities in the activities. The design principles and activities also ensure that the students receive expert support and that there is the possibility for them to articulate their understanding in public, in small groups and as individuals in the classroom. Saxe et al. emphasize the usefulness of collecting overlapping sets of empirical evidence for the different geneses. They also argue that analysis should be based on empirical techniques and methodological approaches rather than fixed solutions and procedures. As analysts, we need to be sensitive to the participants' orientation and how they choose to act. This means developing an understanding of the emerging social interaction under specific historical conditions (see also Roth, this volume).

The idea of the three levels of socio-micro-ontogenesis provides a framework of interpretation that allows us to integrate different types of findings. This framework is a necessary step for making further progress in the learning sciences: Reducing learning to a progression in more scientific mental models (at the level of ontogenesis) simply leaves us with too many questions unexplored and unexplained. The three levels of genesis are open enough to include different approaches and the use of different concepts. We cannot expect the community of scientists in the learning sciences to converge on a set of common concepts. I would even question the idea of common concepts, since we need a variety of analytic approaches in order to understand empirical phenomena. The three types of genesis would always need to be specified to a certain extent. It is noteworthy that almost all the chapters in this book chose to struggle with these different layers of analysis.

When we deal with sociogenesis, the timescale shifts to longer periods of time. Cultural-historical activity theory (CHAT) is a good candidate for demonstrating development and change in social practices (Engeström 1987, 2007; Roth, this volume). The idea of object-oriented activities is connected strongly to change at the level of social systems and cultural norms, rules and conventions. CHAT explicitly focuses on change through concepts like breakdown, rupture and contradiction. CHAT provides us with an explicit account of why radical change takes place. Furthermore, socio-cultural approaches that take a historical perspective can give us important insights as to how institutions and society gradually transforms (Säljö 2000; Wertsch 1991, 2002).

Microgenesis has different intellectual roots, from Piaget to Vygotsky and approaches like ethno-methodology. The interaction between humans as well as between humans and artefacts is what constitutes this level of analysis. As I see this, it is the intellectual tradition and the researchers' interests that decide which direction the line of interpretation will take. For researchers with a background in cognitive science, the unit of analysis remains the individual or ontogenesis. For researchers with the social self as the starting point, the interaction itself and the social organization of knowledge will become the unit of analysis. What can vary for the different types of research is the level of description or, in other words, the level of detail that could be included. What seems to unite different types of analysis on this level is that students engage in sense-making activities, and that meaning is constituted as a situational accomplishment.

Generalization is an interesting problem. Phillips (2006) argues that the only way we can generalize beyond cases and statistical data is to reason in an intelligent way. My interpretation of his position is that we need to think 'smarter' about what the different theoretical stances give us and what can be gained from empirical analysis. In this integrative chapter, I have tried to generalize in three different ways. First, I have read all the chapters based on an overall theoretical position: the three levels of genesis. Second, I have tried to sort out which level the most frequently used concepts aim at in the analysis given in each chapter. Third, I have read the empirical data and the analysis in line with the frequently used concepts. Together, these three types of interpretations provide the opportunity to compare and contrast the different chapters. I have tried to be explicit in using the three levels of genesis in order to raise critical questions about the approaches used in the different chapters. The three levels of genesis do not constitute a normative model, but rather an analytic approach that opens up the possibility of understanding learning and the construction of knowledge at different levels.

The last claim I will make in this chapter is that according to my view, the relation between sociogenesis, which involves institutional aspects, and students' conceptual change is a relationship that we, as a community of researchers, still have problems understanding in a robust way. We need to develop sensible analytical models to describe and characterize this relationship. The relationship between the individual and the institution involves zones where long-term cultural development meets the socialization processes of students. These are zones in which change and stability, at all three levels of genesis, unfold and are played out.

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