Mathematics Education and Language

Interpreting Hermeneutics and Post-Structuralism

Tony Brown

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Interpreting Hermeneutics and Post-Structuralism

by

TONY BROWN
Didsbury School of Education,
The Manchester Metropolitan University, U.K.

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Tony Brown
Manchester
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ABOUT THE AUTHOR

Tony Brown studied mathematics and economics at the University of Kent at Canterbury, England, receiving his Bachelor's degree in 1978. After three years teaching at Holland Park School in London he moved to take up a post with Voluntary Services Overseas at the Teachers College in Dominica. He returned in 1985 to commence full time studies at Southampton University which resulted in him receiving in 1987 his PhD which focused on interactions between children in mathematics lessons. Following a brief spell teaching in a middle school he was appointed in 1989 to his current post of Senior Lecturer in Education at the Manchester Metropolitan University.

INTRODUCTION

> A demanding, prudent, "experimental" attitude is necessary; at every moment, step by step, one must confront what one is thinking and saying with what one is doing, with what one is. (Foucault, 1984, p. 374)

As teachers, we rightly value the ways in which our students bring meaning to the mathematical situations they encounter. There is much scope for judgment, insight and creativity in the style of mathematical work being introduced in many schools and we may aspire to encourage these qualities in the students' learning. Yet there is still a need for an individual to reconcile her own personal mathematical understanding with the ideas and traditions which have grown out of centuries of mathematical exploration and invention (cf. Ball, 1993). Whilst students can be creative mathematicians there is still a need to be able to do everyday calculations and understand aspects of conventional mathematical thinking. We are often tom between attempting to focus on our students' own way of seeing their mathematical endeavours, and seeing these endeavours with our own eyes, inspired perhaps by a "correct" view of mathematics. There are inevitably difficulties for us in making sense of students' own developing understanding without using our own "expert" overview as a yardstick, especially when we pose the tasks that they are working on. Teacher descriptions of students' learning often presuppose an adult overlay framing the mathematical ideas supposedly being addressed. Meanwhile, the sort of constructions which students are likely to generate for themselves are a function of their own particular concerns in relation to the sorts of tasks with which they are presented. Whilst we may wish to encourage students to pursue their own mathematical concerns at times, we retain the option of blowing the whistle and denying that their work is indeed mathematics. Furthermore, as the apparent relevances of different aspects of mathematics, as perceived by society, grow or decline, the nature of tasks on offer, and the values associated with them, will alter.

In the climate of rapidly accelerating social change that we now face, conventions get replaced with alarming regularity and, as teachers, we face challenge in any supposed role as experts in the worlds our students will encounter. In meeting this we need to continually reassess how our intentions might be concealing yet promoting initiation into existing structures which in turn support the reproduction of those structures. Our ways of describing the world need to constantly readjust to meet new demands. At the heart of this is a readjustment in the relationship between the rate at which we grow and the pace at which our environment changes; the very relationships between adult-child, teacher-student, mother-daughter are brought into question, as are the ways these relationships underpin assumptions about approaches to teaching. When social change was slow it was reasonable to suppose that what was good for the father was good enough for the son. As it speeds up, however, conflicts arise; the weakening of the family being but one example. Brookes (1994, p. 45) suggests that for education systems to be compatible with the world as we experience it we need to

> accept the twin constraints of an environmental framework that is changing non-repetitively and accelerating and a generational framework which is cyclically repeatable and only gently changing.

This entails an on-going renegotiation of social roles and reevaluation of how we construct and utilise knowledge. The task of education becomes ever more concerned with enabling us to understand the changes of which we are part and to see how we might have some influence over them.

In academia at large it is no coincidence that the study of language has become so prominent in our examination of the social world; the world does not rest for long enough to allow descriptions of it to settle and become familiar. We become immersed in multiple "feedback" as our attempts to describe things grapple with a world entangled in its own self descriptions. As mathematicians we have often sought immunity to these shifts. The apparent sturdiness of mathematics has somehow resisted pressure to be more responsive to changes in demands made upon it and mathematical activity has retained an image of being anchored by various mathematical truths. But it is now becoming more apparent that mathematics is created and utilised through history according to time dependent needs which change ever faster. Mathematics education research has responded by promoting less "positivistic" understandings of its host discipline, but as yet has seemed reluctant to go the whole way to understanding itself as an integral part of a social web subject to fundamental and continuous reevaluation. As such one might argue that we have not fully addressed the "strong programme" presented by Bloor (1976), which sought an extension of the practices of sociologists of scientific knowledge to include an examination of the content and nature of scientific knowledge, not just the circumstances surrounding its production.

Broadly, this book concerns the way in which language and interpretation underpin the teaching and learning of mathematics. In tackling this theme I wish to position some issues arising from research in mathematics education in relation to some major writers in continental philosophy. In particular, issues of language, understanding, communication and social evolution, all of which are tackled by recent mathematics education research under the banner of constructivism and related areas, are central themes in post-war western thinking on philosophy and the social sciences, yet research in mathematics education seems to under-utilise the resource of work done in the broader context. Whilst there is a growing recognition that such work is of importance (for example, Walkerdine, 1988; Skovsmose, 1994), we are still in the early days of such developments. In developing my theoretical framework I will be calling on certain key writers such as: Gadamer and Ricoeur on hermeneutics, Habermas on critical social theory, Saussure on linguistics, Derrida, Foucault and Barthes in post-structuralism and Schütz on social phenomenology. I seek to show how language is instrumental in developing mathematical understanding and also how both chronological and spatial dimensions of classroom experience condition ideas being met. I examine how language functions in orienting action within the normative constraints of a given situation and suggest that the task of the learner could be seen as reconciling experience with both conventional and potential ways of describing it. Classroom examples offered show school students seeking to capture their understanding in symbolic form. Meanwhile, college examples show teacher education students capturing their understanding in reflective writing. Recognising that the perspective of participants is becoming more central within analyses of social situations, this book offers a theoretical approach to discussing the world as understood through the eyes of participants. Within mathematics education research this means attending to the way in which students and teachers experience the classroom situation which they are in. These perspectives, it is suggested, are imbued with culturally derived structures, present both in the words used by inhabitants and in the physical space they occupy.

Most of the examples offered will describe students in the age range six to twelve learning mathematics in schools, although there will be some discussion of work taking place within teacher education courses including some detailed descriptions of approaches taken. I am writing as someone immersed in the British educational system and drawing on research published in English. Nevertheless, much of the empirical work was carried out in an English speaking Caribbean country (Dominica). The remaining school based material was collected in England from lessons observed in culturally diverse classrooms in London and Manchester, and some from my own classroom teaching in the Isle of Wight. The work reported with initial training students and practising teachers took place at Dominica Teachers College and the Manchester Metropolitan University. As such the majority of situations reported are connected to my own professional concerns as a teacher and teacher trainer. These situations are not typical of classrooms in general; rather my observations were motivated by an intention to locate and analyse examples of students expressing their understandings verbally or in writing. In a sense I am identifying what I see as examples good practice based around a struggle by students to combine mathematical activity with reflection on it. It is hoped that these examples be interpreted as possible strategies for developing the linguistic dimension of mathematical activity in line with a major theme of this book which sees this broadening of mathematical concerns as an approach to situating mathematics in a more developed notion of society.

PART ONE presents a largely theoretical account of the individual confronting mathematics and representing her understanding to others.

Chapter one presents an exposition of hermeneutics together with an outline of its roots in phenomenology. It emphasises language as manifested in action rather than as a transcendentally existing system. This provides a platform for a social scientific analysis of mathematical learning seen as interaction, linguistically mediated and governed by social norms, that seeks to reconcile evolving understanding with static forms.

Chapter two further examines how language functions in orienting mathematical thinking and acting, and offers practical examples of how this is achieved. After outlining how Saussurian linguistics provides the roots of post-structuralism, it employs this theoretical perspective in developing the notion of linguistic framings of mathematical ideas being more stable than the thinking generating them. Further, the self-reflexive qualities of language that position the individual in her society are examined as an important dimension within that individual's self formation. This is used in addressing the difficulty of distinguishing between the individual learner creating and inheriting mathematical ideas.

Chapter three focuses on sharing mathematical perspectives and questions the potential which language displays in locating and holding on to mathematical ideas in doing this. For example, it examines the difficulties teachers face in alerting their pupils' attentions to mathematical ideas that the teachers seek to share. It also asks how students represent their understandings. A detailed description of a lesson is offered in which students seek to reconcile their mathematical experiences with various ways of capturing it for sharing with others. This commences a substantive analysis of children doing mathematics which spans the next two chapters.

PART TWO focuses on the learning of mathematics in the classroom.

Chapter four seeks to capture the classroom through a preliminary study of how students experience mathematics in their classroom environment. It is suggested that the physical and social dimensions of the classroom frame and so condition the mathematics being encountered. A number of examples are offered of students being guided by verbal instructions, peer interaction, physical apparatus, learnt rituals, and so on.

Chapter five introduces a more theoretical treatment of this classroom experience. Utilising a framework from social phenomenology, it examines how students orientate themselves within the space within which they see themselves working. Their learning is described as a hermeneutic reconciliation through time between their expectations and their actual experience; between this experience and their attempts to capture and share it.

PART THREE considers the teacher's own perspective in the classroom.

Chapter six builds on the previous two chapters by examining how the teacher maps out a picture of the individual student's understanding through the evidence available in the immediate classroom situation, with view to guiding and shaping the emerging mathematical thinking of the student.

Chapter seven sees the teacher's practice as being governed by a broader professional intent and examines the processes through which this might evolve. It is shown how practitioner research paradigms can be functional in enabling practitioners to examine critically the linguistic frameworks employed in describing their practice, which, in turn, shape the space encountered by their students. Examples are offered of student teachers on initial training courses and of practising teachers working for a professionally oriented masters degree. As such this chapter provides the book's focus on issues of teacher education.

Chapter eight provides a conclusion. By combining and developing themes from all parts of the book the chapter points to an overarching framework for analysing the cultural functioning of language in mathematical learning and teaching. In particular, it argues for seeing all mathematical experience as being linguistic. Also, it examines the way in which accelerating social evolution results in a fragmentation of mathematics into more localised subcultures with specific needs, resulting in further strain on any notion of universal mathematical truths. Finally, it revisits the perspective of the teacher and considers her task of preparing students for an increasingly uncertain future.

Before commencing however, I shall provide a very brief review of some recent research which I see impacting on the themes I wish to develop.

Caught between the individual's performance of mathematics and society's overview of what the discipline is, we are faced with the task of providing a coherent account of mathematical learning. How do we reconcile the social and individual dimensions of developing mathematical understanding? Habermas' recent work in social theory has sought to combine two traditions that dominated theoretical thinking during the nineteenth and early twentieth centuries, namely, positivism (Comte, 1864, see Habermas, 1972, pp. 65-90; Williams, 1976, pp. 238-239) and hermeneutics (Dilthey, 1976, see Habermas, 1972, pp. 140-186; Gadamer, 1962). This can be seen as an attempt to reconcile scientific overviews of social situations with the experience of the people living within these situations. It displays a growing recognition of a need to integrate a fuller account of the participant's understanding within analyses of social situations. Until recently, positivist approaches dominated the research horizon. This style of enquiry was concerned with scientific overviews where action is seen in terms of techniques or means-ends strategies that are not personalised in any way. Positivism is concerned with "is" statements rather than "ought" statements and as such offers no guidance to revising ways of doing things. Meanwhile, interpretivist or "insider" perspectives focus on the world as experienced by inhabitants in particular situations. The hermeneutic approach underlying this concerns a reconciliation between experience and ways of describing it. Habermas (1972, 1984, 1987) argues that neither of these two traditions enable us in bringing into question the current status quo. This is evident in the way in which the two traditions employ language. Positivism assigns a neutral role to language - it merely labels preexisting things. Meanwhile, for early writers in hermeneutics focusing on text interpretation the task was simply to understand what the writer intended. As such it focuses heavily on existing use of language and in so doing reproduces existing practices and is resistant to introducing new ways of seeing things. Thus in both traditions there was a tendency to constrain activity within existing practices. Habermas promotes an approach to social understanding which transcends both positivism and hermeneutics. He bridges the divide by developing an "insider perspective" with "external attitude" (McNamara, 1995 b, p. 17). He favours critically examining the language and norms which underpin the practices we engage in, so as to understand their genesis and how they serve the interests of the people who preserve them. He sees the task of post-positivist methodology within social inquiry as being to combine the philosophical and practical with the methodological rigour of positivism, "the irreversible achievement of modern science" (Habermas, 1969, p. 79, 1991, p. ix). For the individual this means working on understanding the functioning, significance and limitations of scientific language within his or her own personal situation. For mathematics education research, I suggest this means examining how mathematics is embedded in the performance of it.

Within the field of education the distinction between positivist and interpretivist perspectives is utilised increasingly, in discussions focusing on educational research methodology. In the United Kingdom, writers such as Carr and Kemmis (1986) and Elliott (1993) have developed theories of practitioner research built on critical insider perspectives as alternatives to positivistic research concerned with overviews. For example, diary entries kept by teachers reflecting on their practice can be seen as an instrument for researching actual practice (Brown, 1996 b). In considering the literature of research in mathematics education I suggest that systemic views of mathematics and its teaching are easy to locate in mathematics education research literature. Until recently, the dominant traditions of mathematics teaching have focused on how mathematics is rather than on how it is seen. Teaching media are customarily treated as if they give access to something actually there. The parameters of mathematical activity are clearly delineated, where the symbols assume an unproblematic relation with the concepts they represent. I see insider views of mathematics as those consciously building in some sort of self -reflective dimension. Since participants are necessarily assuming a specific task I suggest that such views are always embedded in a culture. Mathematics only manifests itself in activity governed by culturally specific norms (for example, school mathematics or shopping mathematics). Recent work in mathematics education research has also focused more on how participants experience the mathematics classroom. It seems insider perspectives are becoming more prominent as the absoluteness of mathematics itself is brought into question. Constructivists have focused more on the individual learner's understanding of the mathematical tasks which they face (for example, von Glasersfeld, 1995). Interactionists have focused on how meaning develops out of interaction and interpretation between members of a culture (Bauersfeld, Krummheuer and Voigt, 1988). Mason (1994) has spoken of researching problems, both mathematical and professional, from the *inside*. Walkerdine (1988) offers a post-structuralist account of how student "produce" mathematical meaning within a process of initiation. My own work (Brown, 1991, 1994 a h, 1996 a c) has sought to analyse the hermeneutic process of students reconciling their mathematical experience with their descriptions of it. Meanwhile work has begun in formulating a philosophy of critical mathematics education to examine ways in which discourses operate within mathematics education (for example, Skovsmose, 1994). Part of Skovsmose's task has been to uncover how mathematics education conceals its intentions beneath the language it employs in declaring its project. Whilst Skovsmose tends to under-utilise major contemporary writers in his analysis, this concern is addressed in a preliminary way by other writers who pursue similar issues, such as Taylor (1996) who discusses Habermas and Zevenbergen (1996) who considers Bourdieu.

How should we approach mathematical thinking? How do we share our thinking with other people? How do language and symbols mediate this? What role does the teacher have in all of this? A key difficulty for researchers in mathematics education results from the task of finding ways of talking about the achievements of students caught between creating and inheriting mathematical objects. Student construction of mathematical ideas happens largely within the confines of an inherited language. Their individual perspectives are conditioned by the cultural apparatus they use in creating and presenting them. Similarly, it is hard for researchers to comment, without importing their own perceptions, littered with artifacts from their own education. As inhabitants speaking of our world, we may describe our experience, yet these descriptions are imbued with our society's preferred ways of saying things and conditioned by our tradition of seeing our world through positivistic frames. Skovsmose (1994, pp. 42-55) speaks of past *technologies, formatting* the space we now work in. For example, we live in an actual physical space conditioned by certain geometrical understandings from past eras. Similarly, results of actions governed by older beliefs (for example, where mathematics was absolute) often infiltrate the grounds of more modem accounts. There are difficulties in speaking of an individual's perspective, when, from the very start, this perspective is culturally conditioned.

Cobb (1994, p. 14) highlights the pedagogical dilemmas which emerge from the tensions between mathematical learning being viewed as enculturation or as individual construction. He locates the former with Vygotskian inspired socioculturalists (for example, Rogoff, 1990) and the latter with constructivism derived from Piaget (for example, von Glasersfeld, 1995). He cites Ball (1993, p. 374) who observes how educational literature is replete with "notions of "understanding" and "community" - about building bridges between the experiences of the student and the knowledge of the expert". Cobb (1994) concludes that

> each of the two perspectives, the socio-cultural and the constructivist, tells half of a good story, and each can be used to complement the other. (p. 17)

> the socio-cultural perspective informs theories of the conditions for the possibility of learning, whereas theories developed from the constructivist perspective focus on what students learn and the processes by which they do so. (p. 13)

We are, he claims, thus left with a task of reconciling, rather than choosing between, these two accounts (cf. Bereiter, 1994). We need, on the one hand, to understand what is entailed in creating an overview, whilst on the other, to understand the parameters of the individual's insider perspective of her world. The pursuit of this task underlies some recent theoretical work in mathematics education research which seeks to resolve this potential dichotomy.

Constructivism, as a movement within mathematics education, emerged in the light of Piaget's work and much work under this banner remains firmly within this tradition. In the last few years, however, various breeds of constructivism have begun to assert their differences. A central tenet of many constructivist writers is that the learner actively constructs mathematics rather than receiving it "ready-made". Yet behind the rhetoric of that slogan there are a host of opinions as to what this means philosophically and the implications it has for actual practice. Various approaches are taken by these writers in clarifying their understanding of how teacher and learner confront mathematical ideas. For example, different perspectives are provided by the various conceptions of social constructivism or constructionism which span a broad range of concerns across the sociology of scientific knowledge and sometimes nudge in to what have become known as socio-cultural approaches. Such perspectives trace their roots by way of Berger et al. (1966), who coined the term "social construction", through to writers like Schütz, Husserl and Mead. Recent work in this field develops ideas of how knowledge is a function of the sociological conditions underpinning its generation. As a flurry of new papers are published, however, with some still calling themselves constructivist, while others leave the term behind, it becomes harder to draw clear distinctions between the range of perspectives on offer. However, I shall endeavour to identify some key writers and outline aspects of the perspectives they assume.

Radical constructivism (e.g Von Glasersfeld, 1984) was arguably the dominant version within mathematics education which put constructivism on the map in terms of mathematics education research. An account of its rise has been given by Steffe and Kieran (1994). This view rejects teaching metaphors such as "delivering" and "receiving", which presuppose a mathematics being created outside of the mind of the individual learner. Von Glasersfeld cites Skinner's behaviourist training as an antithesis to the constructivists' quest for understanding (1991, pp. xiii-xix). Whilst the central constructivist assertion, that the learner actively builds up her knowledge rather than receiving it ready made, is derived directly from Piaget's work, Lerman (1989, p. 211) argues that radical constructivism asserts itself as a distinct movement to Piaget's genetic epistemology by affirming its commitment to the notion that coming to know "does not discover an independent, preexisting world outside the mind of the knower". Von Glasersfeld (1984, p. 25) admits that "Piaget's position is somewhat ambiguous" on this point. The task for the teacher who assumes a radical constructivist view of learning is to set tasks and to create an environment conducive to the student constructing her own meanings. Coming to know is seen as an adaptive process that organises experiential reality. Nevertheless, radical constructivist writers are concerned with showing how inherited mathematical symbols and forms can be reconciled with their philosophy. Mathematical notations and physical instructional apparatus are seen as guiding thinking in the same way as furniture guides movement around a room (for example, Kaput, 1991, p. 55). Notations offer support to the conceptualising done in respect of them. The learner needs to build an understanding of how mathematical symbols are used. However, the material effect of this architectural environment only exists in respect of the individual learner's consciousness and the environment cannot be seen as having an independent existence outside of this consciousness. There have also been moves within radical constructivism to accommodate the social. For example, Richards (1991, p. 18) develops Maturana's notion of "consensual domain" which comprises a patterning of "mutual orienting behaviour". Here consensus with others is dependent on it appearing that there are agreed ways of bringing linguistic expressions to experience. But within this any individual observer sees the world as comprising phenomena with certain meanings to him and so is unable to objectify the world in an any more universal way. In line with other constructivist writers, mathematical ideas are held in the minds of teacher and students, without the anchoring of "actual" ideas. Relative stability is only brought about through ideas being "taken-as-shared'' (for example, Cobb et al., 1995) within a particular classroom or in the broader community.

Social constructivism, in its various conceptions, spans the construction of epistemologies, theories, social objects and of things (Sismondo, 1993, p. 547). The various debates straddle concerns regarding social realities and material ones, where in both cases the "descriptions scientists construct are... of newly created entities, rather than pre-existing natural ones" (ibid. p. 531). For example, Knorr-Cetina (1982, p. 119) considers how laboratories have been misunderstood as places where ideas are tested or generated. In contrast she sees laboratories as places were things are made to work within a highly preconstructed artificial reality where the source materials with which scientists work are also preconstructed. Meanwhile, Latour and Woolgar develop Derrida's notion of inscription in examining the ways in which representational devices are employed in creating scientific objects. Indeed they counter conventional sociological assumptions (for example, Barnes, 1988) by claiming that scientific knowledge leads to the construction of the natural world.

> We do not wish to say that facts do no exist nor that there is no such thing as reality. In this simple sense our position is not relativist. Our point is that "out-there-ness" is the *consequence* of scientific work rather than its cause. We therefore wish to stress the importance of *timing*... Once the controversy has settled, reality is taken to be the cause of this settlement; but while controversy is still raging reality is the consequence of debate, following each twist and turn in the controversy as if it were the shadow of a scientific endeavour. Latour and Woolgar (1986, pp. 180-182).

Social constructivism has also been pursued, as a descriptor, more specifically within mathematics education, sometimes as a latter day response to radical constructivism. The individualistic centring of radical constructivism became to be seen as an increasingly troubling limitation, provoking a broad range of work seeking to combine the power of radical constructivism with an adequate account of how mathematics was shared. For example, Ernest, who holds on to the term "constructivism" (1991, 1994), uses the social plane as his home base in finding a more complementary association between object and subject. Here, a notion of "objective knowledge" is retained, where "objective" mathematical knowledge is seen as being that which is both published and socially accepted by "the majority". The notion of mathematical objects existing independently is accepted (Ernest, 1991, p. 55; cf. Goldin, 1990, p. 45). Whilst the learning of mathematics itself comprises constructive acts, this is seen as being consequent on the learner internalising some aspect of "objective" knowledge and reconstructing it. The chief consequence of this formulation is that it permits a notion of mathematics existing outside the mind of the individual learner. Rather like the Platonic view where mathematics is created by the gods, existing independently of humans, here we have mathematics created by recognised experts. Whilst this perspective may have an unease with the notion of mathematics being absolute, it seems to accept the possibility of an absolute overview of mathematics at any particular point in time. Individual actions are governed by supposed over-arching structures. As such it seems more concerned with initiation and is in this respect is closer to socio-cultural perspectives than to radical constructivism.

Whilst this form of social constructivism seems to allow mathematical phenomena posited outside the mind of the individual knower, other recent constructivist writing, pursuing more socially oriented concerns, has challenged notions of the human mind where mental representations mirror externally defined phenomena (for example, Cobb, Yackel and Wood, 1992; Bauersfeld, 1992). These writers, who prefer to see constructivism as one component of their formulation, reject the apparent dualism implicit in such views since this would presuppose connections between stable entities existing independently of human constructions. In recent work, Yackel and Cobb (1996) more explicitly combine their radical constructivist roots, where mathematical ideas are seen as being held in the minds of teacher and students, with social interactionism and ethnomethodology, developing themes I highlighted above in discussing Cobb's earlier paper (1994). In particular, they argue that "sociomathematical norms" are interactively constituted between teachers and students. Drawing on the interactionism of Bauersfeld (1988, 1993) and Voigt (1992, 1995) they pursue the assumption that cultural and social processes are integral to mathematical activity, and that the teacher has a role in facilitating initiation into these, yet at the same time set as their goal that children become intellectually autonomous. In a joint project (reported by Cobb and Bauersfeld, 1995) these two groups of writers develop their common concerns, based around a reconciliation of what they call "individualist" and "collectivist" perspectives. They argue, however, that this does not result in a seamless theoretical framework, since when the individual is in focus the social fades into the background and vice versa (op cit. p. 8).

Meanwhile, Lerman (1996) pursues similar concerns but with different points of reference, nudging closer to what is becoming known as a socio-cultural perspective, thus downplaying the individualist perspective which Cobb and his colleagues wish to retain within a broader model. Having reevaluated his earlier radical constructivist stance Lerman counsels a move away from the individualistic psychology of Piaget, and the associated label of "constructivism", towards the more socially oriented model of Vygotsky. In the former the source of meaning is the cognising individual, in the latter, cultural and discursive practices. Arguing that they cannot be combined in a coherent fashion, Lerman comes out in favour of an "intersubjectivist" framework based around developments of Vygotsky's work. He sees this embracing a number of contemporary perspectives including; subjectivity constituted through social practices (for example, Walkerdine), cognition as situated in practices (for example, Lave) and mathematics as cultural knowledge (for example, Bishop).

Yet more firmly, writers in "situated cognition" (e.g. Lave, 1988; Brown, Collins and Duguid, 1989; Lave and Wenger, 1991) focus on social initiation and the relationship between learning and the situation in which it occurs. This work, which follows an anthropological perspective, questions what kind of social engagements provide the proper context for learning to take place. Greeno (1991) has developed these themes within more overtly mathematical concerns where knowing is seen as being "situated" in specific conceptual domains. Recent critics of this perspective include Anderson, Reder and Simon (1996).

As a final point in this all too brief survey of this debate, Woodrow (1995) seeks to intercept the grounds on which it stands by asserting the cultural dependency of the learning theories themselves. For example, he sees radical constructivism "as a theory created to be in concert with the societies in which it is assumed, societies for which individual autonomy rather than social responsibility is preferred (i.e. America and England)". Indeed, such an approach has become closer to official educational policy in the United States in the light of current reforms (National Council for Teachers of Mathematics, 1989). Further, radical constructivism "assumes the possibility of a negotiated position between teacher and pupil" (Woodrow, ibid), an assumption alien to many eastern cultures. Indeed such "student centred" approaches have recently been attacked by rightist critics, blaming it for the break down of discipline in British schools. Woodrow suggests that protagonists in such debates speak quite different languages, rooted in faiths, which prevent any easy reconciliation, through the acceptance of multiple beliefs, as suggested by some (for example, Cobb, 1994).

In my own work, which I intend to develop in this present volume, I have explored these issues from the perspective of contemporary hermeneutics (Brown, 1991, 1994 a, 1996 a c). I questioned the radical constructivist emphasis on construction by suggesting that this downplays the social parameters built into tasks as framed by the teacher (Brown, 1993 a, 1994 a, pp. 80-83). I showed how Husserl's phenomenology offers an approach to describing how the individual confronts and works with mathematical ideas. In this perspective mathematical ideas, as located through notation, are not endowed with a universal meaning but rather derive their meaning through the way in which an individual attends to them. This is achieved by softening the distinction between "object" and "subject" and seeing them in a more complementary relation as part of each other. The emphasis in this phenomenological formulation is on the individual's experience of grappling with social notation within his or her physical and social situation. This provides a framework, seen from the individual's point of view, in which the distinction between the individual and the social is also softened.

Implications for the mathematics classroom

In discussing the practicalities of mathematics teaching I shall draw on some descriptions of classroom work by constructivist writers flying the radical banner to highlight some of the concerns I wish to address. They have given considerable attention to how students learn and some to how teachers might facilitate this view on learning. For example, Sinclair suggests that from a constructivist point of view “the essential way of knowing the real world is not directly through our senses, but first and foremost through our material or mental actions” (quoted by Steffe, 1991, p. 178). Such a description of the acquisition of mathematical knowledge, whilst presumably being applicable to all learning, activates definite implications for teaching style. It is in the light of this description that Steffe suggests teacher’s goals consistent with such a description of learning, including for example; “to learn how to communicate mathematically with students, to learn how to organise possible mathematical environments, to learn how to foster reflection and abstraction in the context of goal directed activity, to learn how to encourage students to communicate mathematically among themselves.” A number of writers associate such sentiments with the reform process underway in the United States. It seems to me however that there is a need for care in moving from epistemologies to recommendations for practice. With the presentation of such constructivist writings one cannot but help feeling that “child-centredness” rides again, where writers discuss notions which, within the United Kingdom for example, have long since been sanitised within the confines of the official curriculum documentation (DES, 1989). For example, in the book “Radical Constructivism in Mathematics Education” (von Glasersfeld, 1991), which gathers together a number of key radical constructivist writers, there are a range of poignant statements:

> ...the teacher’s role has changed considerably from a transmitter of mathematical knowledge to an organiser, planner, facilitator, questioner, helper, monitor. (p. 247)

> In general the teacher should spend more time listening to the students than the students spend listening to the teacher. But before a teacher can be convinced of the need to reverse the usual ration of listening times s/he must recognisejust how ineffective certain forms of telling tend to be. (p. 78)

> In open ended classrooms, teachers are less secure, their authority is more apt to be challenged, and they may not know the answer to a question. (p. 40)

> ...one notices that the teacher spends the entire time moving from one group to the next, observing and frequently interacting with them as they engage in

mathematical activity. ...the children are encouraged to take responsibility for their own learning... (p. 159)

Whilst I agree with all of the above statements, the writers seem oblivious to setbacks experienced in introducing student-centred approaches in schools in the United Kingdom over the last three decades (Brown, 1993). From the right corner, successive ministers have had considerable success in undermining educator's attempts at introducing mathematics along these lines claiming that the "acid test" is the average teacher's ability to teach according to such high minded educational ideals (Kenneth Clarke). Such approaches which tend to be associated with "progressive" methods elicit especially scathing attitudes from the current administration. From the left meanwhile, Walkerdine (1984) has suggested that student–centredness merely replaced overt regulation with a form of covert regulation. She would argue that the students are constructing in a fairly restricted way since only a limited range of responses is possible given the framework set up by the teacher, and that this framework conceals multiple assumptions about social roles, the style of learning seen as desirable etc. Walkerdine sees the student's tasks as necessarily framed by a multitude of such factors which serve to constrain as well as enable the autonomy of students in their mathematical learning.

Seen in this way the radical constructivist view emphasises the student's construction whilst appearing to downplay both the environmental effects on the constructions which students make and the teacher's role as environmental manager contained within these effects. Such environmental management, however, seems evident throughout the descriptions offered of constructivist teaching styles. This arises firstly, in framing the mathematical activity in language and, secondly, in setting up the physical and social frameworks within which this happens. I suggest that the framing of the mathematical tasks and the accommodating environmental design in lessons described by radical constructivist writers is such that the "ready madeness" of ideas which they apparently wish to reject, is already highly developed before the student is invited to engage in constructive activity. The mathematical components within any curriculum are culturally defined resulting in the building blocks of any construction being in place before the student joins in. By providing such a frame these writers are offering different aspects of the task to that which might be offered by more traditionalist writers. Even in such a traditional teaching regime the learning would, in the eyes of a constructivist, be the result of the student making constructions. The constructivist teacher then, may be seen as emphasising the constructive component of the student's learning that would arise in any style of teaching.

As an example, Kaput (1991 p. 63), in assuming a radical constructivist perspective, considers the problem of finding the general formula for the sum of the first n consecutive integers. He suggests a few methods including numerical tables and Cuisenaire rods as representations prior to formulation in algebraic notation. His account is of a highly structured presentation targeting a particular conclusion. Clearly, such an approach results in the students constructing meaning but within a very specific framework. An alternative formulation presented by Billington and Evans (1987), which is more in line with the approach I wish to pursue in this book, broadens the scope of the activity. Here the problem is posed in terms of counting the number of handshakes when all of the people in a room each shake hands with each other. The task in each of these presentations may be seen as leading to the result: *Sum of first n consecutive integers* = $n\ (n + 1)/2$ but this can be achieved in variety of ways. Each route provides a different context for the production of the final result and thus a different meaning for the statement. Kaput targets the activity at the production of this result and towards this builds a tight framework which restricts possibilities for the introduction of descriptive language by the student within which mathematical expressions can arise. Billington and Evans seem more concerned with the journey taken to the final result and place emphasis on other mathematical aspects of the activity such as: processing information, making predictions, symbolising, tabulating, finding and investigating patterns, seeing connections, generalising, establishing a proof. That is process and product are seen in a more complementary relationship with each other. In my own work with students on initial teacher education courses I have set the problem as a group task concerned with creating an illustrative poster to describe the nature of the problem to a friend who did not witness the actual handshaking. In all of these situations the task for the student is to make sense of the task and make statements in respect of it. The final algebraic relationship is a reductive outcome of this activity whose meaning depends on the nature of the reduction and the experience of making this reduction.

For the teacher setting up such a task there is a need to decide on the style of posing the task and the structuring of the task that this creates for the student tackling it. To guarantee the achievement of a particular result may require a high degree of structuring on the part of the teacher and, perhaps, a lower contribution in terms of framing the problem by the student. The formulation and solution of any problem can be seen as a joint action by teacher and student, composed of (from the student perspective) a "ready made" part contributed by the teacher and a student construction made in respect of it. However, the resulting space provides components and offers a frame for what follows and so on. Similarly, any student construction is, in a sense, "ready-made" for the teacher to work with (cf. Wheatley, 1992). Such a view moves away from a mechanistic cause and effect model in the teacher-student relationship. The teacher's contribution to the joint process of constructing is necessarily implicit in any teacher/learner exchange. Any assertion of a "mathematical" domain is an assertion of a culturally bound form of structuring. In the example above, the task is as much a negotiation about the language to be used as a problem employing conventional mathematical terminology. The teacher, in asserting the conventional structuring, is demanding that the student's constructions be made using socially constructed building blocks. The teacher is thus an accomplice in any construction by the student. I suggest that the constitutive argument of the radical constructivist view can be resisted by an alternative argument assuming the opposite side of the same debate. This argument asserts the existence of the environment, and the teacher's constructions within this, acting on the student's developing understanding.

Although the debate concerning constructivist teaching practices is surely moving on as I write (see, for example, Cobb and Bauersfeld, 1995; Yackel and Cobb, 1996), I concur with those who suggest that radical constructivism provides an inadequate account of how the social web of discourses intervenes in the process of individuals declaring how they see things. Links into this web will provide a principal theme in this book while radical (von Glasersfeld) and social constructivism (Ernest) will provide occasional points of reference. As such I see myself building on the post-structuralist analysis of Walkerdine by building firmer links both with contemporary research in mathematics education and teacher education, by situating her work in a broader theoretical domain, and by referring to newer styles of teaching mathematics. I develop the theme of critical mathematics education as initiated by

Skovsmose by developing his theoretical frame around the experience of individual students and teachers, by providing classroom examples and by demonstrating an approach to teacher education which facilitates the introduction of such a style of work. I seek to complement the work of the social interactionists and socioculturalists by contextualising their theoretical themes in relation to the hermeneutic, critical and post-modernist traditions and so making stronger connections with contemporary philosophy and social theory.

PART ONE

EXPERIENCING MATHEMATICS

How far can mathematics be drawn into a linguistic domain and how far does such a move enable us to clarify the way in which students share their mathematical thinking with their peers and with their teacher? Such concerns will be addressed in this part of the book. Before proceeding, however, I feel an example might help in further clarify some of the issues of concern. I offer an account of someone working in a university seminar specifically concerned with exploring how words are introduced in holding on to evolving ideas (first reported by Brown, 1996 a, pp. 62-64). In the seminar there was an invitation to imagine a particular geometrical configuration. In an attempt to give a more graphic example of the issues of concern in this book I offer the three sentences which initiated the exercise and an account of someone's developing understanding during the exercise, written immediately afterwards.

"Imagine a circle with two tangents meeting at a fixed point. Imagine the circle getting bigger and smaller. Attend to the locus produced by two points where the tangents meet the circle."

I remained unsure of whether the centre of the circle was fixed or movable. My initial sense was that if it was fixed then the circle could only enlarge or shrink within a very limited range. It stopped growing as it hit the point (which I had located above the circle) and finished shrinking as it became no more than the point at its centre, at which point the two tangential points met. The radius could not exceed the distance between the centre of the circle and the top point where the tangents met. I was having diflculty in picturing the locus, mainly because there seemed to be more degrees of freedom than I could cope with. At first the circle growing seemed to be associated with its centre moving down. I could not easily hold the centre as fixed as the circle moved. I could not be sure about the shape of the locus as too many things were moving at once. I thought of a U-shaped curve, a parabola maybe. Or, possibly an ellipse since I had some sense of the locus *joining at the top as well.*

I reported on this to my colleagues. Some seemed surprised that my centre could move. A discussion between my colleagues ensued which left me somewhat bewildered. There was talk of fixing radii or of fixing the centre. I heard one colleague speak of a tulip. It was this latter image which I used in firming up my image of a parabola - I could picture a tulip as being rather like the base of the truncated parabola I had in mind (see Figure 1).

I commenced the exercise feeling comfortable with the words circle and tangent as used in the original direction. These terms were very familiar to me and I regularly use them in describing things I see. Ellipses and parabolas became things I attempted to fit on to my picture of the locus. This, however, was unsuccessful since the image was too dynamic for me to capture it clearly in words. After the brief discussion I became aware of not being able to follow my colleagues and began working on clarifying my original picture. I began attempting to fit a "tulip" onto the images I was having. At this point I felt I was placing my images on the words of others. I caught fragments of the discussion as it continued; like - "you take one of the lines and you get half of the tulip" but I had too few straws to grasp hold of and continued working on my own. It later turned out, in fact, that my colleague had not said tulip but rather had said "tulip-vase".

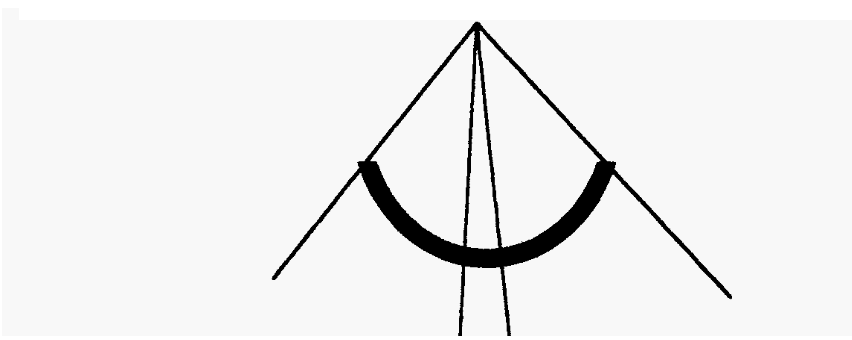


Figure 1.

Initially he works with the words offered in the original description. His thoughts are framed in words with which he is familiar as he seeks to build up his own picture of the locus. "Circle" and "tangent", are among the conventional mathematical terms used in the original description which guide his thoughts. "Radius", "ellipse" and "parabola" are conventional words he seeks to introduce in making sense of the picture. "Tulip" is introduced into his thinking both as an aid in developing his own picture but also in an attempt to share the images offered by his colleagues. The person offering the original "imagining" may have had an image in his head but the significance to participants depended on a number of issues;

- he will have a particular understanding of, and ways of working with, words like "circle" and "tangent",

- he will have the ability to introduce other conventional mathematical terms and personal images into his analysis to support his thinking,

- he will introduce particular interpretations into the original wording, for example, allowing the centre of the circle to move or not,

- he has to find ways of capturing the shapes and movement he senses in a way that can be stored and communicated,

- he also had a need to reconcile his own mental imagery with descriptions offered by others about how they were seeing things (for example, "Tulip").

In tackling this exercise the participant is attempting to reconcile his mental dynamics with descriptions in words and in so doing move from his personal account to one that could be shared with his colleagues. Not only is he trying to bring images to the words of the seminar leader and his peers but also find ways of capturing his own imagery in words. This locates the struggle between understanding the world as it is and operating on it to make it different - the very attempt to describe the world as it is, changes the world. The participant who introduced "tulip-vase" as a way of describing what he saw affected the perceptions of others and the descriptions which they offered. Any attempt to move from the personal to the social or vice versa requires such a *moving forwards* as interpretations combine. The geometrical construction lends itself to formal statements yet such statements are arranged by the individual user. The statements have an interpretive feel and are offered with varying degrees of certainty and one is only able to focus on specific parts of the construction at any one time.

In more general terms the individual is seeking to share some images with his colleagues but, on this occasion is obliged to tackle this task through the medium of spoken words. He attempts to connect his own mental imagery with the words of his "teacher" and, in turn translate this imagery into his own words. Both "teacher" and "student" are using the language of their culture to capture their personal way of seeing things. But what does "personal" mean when it has to be so carefully channelled through the social filter of language? In the first chapter I introduce mainstream hermeneutics and consider how it might assist us in describing the role of language in the process of understanding and, in particular, in developing mathematical thinking. The second chapter takes us more firmly in to the realm of language by considering post-structuralist perspectives on how mathematical reality is constructed in language. In the third chapter, the emphasis shifts to considering how language is employed as a medium of exchange within shared mathematical activity. It further examines how the development of mathematical thinking can be understood as a reconciliation of alternative linguistic forms.

CHAPTER 1

HERMENEUTICS AND MATHEMATICS EDUCATION

> The language used can only be the language we have. So one must not look for meanings - only for the things that are meant. (Brookes, 1971, p 159)

In this chapter I discuss how interpretation might assume a more central role in mathematical understanding. By shifting the emphasis from what a statement might mean to how it has been used by someone in a specific situation, the human agent becomes implicated, and a certain perspective gets revealed. This is reflected in recent discussion in mathematics education research which seems generally opposed to notions of absolute meaning and has shifted towards understanding meaning as an individually or socially constructed phenomena. Nevertheless, whilst such a move seems consistent with theoretical shifts in other academic fields there is still an underlying difficulty resulting from a history of seeing mathematical meaning as in some ways independent of time and context, a notion associated with concepts rather than with conceiving, a fixed point to which the learner converges. Whilst radical constructivism, for example, tries to move away from such stable notions of meaning, in doing this they explicitly avoid entering into theories about how things are. I will argue here that by using notions of objectivity derived from phenomenology, a theory of how things are can be introduced without undermining the constructivist theory of how we know.

Hermeneutics, the theory and practice of interpretation, is governed by a belief that whilst the world may exist independently of humans, it cannot present itself directly to the human gaze. It attends to the process through which we develop an understanding of the world. The hermeneutic task can be seen as an uncovering of meaning, but a historically situated meaning dependent on the media and experiences through which it is observed. Further, we can never uncover a meaning free of the conditions that gave rise to us and of the particular perspective we assume. Hermeneutics readily lends itself to the disciplines within the human sciences, which in general, "deal with the world of meaningful objects and actions (as opposed to physical objects and events in themselves)" (Culler, 1976), where the human subject is assumed to have a particular position and perspective rather than some God-like overview.

The principal task of this chapter is to outline the phenomenological roots of contemporary hermeneutics and show how such a perspective can assist us in describing mathematical activity. By seeing mathematical expressions as being used by humans in particular situations, rather than as things with inherent meaning, emphasis is placed on seeing mathematical activity as a subset of social activity and, as such, is subject to the methodologies of the social sciences. This will be organised as follows: Firstly, I shall outline some issues arising through seeing mathematical expressions as being necessarily contained in action, resulting in meanings that transcend mathematical symbolism. This is followed by a discussion of phenomenology and hermeneutics. I then show how different conceptions of hermeneutics lead to alternative educational practices. Finally, I consider how we might understand mathematical learning as a hermeneutic process and show how the creation of mathematical phenomena in human understanding is a consequence of a linguistic process of classifying according to an individual phenomenology. A reader wishing to minimise a more general theoretical development may prefer to skip the next three sections and move directly to the final two sections of this chapter where concerns more specific to mathematics education are introduced.

ACTION AND MEANING

In his book *"Keywords"* Raymond Williams (1976) took a hundred or so words from the contemporary scene and discussed their usage over the last century or so and in each case demonstrated how this usage had evolved over the years. In commenting on this book Brookes (1978) pointed out how the word "meaning" simply was not used by Williams. It seems that this was a deliberate attempt to show how the use of a word is more important than any supposed intrinsic meaning. Brookes argued that to assert an intrinsic meaning is to underplay the context of what one says and to presume the universality of certain ways of seeing things.

This emphasis on usage echoes Wittgenstein (1958, p. 20) who in *"Philosophical Investigations",* suggested that the meaning of a word might be seen as its usage in language and is thus dependent on both situation and time. This offers an alternative to seeing words as having inherent meaning and a key to analysing expressive activity as action; to say a sentence is to perform an action, an action that takes place through time. The meaning of a sentence, seen as an action, is related to its perceived effect in a social situation (Thompson 1981, p. 126). Shortly, we will see how Ricoeur sees this effect, the way in which action leaves its mark, as the "objectivity" of the action. He follows Husserl, the founder of modem phenomenology, in seeing objectivity and subjectivity in a more complementary relationship with each other.

If then the production of any mathematical expression can be seen as an action, the meaning of such an expression is necessarily subject to an interpretation that transcends any meaning in the expression itself. This necessitates looking at how the expression is being used by the individual in a particular context. A distinction needs to be drawn between "mathematics" (as an independently existing set of ideas) and "activity seen as mathematical". In this perspective, the meaning of any mathematical action goes beyond that which would be found in a purely literal or symbolic investigation and cannot be separated from its agent or the context in which it arises.

In a social situation encompassing mathematical learning, a variety of linguistic forms will be used within a broad communicative environment. Aspects of the language used will be specifically associated with conventional mathematical ideas, but much will be less precise, supporting other facets of the exchange. In most learning situations we are concerned with activity taking place over periods of time comprising personal reflection making sense of engagement in this activity. The learning process entails more than a local concern of getting to grips with clearly defined "mathematical" concepts. Indeed there are many forms of mathematical discourses each flavoured by their particular social usage. For example, a university lecturer might speak as if she were a Platonist, with utterances being made as if they were extracts from a transcendental world of mathematics. A government representative, meanwhile, might understand mathematics in terms of how it might be partitioned for the purposes of testing. A civil engineer might regard mathematics as a subject whose prestige is consequential to its ability to be applied in "real life" situations. Each of these people would enter into a dialogue placing varying stresses on their mathematically inclined utterances according to some ideological tainting. (Richards, 1991, pp. 15-17, discusses the way in which specific linguistic domains condition the way in which mathematics is spoken about.) A string of symbols, no matter how neutral it may appear, cannot be seen independently of the context into which it is being issued, nor its selection by someone with a particular purpose. The style of describing mathematics is necessarily interpretive according to some mode of signification; that is, with a particular way of bringing words and symbols to experience. Certain styles of signification become naturalised in the sense that they become culturally conventional in a way that seems to be entirely neutral, just like a realist painting might be regarded as the most straightforward representational mode. Mathematics becomes locked into certain conventional symbolisations which dominate thinking about it, whilst introducing an ideological layer which mediates our thinking about it.

In speaking mathematics, or of mathematics, my delivery is never neutral. I am necessarily acting in time according to a particular agenda. I refer, by implication (through the perspective that I reveal), to myself, to the world I see, and to the person(s) to whom I am talking. Mathematics can only be shared in discourse and the act of realising mathematics in discourse brings to it much beyond the bare symbols. The mathematics which I intend to communicate is always mediated by the explanatory procedures of such a social event. My interlocutor is obliged to interpret my speech, reconciling parts with the whole, stressing and ignoring as he sees fit. The distinction between knowing through facts and knowing through signs becomes blurred in this process since the facts of mathematics are immersed in the usage of them. The expressions of mathematics only arise within actions in social events. Ricoeur (1981) emphasises these qualities of language usage in combining Saussure's linguistics with the speech act theory as described by Austin (1962) and Searle (1969). Austin sees the effect of a sentence offered in a social situation as having three levels; a) the locutionary effect (the literal meaning), b) the illocutionary effect (the intentional action implicit in the statement, for example, "I define...", "I add...", "I offer...") and c) the perlocutionary effect, (the action actually effected by the sentence). Ricoeur (1981, p. 199) suggests that these levels form a hierarchy according the degree of interpretation needed. The locutionary meaning can be checked in a dictionary. To describe the perlocutionary meaning, however, requires the subject to have experience of living in an appropriate language using community and to be accustomed to using words in given situations in a conventional way. In this book, by focusing on the activity of mathematics, I shall be examining locutionary, illocutionary and perlocutionary dimensions of mathematical statements. The perlocutionary level, for example, attends to the social embedding and effect of the linguistic act. For a student learning mathematics, the ideas she seeks are always located in social practices, and she needs to become adept at these practices if she is to understand the mathematics that they hold. As I proceed I will question how far mathematics can be understood purely at the locutionary level. I will argue that the student's principal task is to learn the practices which host ways of seeing mathematics.

PHENOMENOLOGY

The radical constructivist assertion, that the student constructs his own knowledge as opposed to receiving it "ready made", echoes the classical debate as to whether the human subject constitutes the world or is constituted by it. Ricoeur (for example, 1966) has developed Husserl's writing on phenomenology and offers some assistance in tackling this constitutive/constituted debate. He does this by seeing subject and object as part of each other; the individual always being part of what he sees. For example, in handling some practical mathematics apparatus I am finding out about myself. The apparatus is only meaningful insofar as it resists and guides my actions. The apparatus and my body become unified in any action.

In suggesting that subject and object are part of each other, Ricoeur softens yet maintains the distinction between them. He speaks of any action as having reciprocal "voluntary" and "involuntary" components. The voluntary component gives rise to the involuntary component which has no independent meaning but rather can be seen as the immediate context, or the resistance, which gives the voluntary component its meaning. The involuntary shapes itself around the voluntary act. This implies a (hermeneutic) process where the subject voluntarily acts in the world he supposes it to be, but this in turn gives rise to (involuntary) resistances which are always at some distance from those anticipated. For the subject to get anything done, however, he needs to make the assumption that he can act on his current understanding. That is, he suspends doubt whilst acting as if his reading is correct.

In his detailed discussion of Ricoeur's work, Thompson (1981, p. 128) identifies this aspect of his work as dealing with the constitutive/constituted dichotomy:

For Ricoeur's attempt to understand the reciprocity of the voluntary and the involuntary is a systematic attack on the dualism of an autonomous self-consciousness exiled from an objective world which it regards as an other. Ricoeur pursues this attempt through a detailed demonstration of how each moment of the will comprises both a voluntary and involuntary aspect, the ultimate unity of which remains an unattainable ideal.

A key notion in Husserl's phenomenology is that of "intentionality". In seeking to clarify this notion Schütz (1962) asserts that:

There is no such thing as thought, fear, fantasy, remembrance as such; every thought is thought of, every fear is fear of, every remembrance is remembrance of, the object that is thought, feared, remembered.

Similarly, Ricoeur asserts that a consciousness is always a consciousness of something. This is not to say that the subject is conscious of a discreet object which it sees as the other, but rather

the basic datum of experience at its most immediate level is the intentional unity of subject and object from which both the concept of a pure subject and of a pure object are subsequently derived by reflexive consciousness (Ricoeur, 1966, translator's introduction, p. xiii).

My understanding of this is that I might wish to talk about the situation within which I see myself, as if it were independent of me, but I can only do this after experiencing myself as part of it. I experience myself "acting" through time but I am unable to talk about this as it happens. Even a sports commentator talks about things that have happened. In my subsequent descriptions, I can speak of "actions", facts after the event, which can be classified in language after reflection. It is in such a description, made in hindsight, that I am able to describe myself as if I am separate to the situation I inhabit.

Zizek (1989, pp. 11-14) in his lengthy discussion of Lacan's psychoanalytic work locates a similar notion. Referring to Freud's work on dreams he distinguishes between the dream (latent content), the memory of the dream (manifest content) and the recounting of the dream in words (cf. Freud, 1991, pp. 381-390). The meaning of the dream cannot be captured as it happens, but rather some retroactive categorisation is necessary to prepare it for description in language. I am very much a part of the dream as it happens and I need to reflect subsequently in order to make sense of it. This reflective process results in myself, and the world of which I am part, being described in language, providing an orientation to the world I have experienced through my senses. As another example, Zizek (1991, p. 100) identifies the Marxist notion of "class struggle" as a "structuring principle" around which we can orientate social phenomena we have experienced through historical processes. In mathematical activity concerned with making sense of certain situations, we are confronted with a similar task of introducing structures around which we can orientate our thinking. By introducing successive linguistic and symbolic overlays the various aspects of our work within an activity can be examined through the "stressing and ignoring process" implied, without which "we cannot see anything" (Gattegno, 1971, p. 11).

Ricoeur uses Husserl's notion of "bracketing", where the existence of objects, and relations between them are assumed and fixed for the time being so that consciousness is directed towards "phenomena", that is, objects having certain meanings to an individual person at a given time. According to Ricoeur (1966, translator's introduction, pp. xiii-xiv),

> there is no consciousness unless it is consciousness of an object- and, conversely, an object presents itself as an object only for a consciousness... by imposing the phenomenological brackets we transform the contents of experience from a physical world of objects into a world of phenomena, that is objects as meanings presenting themselves to a consciousness.

Coward and Ellis (1977, p. 132), in discussing Husserl's work, suggest that

> phenomenology disputes the so-called "natural attitude", the existence of the external real world. It is not concerned with the spatio-temporal existence of things, such concerns are simply bracketed out: if it is real to consciousness, then it is real.

Fuller accounts of Husserl's phenomenology are offered by Schütz (1962, pp. 99-149), Gadamer (1962, pp. 214-234) and Pivcevic (1970).

Objectivity

Husserl's work disrupts notions of objectivity where it is seen as the antithesis of subjectivity.

The naivete of talk about "objectivity" which completely ignores experiencing, knowing subjectivity, subjectivity which performs real concrete achievements, the naivete of the scientist concerned with nature, with the world in general, who is blind to the fact that all truths that he acquires as objective, and the objective worlds itself that is the substratum in his formulation, is his own life construct that has grown within him, is of course, no longer possible when life comes on the scene. (Husserl, quoted by Gadamer, 1962, p. 220).

Whilst the independent existence of the material world, prior to any classification and outside of individual consciousnesses is not denied, the world of material <u>objects</u> existing, in a fixed way, independently of an individual consciousness, is denied. The existence of a world of material objects presupposes some prior classification. My ability to distinguish shapes, colours, smells, textures, objects, is dependent on my senses, is culturally conditioned, and emerges through time. In describing perception through a semiotic framework Peirce identifies three ascending levels, known as Firstness, Secondness and Thirdness. These are; quality (the initial sense, iconic), fact (the identification of objects, indexical), law (the relations between objects as understood through a specific cognitive state). (This aspect of Peirce's work is discussed by Groden and Kreiswirth, 1994, pp. 569-562; Hookway, 1985, pp. 90-97 and also more fully by Feiblemann, 1960). It seems to me that phenomenology intercepts between the first two levels. Perceiving the world requires categorisation of it, which involves differentiating and relating aspects of this world. It is towards this end that we use language. The material world is only describable within linguistic categories to an individual consciousness and any such description is the result of an interpretation. The voluntary action brings into play involuntary resistances that only have meaning in subsequent descriptions of their effect. Ricoeur (1981, pp. 197-221) talks of the "meaningful effect" of an action as being its "objectification"; the mark it leaves on time. The meaning of an action, that is, its objectivity, is related to how it is described in retrospect, i.e. after it has been organised in linguistic categories, as if in some historical account. Ricoeur explores this in terms of an analogy with the objectification which speech goes through in being committed in writing. Meaning is dependent on the categories that we introduce and objectivity is a function of describable traits. I shall introduce the particular traits Ricoeur identifies as being retained when speech is fixed in writing. As an example, I consider the nature of the reduction that takes place in attempting to capture a lesson in the form of a transcript. I include here a brief extract of a transcript created during a lesson given by a teacher involved in a project to be discussed later (Brown, 1987 b, 1994 b). The account was recorded by the teacher herself and is reproduced here in its original form.

Four six year olds sit around a table together.

Teacher:	Take your exercise books and pencils. Work together using the counters and make as many stories of 12 as you can. Write the number stories.
Richardson:	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12! (as he touches each counter).
Clifford:	3 plus 1, 2, 3, 4, 5, 6, 7, 8, 9, 10! (as he touches)
Richardson:	11, 12. (to Clifford)
Richardson:	3 plus 1, 2, 3, 4, 5, 6, 7, 8, 9 (to himself as he points)
Roger:	Nine plus what? (He then writes 9+3=12 since he was ignored)
Chester:	How 12 going?
Roger :	A one and a two.
Chester:	10 plus 2 is 12 (He then writes it in his book)
Clifford:	10 plus 2
Chester:	And 2 plus 10
Richardson:	2 plus 10? Boy, you do matching (they then write 2 + 10 = 12)
Clifford:	1 plus (as he manipulates bottle tops he looks to Richardson)
Richardson:	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. 1 and 11. (They all write it)
Clifford:	Look it 11, a 1 and a 1.
Chester:	12 plus 1 ?
Roger:	12 plus 1 ? Whey! (sarcastically) 11 plus 1. 11 plus 1 is 12.

These few sentences from the report evoke, to some extent, the lesson which “actually” occurred. They were subsequently used by the teacher and me in talking about the teacher’s practice and the student’s learning. The creation of this transcript was an attempt to capture the flow of experience into something we could stay with and reflect on. It focuses on the things that we saw as important to our research; it does not mention the teacher’s opinion, the curriculum being followed, the weather, their clothing, and so forth. The report characterises the lesson in very specific way. Some elements of the lesson perceived have been transformed reductively into the elements of the lesson report. In reading the report we invert this transformation so that we can imagine the actual lesson. The nature of this inversion, however, will depend on who the reader is. The teacher, one of the boys, or I, would all have different ways of fulfilling the reference of the text in the report. The transformation that takes the actual lesson into the teacher’s report is many-to-one whilst the inversion is one-to-many. The teacher, as the writer of the report, determines the rules of the original transformation by selecting the elements of the lesson that she records and how. The reader of her report chooses the rules by which the inversion is made according to their own scheme of interpretation. This interpretation will depend on the history and motives of the reader, along with the expectations associated with these.

Ricoeur (1981, pp. 197-209) sees text as a “fixation” of a speech event whereby elements of the event are reduced into a set of written words. He explores this transformation to consider what is lost from the “actual” event and in so doing identifies various traits that differentiate the speech-event from its fixation in writing. I shall adapt my interpretation of these four traits to the example of this lesson in identifying that which is lost by reading the report rather than attending the lesson.

a) The fixation of the discourse.

A transformation is made from the speech and writing in the lesson to the report. The writing fixes the speech which in the lesson was a fleeting event, but what exactly is fixed? In the report the “said” of the speech event has been recorded and also that written by the teacher and students. There might be a one-to-one mapping from the ”said” of the living speech to the report. However, the speech existed in a social and physical context that was not so readily transformed in this way.

b) Face to face observation of the speaker by the interlocutor.
It is possible to identify some of that lost in this reduction. In the lesson we would be conscious of who is speaking. For example, the gestures of the boys have only their spoken element represented in the report. Intonations, delays, facial expressions and pointing fingers are only sometimes implied. The possibility of interpreting such gestures is lost. Because of knowing the boys and their teacher I can speculate on further elements missing in the reduction, such as the comedy of the whole situation, the way in which the four boys regard each other, the earnestness of their commitment to the task. In short, the individuality of those present is largely lost. Such features would contribute to any personal interpretation but they are not identified in the report.

c) Speaker and interlocutor in a shared world.
Furthermore, the speech refers to a world and the reference of the speech reported is represented in a reduced state. Much of the speech seems to refer to the bottle tops and the act of counting them. Being absent deprives us of the full context of the speech and of the context of that to which it refers. Consequently, in considering the lesson from the outside, we fulfil the reference of the recorded speech from our own experience. The references within the "closed" world of the boys and the teacher are not shared. In addition, the time structure and pacing is lost. Our only time markers are inherent in the sequencing of the sentences of the report. The nature of the spacing between the words, however, is not clearly represented.

d) Accommodation of interlocutor by the speaker.
I was conscious that my presence as an observer sometimes caused disturbances, such as, students being shy, or asking me questions about my work or their work. Sometimes I became the topic of conversation. In certain situations I spoke to the students, perhaps to relax them or to enquire about their work. Similarly, because I was a teacher trainer or a researcher, the teacher sometimes felt uneasy about me being there which sometimes led to the teacher behaving differently. In a sense the speech and behaviour of the teacher and students were acknowledging my presence. I was being accommodated as a person in attendance. Such accommodation would occur in some form for anyone in attendance but clearly not for someone reading a report of the lesson. That the interlocutor disturbs the lesson in such a way is

inevitable. Further, the interlocutor may be more able to come to terms with a situation that responds to his actions. In short, it is not possible for the teacher or students to influence the opinion of someone reading the report of the lesson, They do not address the reader in their actions.

For both the students writing the numbers, and for the teacher and me trying to capture the lesson in a transcript for a research exercise, some reduction is inevitable as events evolving through time are squeezed into a fixed form. In doing mathematics, my thoughts are always modified as I attempt to frame them in words and diagrams on a page, but in doing this I mark time in a way which my thoughts do not. Similarly, for a teacher reflecting on her practice, any attempt to pin down what she actually did only partially reflects lived experience. Objectivity within phenomenology emerges through an interplay between the experiencing of the world and making statements in respect of it. Whilst my experiences lead to me describing them in a particular way, these descriptions which I offer condition the way in which I subsequently experience things. Here objectification is a consequence of a reflexive process where notions, and relations between them, are bracketed (i.e seen as phenomena and assumed for the time being), and through this bracketing, regulate the way in which the world is described. In a sense the objective world can only be seen through successive linguistic overlays; or rather, particular time-dependent partitionings. It is through this route that phenomenology can be seen as offering the opportunity for providing an ontological grounding for action. By pursuing the paradigm of text interpretation, through seeing meaning as being held in descriptions of actions, Ricoeur sees acting as analogous with writing and interpretation of this action as reading. It is through this sort of fixation that we can employ techniques of interpretation in facilitating both understanding (through signs) and explanation (of “facts”). It is this close relation between understanding and explanation which hermeneutic enquiry seeks to unfold.

HERMENEUTICS

The hermeneutic circle

Hermeneutics was originally developed and employed in the analysis of biblical texts but was extended, largely by Dilthey working at the turn of the century, to cover the whole of human existence. Heidegger, the most celebrated student of Husserl, introduced an hermeneutical dimension into his teacher's work in phenomenology by insisting that meaning is historical rather than transcendental; a consequence of interpretation through time rather than mere consciousness. Leading modem exponents of hermeneutics, whose work guides us here, are Gadamer and Ricoeur. For these two writers, whose work has included a strongly theological dimension, there are certain truths that orientate our way of seeing things. Hermeneutics, permits a range of interpretations, some of which may be seen as being closer to the truth. However, no interpretation is ever final. Hermeneutical understanding never arrives at its object directly; one's approach is always conditioned by the interpretations explored on the way. Whilst one's understanding may become "fixed" in an explanation for the time being such fixity is always contingent. In choosing to act as if my explanation is correct, the world may resist my actions in a slightly unexpected way, giving rise to a new understanding, resulting in a revised explanation, providing a new context for acting and so on. This circularity between explanation and understanding, termed the *"hermeneutic circle",* is central to hermeneutic method.

Dilthey (1976) described a hard distinction between explanation, as might be offered within the natural sciences, and understanding (or more specifically an interpretation), as might be offered in the human sciences such as history. The former could be offered as a statement of fact whilst the latter could always be subject to personal interpretation. More recent writers, notably Ricoeur (1976, 1981) have challenged this, bringing understanding and explanation into a more complementary relation under the umbrella of interpretation. The way in which I experience the world governs the way I talk about it. The way in which I talk about the world, however, now informs the way in which I see it in the future. So we have the two arcs of the hermeneutic circle, from understanding to explanation (i.e. from understanding the world through a process of categorising it, to making constructions in language in respect of it) and vice versa (Ricoeur, 1981, pp. 145-164, 197-221). (As we shall see shortly this recognition also underpins the practical approach of ethnomethodology, for example, Garfinkel, 1967.)

By resisting firm distinctions between explanations of the natural sciences (knowing through facts) and understanding of the human sciences (knowing through signs), Ricoeur sees both forms of knowing as subject to an interpretive framework. Within history, for example, whilst it may be possible to continue offering ever more interpretations of "what happened?" if we are to act in the light of this knowledge we have to suspend doubt for the time being and assume a certain position towards getting things done. Such a closure might be seen as complementing the phenomenologist's suspension of belief whilst thinking (see Schütz, 1962, p. 229, and discussion of this in chapter 5). Conversely, taking physics, as an example from the other end of this scale, while we may have statements which "on the surface" seem entirely incontrovertible, it is still necessary for an individual human to decide how such statements will be used in the social space or how they have been used. In both situations we fix the flow of time by making statements that hold for the time being. In history the fixity is in the decision to pin down an interpretation and act on the basis of it. In physics the fixity is embedded in the linguistic holding of physical phenomena. Nevertheless, hermeneutic analysis sees both "fixities" as subject to revision.

All of our own creative utterances are in a language which conceals an inherited way of classifying, choices made by our ancestors, of ideas that seemed worthy of a name of their own. These historical choices can never be eradicated and will forever condition and mediate our experiences. Further, we cannot discuss language we use from outside language. Unlike certain forms of analytic philosophy which see language as "picturing" the world, hermeneutics sees language as being part of the world. The unknown is unrecognisable unless the structures of the known can be grafted on. This notion of always revisiting, although fundamental in modern versions of hermeneutics, can be traced back to Plato and beyond. As we are always already immersed in meaning we are, as a consequence, unable to enter any situation free of the traditions from which we emerge.

Critical hermeneutics

Critical social theory, sometimes referred to as "critical hermeneutics", is associated with the Frankfurt school, with Habermas as its foremost writer, in some ways developed as a reaction to the more moderate line pursued by Gadamer. As in the hermeneutics of Gadamer, however, critical hermeneutics presupposes a truth to be found. Ricoeur (1981, p. 78) suggests that,

> (whereas) Gadamer introduces misunderstanding as the inner obstacle to understanding, Habermas develops a theory of ideology, construed as the systematic distortion of communication by hidden exercise of force

Another key point distinguishing moderate from critical hermeneutics is that for Gadamer the hermeneutic task is based on a "dialogue that we are" whereas Habermas has a quest for "an unrestricted and unconstrained communication that does not precede us but guides us from a future point." (ibid.) Habermas (1970) follows Freud's work on the way in which language can become distorted as a consequence of covering up some previous disturbance. Like Freud he sees this as a possibility in collective behaviour within social systems where some underlying truth has become concealed.

Some see Habermas' "rationalist" enterprise as a somewhat belated contribution to the Enlightenment, the revolution which sought to free human thought from the shackles of religion, guided by the belief that if we think about things we can make them better. Indeed, it was spurred by a recognition that it was man who had created a Christian God rather than vice versa and so man was best equipped to decide his own fate (Fukuyama, 1992, p. 199; Harvey, 1990, pp. 12-15). Habermas follows a long line of critical tradition in asserting that there is always something outside of the structures that humans bring to their world - a realm of mystique formerly associated with God but now, according to Habermas, occupied by modern day mysteries that impinge on the life of individuals, such as the concealed exercise of political force. Habermas focuses on such modem day mystique and sees the individual's task as being concerned with understanding the powers that operate against her, a task which cannot be fully executed from within the realm of language. Through bringing into question existing ways of doing things we can evolve towards a broader realm of human action that captures ever more of the space previously governed by mystique. He sees this evolution as being closely associated with how things are described. For example, as an individual if I describe the current prime minister I say something about myself as well. As I describe other things I reveal yet more about myself, I constitute myself through the way in which I describe the world around me. Societies similarly participate in such self-formative processes. Through commenting on the world around they reveal their perspective. However, these ways of describing can change. Individuals or societies evolve as they operate on their ways of describing things around them and perhaps seek out distortions in the language we employ. By taking control over how phenomena are described we open up avenues for new possibilities.

A particular feature of critical hermeneutics is a strict separation between what Habermas calls "monologic" languages, within which he includes mathematics, and natural languages (1972, pp. 161-163). He sees formal languages as being "divorced from a concrete life context." An expression "like a vehicle, .. goes unaltered from the possession of him who states it to the possession of him who understands" (Dilthey, quoted by Habermas, ibid.). Such monologic "(linguistic) expressions appear in an absolute form that makes their content independent of the situation of communication" (Habermas, ibid.). For this reason he excludes formal mathematics from hermeneutical understanding and so asserts a further difference with his more moderate counterparts.

The role of language in developing understanding

Gadamer's "moderate" and Habermas' "critical" hermeneutics have different perspectives on how language underpins developing understanding. A review of a classic debate between them to be offered now will serve us later in considering how far developing mathematical understanding can be viewed as a personal activity when the components of mathematics are largely derived from social constructs. I suggest that this clarifies some of the issues raised above to do with how mathematical notions are framed.

Gadamer and Habermas each address the scope of language and its role in developing understanding. In particular, they ask if it is possible to frame all experience in language. Both writers presuppose some notion of "how things are"; but for any individual this is always mediated by experience since we understand things through the way in which they respond to our actions. We build our understanding through time. Gadamer (1962, 1979) is an advocate of seeing all experience as being mediated by language, that is, all interpretation is linguistic. It is in describing the nature of this mediation, however, that we meet a major source of controversy. Habermas (1972) cautions against what he sees as an anarchism implicit in Gadamer's insistence on the universality of language. He sees this arising through a failure to regulate the validity of meanings produced linguistically. For Habermas, language is always limited by extra-linguistic experience. It is for this reason that he draws the line between the formal languages of science and natural languages. Such a distinction places limits on the individual's ability to construct meaning without reference to cultural constraints. Gadamer contends, however, that all experience is mediated by language and that even scientific languages are subject to hermeneutic processes since there is always a need for someone to decide where and how to use them. The controversy is rooted in there being two different conceptions of language. It seems Gadamer's view of language, if applied to mathematics, extends to include hand gestures, drawings, silences, actions with mathematical apparatus, all of which are susceptible to broader interpretation than are formal symbols. Extending this interpretation, we might suggest that educational practices and institutions can be seen as either promoting or preventing domination by the authority of traditional social structures and ideologies. The more that we assert the authority of conventional notation and procedures over more personal formulations the more we move towards tightening the grip of society's modes of description and the ideologies built within them.

In short, it is the way in which the linguistic is defined in relation to the extra-linguistic that differentiates the positions held by Habermas and Gadamer. In discussing this debate Gallagher (1992 a, pp. 70-73) brings Heidegger to the aid of Gadamer and Piaget to the aid of Habermas before introducing Merleau-Ponty to adjudicate. When Heidegger (1977, p. 324), says "man acts as though he were were the shaper and master of language, while in fact language remains the master of man", he sounds close to Gadamer (1962) contending that "language always forestalls any objection to its jurisdiction. Its universality keeps pace with the universality of reason" (p. 363). The "intimate unity of language and thought" of which Gadamer speaks (p. 364) precludes the possibility of extra-linguistic cognition. In contrast, Habermas examined the possibility of maintaining a critical distance from both language and the process of tradition. Meanwhile Piaget asserts that interpretive intelligence precedes language acquisition and that there is already a certain logic built into the undifferentiated signifiers of pre-linguistic action which helps to structure linguistic behaviour. "Language does not constitute the source of logic but is, on the contrary, structured by it" (Piaget; Gruber and Voneche, 1977, p. 507). However, as we shall see in the final chapter, Derrida argues, more strenuously than Gadamer, that the interpretable world is imbued with textuality from the outset. The history of the world, its social organisation, its physical structure etc. is closely associated with the history of language. We are born into a world which is already conditioned by language.

So then, we have Piaget's semiology is about attaching signifiers to already constituted signifieds (Walkerdine, 1988, p. 3; to be discussed more in the next chapter), Meanwhile for Gadamer, it is language itself which frames and thus brings into being the signifieds. Merleau-Ponty addresses this apparent dichotomy in two ways. Firstly, he sides with Gadamer by pointing out that even the child's pre-linguistic experience takes place in an environment where there are people talking and the baby's murmuring shows cultural bias from a very early age. The suggestion being that language is a more basic experience than Piaget would allow.

> The child receives the 'sense' of language from the environment... The child's relationship with his environment is what points him toward language. It is a development toward an end defined by the environment and not preestablished in the organism (Merleau-Ponty, 1973 a, p. 14)

Secondly, by denying the possibility of a transcendental view mediating between the two linguistic formulations there is necessarily a resistance to a final answer being reached - what Ricoeur (1973, p. 163) calls "irreducible perspectives" (Merleau-Ponty, 1973 b, p. 225). You need some sort of perspective as a "home-base'' where alternatives may seem alien to the adopted "frame" (cf. Bateson; Goffman, 1975, pp. 10-11). As an example of this difficulty, Pool (1989) describes the "incommensurability" of two teaching traditions. After a spell working as a mathematics teacher in English schools he moved to, in some sense, a parallel job in Sierra Leone but with very many radical differences. Given his background in the English education system the culture seemed very curious. However, gradually he became clearer about the way in which schools operated and began to understand. Indeed, upon his return to England it was English schools which seemed peculiar and it took him some time to readjust back. The two cultures could not be compared "objectively" side by side, since to understand either culture required living in it towards making it part of oneself. It was not possible to mediate between the two cultures and declare which had the most effective approach to education. These issues are discussed in more detail by Mehan and Wood (1975, pp. 14-31) who have carried out a broad examination of how cultures sustain their own realities and resist objections from other cultures.

Is there is any virtue in reconciling the positions taken by Habermas and Gadamer? Both views are needed in framing equally valid alternative perspectives. Ricoeur offers some sort of middle course which accommodates both positions in his formulation of the "hermeneutic circle". According to Ricoeur (1981) the individual switches between seeing the world as something of which he is part and seeing it as something he can objectify and operate on. To put it very crudely, Gadamer stresses the first, Habermas stresses the second.

A short detour: Ethnomthodology

Before turning to an examination of how hermeneutic approaches assist us in addressing educational issues and, in particular, the task of developing mathematical learning, I should make some mention of the field of ethnomethodology since there are many parallels to be drawn here. Derived from the hermeneutic tradition it provides an approach to practical engagement. Early work in this area (for example, Garfinkel, 1967, Mehan et al. 1975, Sacks, 1992, the related concerns of symbolic interactionism, Blumer, 1969, and of frame analysis, Goffmann, 1975) began to appear in the United States around the same time that Gadamer and Habermas commenced their long running dialogue on the other side of the Atlantic. Within mathematics education research ethnomethodology, symbolic interactionism and frame analysis have proved particularly influential among German researchers in mathematics education pursuing an interactionist perspective (Bauersfeld, Voigt and Krummheuer).

Ethnomethodologists (for example, Mehan et al., 1975, pp. 192-215) acknowledged that they shared a common theoretical perspective with the hermeneutic-dialectic tradition. They however, preferred to assert what they saw as an important difference, namely, they claimed that writers in hermeneutics (they cite Habermas) believed they could "reflect" their way out of practical difficulties. Influenced by Schütz, ethnomethodologists argued for a more practical engagement which acknowledged the shifting perspective of the investigator in the field. Sacks (1992, p. xxxi), for example, argues that contributions to science are composed of two parts; the account of the findings and an account of the scientist's actions by which the findings were obtained. He argues that both of these parts are "science" and not just the findings. For ethnomethodologists the obtaining of data is a process requiring many (revisable) ontological presuppositions and entailing continuous reflectivity by the investigator along the way. In this view the objective reality of social facts is an on-going accomplishment of the concerted activities of daily life; interpretation is constitution (Garfinkel, 1967, p. vii, Mehan et al., 1975, pp. 196-198).

Blumer's (1969, p. 2) social interactionism, drawing primarily on the work of Mead, echoes phenomenology in seeing human beings as acting towards things on the basis of meaning the things have for them, but also that this meaning arises out of social interaction with others, and that these meanings are modified through an interpretive process used by the person in dealing with the things he encounters.

Meanwhile, Goffman, reacting to themes initiated by William James and Schütz, is motivated by the question: *Under what circumstances do we think things are real?* His "frame analysis" examines the way in which our definitions of situations are built up in accordance with principles of organisation which govern social events and our subjective involvement in them (Goffman, 1975, pp. 10-11).

Later work by writers in hermeneutics (for example, Habermas, 1984, 1987) builds a more overtly practical dimension into their analysis and has developed a more sophisticated view of how understanding is conditioned through social understandings. Meanwhile, various writers in the field of education (for example, Carr et al, 1976; Elliott, 1987, 1993) have examined the the practical potential of work by Gadamer and Habermas and built their own models of practitioner oriented educational research around them. For example, the Action Research cycle developed through the work of Stenhouse (1975), Schön (1983), Elliott (1991) and many others bears a close resemblance to the triadic "elicitation-reaction-evaluation" pattern present in ethnomethodology. Similarly, Freire's (1972) notion of *praxis,* influenced by Hegel and Marx, allows for a time dependent generative understanding of the world built through reflection. Consequently I feel that at least some of the objections of the early ethnomethodologists have been met by the writers they criticise and by the followers of these writers pursuing practical concerns.

Habermas himself (1984, pp. 124-130) has evaluated ethnomethodology through the perspective of Garfinkel. He claims that it sits uneasily between "the processal and merely particular character of the everyday practice interpretively generated by participants" and "the methodological consequences from the fact that the social scientist has in principle the status of a participant". Having, in their analysis, rejected the notion of a "disinterested observer" ethnomethodologists still face a task of reflectively grasping the implications of the investigator's participation. Habermas sees them as only tackling part of the picture in his own grand enterprise of reconciling what he calls "systemic" (overview) and "lifeworld (insider) perspectives. With respect to the Habermas-Gadamer debate discussed in the last section I suggest ethnomethodologists pinpoint the tension the two Germans discuss, which I see as concerning the way in which experience, which always flows through time, is captured in relatively fixed statements. Similar problems are encountered by all whether they be traditional sociologists, hermeneutists, ethnomethodologists or "just plain folks" (Lave, 1988).

Nevertheless, the ethnomethodologists' concerns with how experience flavours outcomes are very poignant in this present study, even if one might suggest their concerns are largely subsumed in the new work of writers in their source traditions. Their pragmatic attitude will underlie attempts in this book to capture the insider perspective of the "process" and "content" of learning and teaching mathematics. Their insights will prove particularly relevant and useful in chapter 5 where I discuss Schutz's social phenomenology. In many important respects they have continued his work and transformed it from processes in the mind to social affairs on the public plane (Mehan et al., 1975, pp. 196-198). Also, in chapter 7 I shall be offering an approach to practitioner research which explicitly discusses reflection in action (cf. Schön, 1983). Similarly, in this present analysis, there will be many parallels with symbolic interactionism in discussing the ways in which mathematical ideas form for the individual through engagement in shared activity taking place through time.

HERMENEUTICS AND EDUCATION

In a field not noted for its simplicity Gallagher (1992 b) provides a lucid account of how hermeneutics offers a powerful way of considering classroom experience. He suggests that writers in hermeneutics have generally been too concerned with text interpretation in a narrow sense and that this has blinded them from the general applicability of the approach within the field of education. He identifies classroom and play experience as being similarly valid objects for hermeneutic analysis. A central theme in hermeneutical thought is how the perspective and motives of a reader affect the meaning of a text. Whilst the writer may have definite intentions, can we see meaning as being coterminous with these? Hermeneutists can be arranged in terms of their preparedness, as readers, to make the journey towards putting themselves in the shoes of the author. They can be seen along a continuum, placed according to how they see the relative importance of the intention of the writer and the significance to the reader in asserting the meaning of a text. Gallagher draws a similar line in the field of education, locating instead the relative weights placed on teacher intention and significance to the learner. For example, if a teacher is presenting some mathematics, are we more concerned with the teacher communicating some specific ideas or with the students building their own individual picture? Although rooted in a tradition which emphasised the intended meaning of the author, modern hermeneutics has moved away from this towards placing much more importance on the significance to the reader.

Conservative views of hermeneutics align themselves with educational discourses that see learning as grappling with tradition. Here the aim of education is characterised by tasks involving a "re-cognition and re-construction of a meaning (towards preparing) the individual for common participation in the state, the church, free society, and academia" (Scheleiermacher, quoted by Gallagher, 1992 b, p. 213). The student's responsibility for constructing meaning here goes little further than offering evidence from a literary text to support an insight about the author's intentions.

Gadamer's moderate hermeneutics, however, does not see tradition as fixed in this way, but rather sees it as being transformed through the educative process. The components of tradition are not seen as fully constituted objects to behold but rather tradition is something of which one is part. Viewed in this way, learning always has a self-reflexive dimension as a consequence of the focus of the learning being on the learner's relation to the "object" of study. To learn language, for example, I need language, something I am already immersed in. Tradition and language are fundamental constraints to any hermeneutic process since the learner is steeped in these and this prevents his action from being seen as being in any way independent. Yet at the same time the learner is responsible for constructing this very tradition which constrains him. Any creative linguistic offerings a student might make are always already partly constituted by virtue of being in an inherited language. Gadamer firmly asserts the centrality of the individual human in the creation of meaning whilst, on the one hand seeing the world as something of which he is part or, on the other, seeing it as comprising elements upon which he can operate. Here learning can be seen as comprising self learning, a learning coming about through experiencing oneself operating on and in the world. So viewed educational practices are principally concerned with enabling the student to construct meaning. The emphasis is not on the student recreating the teacher's intention but instead is on the student's production of meaning in respect of their given task. Interpretation is seen as producing something new.

Critical hermeneutics follows Gadamer in offering an alternative to more conservative views which stress the teacher's intention. It aims at unconstrained communication and associated educational discourses seek to avoid reproducing in the student's mind the structures of society and the ideological distortions that go with them. According to such writers education is seen largely as a transformative process, principally concerned with the "emancipation" of the student from the ideological structures which bind his action. Such notions of emancipatory education have also been discussed by, for example, Freire (1972), Carr and Kemmis (1986) and Lerman (1992). See also discussion concerning the ideological underpinning of the sociology of scientific knowledge (for example, Barnes, 1977; Lynch, 1994). Within the mathematics education research literature Skovsmose (1994) has considered the notion of "critical mathematics education" where he questions whether mathematics makes us see reality in a distorted way (op cit. p. 4). Inspired by Adorno's *"Education after Auschwitz"* he suggests that "critique" and "education" need to be separated to ensure that education is more than "delivery of information'' or "socialising young people into an existing culture" (op cit. p. 11). He is thus motivated by finding ways of enabling students to understand how mathematics is functioning in the social world with view to adopting a critical attitude and considers strategies for situating mathematics in real life situations. In this way he sees mathematical learning not so much as learning about the world as it is but rather as an intrinsic part of the process of generating the new world. Towards achieving this he advocates an increased emphasis on thematic project work within mathematical learning to enhance student awareness of how problems may be contextualised.

Critical hermeneutics sees its scope extending beyond the universal linguistic dimension which characterises Gadamer's version of interpretation and addresses extra-linguistic factors such as economic status and social class which it sees as distorting interpretations. Within mathematical activity, for example, certain aspects may be valued more highly because of their functionality in particular social practices. Or, as another example, the style of school mathematics demanded of all students might be seen as being overly influenced by university mathematicians focusing on the needs of potential mathematics undergraduates rather than on the needs of potential office workers.

TOWARDS AN HERMENEUTICAL UNDERSTANDING OF MATHEMATICS

Knowing occurs in an environment full of historical leftovers. Similarly, mathematical thinking revolves around a culturally defined set of ideas. Indeed the very language we use is rife with conventional ways of classifying things. Within such a view, mathematical truths, or rather the collection of mathematics' key ideas, that are mathematics to most school students, do not preexist their identification by humans. They are cultural artifacts which have begun to appear "natural" through having been around for so long. Human linguistic constructs have been responsible for forming mathematical phenomena rather than vice versa. Mathematical meaning is produced in discourse and we are always trapped between producing or reproducing meaning.

Mathematics, I have suggested, can be seen as a subject of hermeneutic understanding if the emphasis is placed on interpreting mathematical activity, which itself might embrace the generation of mathematical statements. However, the making of these statements might cause a change in our perception of the context in which we see them arising. Such circularity emphasises the individual relation of the learner to mathematics where the learner can only see mathematical phenomena from an individual perspective. Thus it might be suggested (echoing the concerns of the ethnomethodologists) that the individual's view of mathematical "content", as represented in the statements he makes, necessarily retains a residue from the "process" through which it has been approached. So notions of hermeneutic understanding as applied to mathematics require a shift in emphasis from the learner focussing on mathematics as an externally created body of knowledge to be learnt, to this learner engaging in mathematical activity taking place over time. Such a shift locates the learner within any account of learning that he offers, thus softening any notion of a human subject confronting an independent object. In this way positivistic descriptions that draw hard distinctions between process and content (or between individual experience and overview) of learning mathematics are avoided since there is no end point as such but rather successive gatherings-together of the process so far, seen from the learner's perspective. The introduction of different interpretations gives rise to the possibility of a productive tension between mathematical activity and accounts of it.

The exact expressions conventionally associated with mathematics only appear in activity, within the context of many other sorts of expression. Such statements are always offered as part of a process of "looking back" concerned with pinning down key points of the event. The reflective dimension inherent in this results in the active generation of mathematical expressions through time being part of the reality described. Similarly, the intention to learn is always associated with some presupposition about that to be learnt and learning is in a sense revisiting that already presupposed. This continual projecting forwards and backwards affirms an essential time dimension to mathematical understanding that can never be brought to a close by an arrival at a "concept", since the very framing of that concept modifies the space being described. The way in which an expression is seen and used is always in a state of flux, being modified as the life experience of the individual affects the contexts in which it is seen as being appropriate. Expressions offered by an individual are necessarily approximations to that which he means, speaking from the perspective of his individual life context. It is this very tension between statements and the meaning assigned to them that locates the hermeneutic circle. This moves away from notions of understanding developing in the mind, as might be offered by disciples of Piaget but, like Walkerdine (1988), focuses more on understanding arising in the social use of linguistic (or mathematical) forms in signifying phenomena. The issue being not so much arriving at concepts in a world that is knowable but rather converging to a cultural usage of linguistic expressions.

As an example, I observed a lesson where eight year olds were working on the program "Reflect" (SMILE) as part of some work on "symmetry". This program allows the students to generate symmetrical shapes. The students were quite able to offer statements on the subject:

"If it goes up there, the other side goes up the other way."
"It's the same both sides, like it's cut in half."
"That line up the middle is like the line of symmetry."
"It's straight in that it divides."

Here it seems the students are not so much arriving at the "concept" of "symmetry" but rather are offering a succession of statements that might be seen as being under the umbrella of this label. Students of this age seemed unable to offer anything approaching a formal definition. By seeing the commonness in the collection of statements the student might move towards more abstract notions of "symmetry" and recognise the appropriateness of the term in other situations. But whatever this commonness might be it is always subject to modification as the set of experiences seen to be embracing it increases. A formal statement thus can only ever be seen as a report on the current view. However "fact-like'' a statement might appear it is always subject to humans deciding where and when to use it. Some might assume their own understanding of such a term is shared with others but this might simply mean that they are sufficiently close in their usage of it for them to be able to say they agree on its meaning. Any permanence supposed here is perhaps an illusion. Surely, it is no more than a way of describing that allows the individual to cope for the time being, until the linguistic categories employed become inadequate in describing the situation within which she perceives herself to be acting. I suggest this imprecision is inevitable and that it should be celebrated. The task is one of understanding how different interpretations relate to each other rather than one of pursuing a quest for locating the best possible representation.

Mason (1989 a) proposes a model which connects with this approach. This comprises a helix where the experience of a mathematical situation is seen as passing repeatedly through the "getting a sense of", the "manipulating of" and the "articulating of" the problem. This has much in common with the notion of the hermeneutic circle which might be used here to describe the tension between interpreting a problem and making statements in respect of it, which in turn influence subsequent interpretations. Mason himself suggests that

> the process of abstracting in mathematics lies in the momentary movement from articulating to manipulating. Articulation of a seeing of generality, first in words or pictures, and then increasingly tight and economically succinct expressions, using symbols and perhaps diagrams, is a pinnacle of achievement, often achieved only after a great struggle. It becomes a mere foothill as it becomes a staging post for further work with the expression as a manipulable object (Mason, 1989 a, p. 3).

I take this to mean that the student, in gaining understanding, moves between emphases; for example, from following through a chain of thought to placing it into some context, or from talking about some situation to declaring an algebraic pattern, or from proposing a formula to checking it out with an example. In working on a problem one may become engrossed in the procedures and restrictions but after a while see a pattern; moving from work with particular cases to a recognition of the general. Thus the task of understanding might be seen as a mixture grasping relations internal to the mathematics and of seeing the problems in some context.

Any act of construction by a student in a mathematics lesson can be seen as part of an event that is at the same time modifying the environment for this action. For example, the production of a mathematical expression by a student would be complemented by the structure, part of which may have been constructed by the teacher in response. The event thus comprises a shared action seen from different perspectives (cf. Maturana, 1978). For some outside observer looking on, a "shift of attention" (Mason and

Davis, 1990) may be required in switching from seeing the joint action as one of the student constructing, to one of the teacher asserting some framework within which this arises - from constitutiveness to constitutedness.

As an example, I shall describe a seven year old girl working with Dienes blocks (see also, Brown, in press b). Five students and a teacher were seated around a table covered with a wide variety of base 10 material. After a period of "free-play" the girl was directed towards counting the number of unit cubes in a 10x10x10 cube. She declared that she knew that on one face there were 100 unit cubes. She then proceeded to count the number of faces and concluded that since there were six faces there must be 600 cubes altogether. The teacher's response was to pick up a 10x10 "flat" and ask how many would be needed to make the big cube. The girl started piling one flat upon another and counted as she did this. After saying "five hundred" she grinned broadly, recognising that the result was not going to be 600. She went on to conclude that there were 1000 cubes. The girl was subsequently able to give two accounts of how to calculate the total yet seemed unable to reconcile them. She opted for the latter result primarily, it seemed, because her teacher had nudged her there.

Whilst engaging in this activity the girl was making "constructions" of various sorts. Her work during the free-play interlude seemed to have her engaging in a variety of voluntary tasks defined by herself. The environmental resistances included; the obligation to stay at the table, the suggestion that the materials be used in a certain way, the other students' use of the materials, the framing of the activity consequential to the physical properties of the materials, the verbal guidance of the teacher. These resistances shaped themselves around the voluntary actions of the girl. The successive overt actions of the student and teacher suggested a space for the other to work in. This space, in being interpreted in particular ways, gave rise to intentional actions made in respect of the individually perceived phenomenological field. The interlude can thus be described as a jointly created sequence of actions where both teacher and student construct but in a way responsive to the jointly created environment within which this happens.

The more specific guidance of the teacher, towards counting the unit cubes in the "block, served to reorientate the girl's gaze redefining for her the phenomenological structure of her perceptual field. My reading of this is that for the girl it seemed the solution was to be found in the "block itself as, in the first instance, she made no reference to the other pieces available. She seemed to count those cubes that were visible to her in taking such a perspective and consequently decided that there were 600. However, once the teacher became aware of this and introduced the possibility of counting by using some of the other pieces available her gaze was again redirected so that the "flats" were used in a particular way towards constructing the "block". Any other potential use of the "flats", however, was being temporarily ignored.

The selection of, and the meaning assigned by the girl to, the various pieces was consequential to the framing of them by the teacher. The student was making constructions but component "objects" within these constructions were suggested in the teacher's actions which stressed and ignored certain qualities. The pieces were not being seen as "units", "flats" and "blocks" in themselves but as phenomena which had certain meanings in relation to each other, in the eyes of the girl. I suggest that for a brief spell the girl was attending to what she saw as two equally convincing accounts. As such she was torn between pursuing one or other project; namely to show that there were 600 cubes or to show that there were 1000 cubes. In a sense the meanings she brought to the different components, "block", "flat", etc, were project-dependent.

Can mathematics be seen as existing outside of the language that describes it? We shall pursue this question as the book proceeds. I see it being related to the more general concern of how the world can be seen outside the language that describes it. Mathematical phenomena are consequential to the way in which the world is sub-divided into categories by individuals. It is this very categorisation which mediates and thus organises our way of seeing.

The symbols and classifications of mathematics are historically determined. They are arbitrary in the sense that the symbols and classifications of any language are arbitrary. As such it can be viewed phenomenologically in that these symbols and classifications have particular meanings for any individual derived from that individual's experience of their usage. The field of mathematics which comes into being in its classification in language can only be perceived retroactively and its very existence presupposes a language. For someone learning mathematics there is a similarity with learning a language in that there is a need to grapple with an inherited mode of symbolisation and classification, arbitrarily associated with some preexisting world.

Learning mathematics can be seen as involving a play between the various accounts offered of mathematical situations. The offering of alternatives might be seen as a way of initiating productive tensions for forcing awareness towards reevaluating supposed natural ways of seeing mathematics. The linguistic overlay given to a situation can be seen as a way of introducing an interplay between the describing and that described. By following an hermeneutic view of understanding, the production of meaning in this activity can be seen as deriving from a dialogue in a continuous process of introducing linguistic and symbolic form into the socially active space in much the same way.

Where then lie conventional notions of mathematical understanding? I suggest a more humble notion of monitoring mathematical understanding may be required, namely; that it can be checked through the ability of the learner to tell convincing stories generated by himself or borrowed from the teacher. Further, this understanding is only demonstrated if the learner can make use of certain aspects of the conventional, inherited system of exchange. By accepting a hermeneutic view of mathematical understanding we emphasise the social qualities of mathematical learning by stressing attempts to hold it in social notation. Consequently, the distinction between mathematics and other disciplines is softened. Mathematics becomes something held in the expressions of participants in mathematical activity, who are asserting their view of, and their relation to, some supposed mathematics. The reality of any supposed transcendental mathematics, at a fundamental level, presupposes people acting as if it is there.

CHAPTER 2

THE PRODUCTION OF MATHEMATICAL MEANING: A POST-STRUCTURALIST PERSPECTIVE

Where therefore is truth? A nobile army of metaphors, metonymies, anthromorphisms... truths are illusions of which one has forgotten that they are illusions (Nietzsche, quoted by Spivak in her translator's introduction to Derrida, 1976, p. xxii)

As we have seen, in recent work in the study of language there have been moves away from seeing words within a language as being a labelling device to denote preexisting phenomena. Rather, the very framing consequential to introducing language is the process through which phenomena are brought into existence; the mechanism through which meaning is produced. I have shown, within an hermeneutic analysis, how language comes to the fore in mediating mathematical experience. Indeed, it becomes difficult to locate mathematics outside of a linguistic frame. Mathematics is accessed through accounts offered about it. Post-structuralism, to be discussed now, stretches this notion a little further. Here, it will be suggested, mathematics has no existence outside the textuality (the system of differentiality) that creates it. Any attempt to locate the underlying truth results in us encountering what Lacan calls the "lack"; the emptiness which emerges after the final layers of description ("stories") are peeled away (Brown, Hardy and Wilson, 1993). Such a view results in educational discourses which emphasise the individual constructions of the student, largely unconstrained by concerns for the teacher's intention, or by any notion of universal "truth". Rather, we enter what Foucault calls "regimes of truth", or discourses governed by their own internal structures, consequential to society's view of itself.

In this chapter, following an introduction to post-structuralism and its implications for education, I will set out the structuralist roots of post-structuralism through outlining Saussure's linguistic model. In particular, I introduce Saussure's notion of the "sign"; an association between a mental concept and a mental image of a word or symbol. I consider the implications of Lacan's assertion that the linguistic signifier is more stable than the concept with which it is associated. An example is given of a particular mathematical expression being used by students where the meaning of the expression evolves as the students' work progresses. I also outline the importance of Saussure's distinction between language as a system and the performance of language. After drawing an analogy with mathematics I conclude with a discussion of how learning mathematics is always associated with performance in mathematical activity, where individuals reveal how they see things and so introduce their own perspectives into the ideas they tackle. Finally, I suggest that this results in a self-reflexive dimension to learning mathematics which further compromises any presumed anchorage in universal truth. Most of the more abstract general theoretical discussion is confined to the first short section.

POST-STRUCTURALISM: A RADICAL FORM OF HERMENEUTICS

Although Derrida and Foucault might themselves question the description, they are generally described as post-structuralist writers. Certain writers (for example, Caputo, 1987, pp. 117-150; Gallagher, 1992 b, pp. 278-284) may well be adding insult to injury by arguing that post-structuralism can be seen as a radical form of hermeneutics. Many others would argue that hermeneutics and post-structuralism are incommensurable traditions. Eagleton (1983) provides an introduction, Spivak (1996, pp. 74-104) a more sophisticated analysis of this question. Nevertheless, whilst I would agree that we are examining here another case of "irreducible perspectives", post-structuralism does, I would argue, display many important similarities with the hermeneutics of Gadamer and Ricoeur as described in the last chapter, which makes it difficult to draw strict boundaries. The universality of language and linguisticality of interpretation are central in both camps. The principal distinction of interest here is Derrida's fairly extreme stand in not seeing interpretation as being governed by the motivation of locating the ultimate "truth". To him, writers like Ricoeur and Gadamer, with their theological leanings, are conservative (or at least metaphysical) because of their reluctance to abandon their supposed points of anchorage. Whilst Ricoeur, for example, acknowledges that it is possible to make many interpretations, he asserts that some will be better than others - some are closer to the "truth". Ricoeur and Gadamer attempt to hold on to some notion of underlying reality, albeit obscured by the media through which we attempt to access it. In post-structuralism, however, this quest is abandoned, the meaning is in the text itself. Nevertheless, Foucault's early work (for example, *"Madness and*

Civilisation") pursued a largely hermeneutic quest, but he later rejected the "promise" of such enquiry. He came to feel that no matter how deeply one penetrates below the surface of the text one could not encounter reality outside the discourse itself. For example, in his book *"The Birth of the Clinic"* he no longer sought "madness itself behind the discourse about madness" (Habermas, 1985, p. 241).

Other writers, such as Barthes (e.g. 1976, 1977, pp. 79-124), Lacan and Levi-Strauss commenced with a more overtly structuralist enterprise, where they attempted to develop the linguistic model offered by Saussure into a "science of signs which goes beneath the surface events of language (parole) to investigate a variety of concealed signifying systems (langue)" (Urmson and Ree, 1989, p. 311). In Levi-Strauss's work on structural anthropology, for example, there was some belief in a structure (say of a particular society) that could be observed from the outside with some fixed relation between its outward manifestations and its inner workings. The task here was to translate the disorder of empirical experience into the order of systematic structures. The post-structuralists, meanwhile:

> rejected the binary oppositions between surface and depth, event and structure, inner and outer, conscious and unconscious ..(and) renounced the structuralist quest for a science of signs, celebrating instead the irreducible excesses of language as a multiple play of meaning (Ibid.).

Given the focus of this book I should perhaps add internalist and externalist, individual and social to this list of rejected binary oppositions. Derrida (1978, p. 287) pinpoints a significant shift in the work of Levi-Strauss which moved structuralist concerns in to more complex territory:

> The study of myths raises a methodological problem, in that it cannot be carried out according to the Cartesian principle of breaking down the difficulty into as many parts as may be necessary for finding the solution. There is no real end to the methodological analysis, no hidden unity to be grasped once the breaking down process has been completed. Themes can be split up *ad finitum.* Just when you think you have disentangled and separated them, you realise that they were knitting together again in response to the operation of unexpected affinities. Consequently the unity of the myth is a never more than tendential and projective and cannot reflect a state or a particular moment of the myth. It is a phenomenon of the imagination, resulting from the

> attempt at interpretation; and its function is to endow the myth with synthetic form and to prevent its disintegration into a confusion of opposites. ... it follows that this book on myths is itself a kind of myth. (Levi-Strauss, 1970, pp. 5-6)

Derrida sees this observation by Levi-Strauss as a challenge to any notion of a "centre" orienting structural analysis. Such moves, which characterised the shifts, in the late sixties, in to what became known as post-structuralism, resulted in the whole idea of a systematic structure being undermined since there could be no agreed relationship, as any such view presupposed a particular individual perspective. Indeed, any supposed meaning itself becomes forever elusive. More importantly however, each individual can only describe the world of which they are part and so there is necessarily, a reflexive dimension to any such description. The observer is not so much describing a structure but rather their view of it, and by implication they are describing a bit of themselves. Foucault's (1972) book *"The Archaeology of Knowledge"* emphasises that forms of knowing occur in an environment full of historical leftovers. Indeed the very language we use is rife with conventional ways of classifying things, concealing existing power relations.

POST-STRUCTURALISM, EDUCATION AND MATHEMATICS

Post-structuralism permits a breaking free from tradition in a multiple "play" in language. Meaning is to be found in the "textuality"; in the play of different accounts offered. For example, Barthes' notion of teaching is akin to a conversation where the "correcting and improving movement of speech is the wavering of a flow of words" (Barthes, 1977, p. 191). There is no delivery and no receipt but rather the learning space is a discussion where "no one, neither teacher nor students, would ever be in his final place" (op cit., p. 205). There are however, definite constraints on this free play. Post-structuralist accounts of education tend to be rooted in the Marxist notion of society forming consciousness, where individuals are absorbed in social norms. For Althusser, (human) individuals are understood as "bearers" of a system of social relations which exist prior to and independently of their consciousness and activity (Urmson et al., p. 7). For an individual seen in this way, knowing, and its evolution, is closely associated with action, since the social practices which host specific actions are imbued with the society's preferred ways of doing things. Whereas critical hermeneutics sees education as risking the reproduction of the institutional power relations through its content, post-structuralism identifies a more all-embracing reproduction, namely the metaphysical framework which conditions all understanding.

> For what can be oppressive in our teaching is not, finally, the knowledge or the culture it conveys, but the discursive forms through which we propose them (Barthes, quoted by Gallagher 1992 b, p. 300).

For example, Walkerdine (1988) describes categories such as "child", "teacher", "learning", etc. as constructions situated within historically and culturally specific discourses - the very fabric of the language we use presupposes manifold assumptions about our classification of the world (see also, Evans and Tsatsaroni, 1994). The reflexivity inherent in the language we use positions us in relation to the world we see our selves in. Foucault (1979, pp. 170-194) describes the regulation implicit in the classifications in every aspect of schools from the architectural design to the administrative structures, for example, examinations categorising students according to ability, dress codes and separate facilities according to sex, as strict codes regulating behaviour. Similarly, Althusser (1971) sees schools as an essential part of the "ideological state apparatus", asserting particular forms of "hegemony", that is, power relations held in place by common assent.

Conventional mathematical ideas are all culturally derived but have become so embedded within the fabric of our culture that it is hard for us to see them as anything other than givens. These historical choices can never be eradicated and will forever condition and mediate our experiences of mathematics. Derrida (1978, p. 281) has spoken of the difficulty of analysing linguistic structures since there is always a need to use the elements of the structures themselves in dismantling these very structures. Similarly, we can never analyse mathematics without using the culturally derived components of this very mathematics. Any "new" mathematical construction is always made within an inherited language which means that it is always already partially constructed. The culture provides the building blocks and the final building is a function of these. This applies to both the "objective" components of mathematics for example, "cos", "+", "Σ" or ">"

and the culturally preferred ways of combining them, as in school maths, or "real world" maths, or university research maths, or whatever (the architectural styles) . We cannot investigate mathematics without being tied into preexisting styles of categorising. The various schools of hermeneutics might be seen as differing according to the status they would give to this mathematical inheritance and the extent to which we are locked in to the language we have created (cf. Brown, 1994 d).

Any form of mathematical instruction introduced by a teacher conceals a socially conventional way of making associations between symbolic forms and phenomena seen as mathematical. Walkerdine (1984), following Foucault's work, has argued that much that went under the banner "child-centredness" was to do with covert regulation, by asserting conventional forms of signification as "natural". This is also partly true of the "neo-child centrists" among the constructivists. Within a child-centred philosophy the child is engaged in constructing meanings and so in the development of signifying practices. Expressions are being combined with phenomena towards producing meaning. However, this constructing arises in an environment where certain conventional discourses prevail and the task becomes to introduce inherited forms in a conventional way as represented by the teacher or published scheme. In this way, the constructions are controlled but in an environment described as if the child is responsible for controlling the agenda. As we have seen when the constructivists talk of students constructing they sometimes underplay the fact that this constructing is being done in an inherited language associated with a conventional way of using it. As I indicated earlier, the ready-madeness they wish to reject is necessarily implicit in the language that the teacher and student use in communicating with each other. In addition, framing any activity for the student, the teacher is selecting the particular domain of discourse that further contextualises and thus conditions any terminology used. Conventional mathematical discourse has classified mathematical phenomena in a particular way that is arbitrary both in its choice of words and in its choice of the things to which it attaches a name (cf. Saussure). The same is true of the procedures employed in respect of these phenomena insofar as there are conventional ways of tackling certain sorts of tasks (for example, the "decomposition" method for subtraction problems).

SAUSSURIAN LINGUISTICS AND MATHEMATICS

"Saussurian linguistics" is a somewhat ambiguous term since Saussure himself did not write a book on the subject, a task he saw as far too complex. The *"Course in General Linguistics"* (Saussure, 1966), the main work credited to him, was put together posthumously by his students compiling their lecture notes at the turn of the century. Within contemporary linguistics Saussure has been superseded by many writers, His book is largely technical and is generally disregarded by modern day writers in the field. However, within literary theory and other post-structuralist writings he is referred to more than any other linguist. Saussure's work was specifically targeted at the task of linguistics but subsequent readings have given his work status as a framework for all of the human sciences. Jackson (1992) has suggested that the Saussure who consorts with Derrida and Lacan is not the empirical linguist at all but rather an idealist philosopher of language invented in Paris in mid-century long after the real Saussure passed away. This character, created out of the stories told about him, thus makes a surprisingly appropriate role model for post-structuralism. Construct or otherwise, "Saussure" and "his" book have provided a framework for many modern writers. The book is also surprisingly clear given the complex nature of the writers who cite it.

Only the key ideas will be outlined here. For a more general critique of Saussure's work, see, for example, Lacan and Wilden (1968), Ellis (1989) and Hodge and Kress (1988). His work has also been discussed in relation to mathematics education by Brown and McNamara (1993); Brown (1994 a); McNamara (1995 a b); von Glasersfeld (1995).

The sign

Saussure's usage and popularisation of the terms "signifier", "signified" and "sign" has been particularly influential. According to Saussure (1966, pp. 65-70) the "signifier" is the mental imprint of the word or sound and the "signified" is the associated concept. As such they are both mental phenomena which, as a pair, form what he calls a "sign". Saussure's sign is not to be confused with the "naive" notion of the sign which associates a physical symbol with a physical object. However, a third element is, nevertheless, implied, although not explicitly identified by Saussure in his formulation (indeed he deliberately excludes it); namely, the "referent" or object itself. Saussure's "sign" is entirely a mental phenomena. We can perhaps illustrate it more clearly by reference to an example. The word "dog" (the signifier) we associate with a certain concept (the signified) and on occasions we might encounter a "real" dog (the referent) with which we can associate this sign. Even in this simple case, however, there are two sorts of arbitrariness present (Ellis, 1989, pp. 45-46). Firstly, the signifier "dog" is quite arbitrary and does in fact change according to the language being spoken. French people use the word "chien". Secondly, the signified concept "dog", is no more than an arbitrary grouping according to certain selected characteristics somewhere on the continuum starting at "animal", passing through "mammal", "canine", "spaniel" to "Fido". In this case, the sign "dog" could be seen as being associated with a "real" referent, for example, a dog named Fido. However, not all referents are real in this sense and this is the reason they are not dealt with in Saussure's model. The signifier "learning" for example, gives rise to signifieds and referents which are rather more problematic. Mathematical phenomena present similar difficulties. Take the chain: spatial phenomenon, 2-D shape, polygon, regular polygon, square, "Trafalgar Square". The referent "Trafalgar Square" is perhaps less convincing and possibly takes us out of the realm of mathematics. But similarly, the chain: number, rational, integer, natural, single digit, 5. In what sense can "5" be a referent; i.e. be something actually there? The practice of mathematics is not always dependent on gearing into the "real" world in this way. Mathematical terms are discussed further by McNamara (1995 a b).

Saussure himself was more concerned with the differences between signs rather than with their association with the material world, which explains his avoidance of discussing referents. Signs were not seen as having meanings in themselves but instead derived meaning from their relation to other signs.

> In language there are only differences. Even more important: a difference generally implies positive terms between which the difference is set up; but in language there are only differences *without positive terms.* Whether we take the signified or the signifier, language has neither ideas nor sounds that existed before the linguistic system. (Saussure, 1966, p. 120)

The meaning of a word is thus dependent on the way in which it is used in a sentence. He saw the sign as unstable in the long term in the sense that the signifier and signified can move in relation to each other. For example, the signifier "omnibus" has become "bus" whilst the signified is in some sense unchanged, i.e. a vehicle capable of carrying a number of people. Similarly, pronunciation might change both geographically and chronologically. Conversely, the phenomena signified by the signifier "Pythagoras" has shifted through time; he used to be associated with theories of number (Tahta, 1991). Meanwhile, the relationship of the complete sign with the referential field is arbitrary until it becomes a convention through socially consistent usage. Barthes (1976, pp. 109-159) has suggested that certain signifying practices become "naturalised" into what he calls myths, particular ways of seeing things in a given community. Such myths become embedded in ideological descriptions, creating reality for those who hold that ideological position. This is also discussed further by Coward and Ellis (1977, pp. 25-44). School mathematics, for example, features a particular way of classifying mathematical activity into components and procedures that themselves become "mathematics" in the eyes of students. For example, a particular procedure for multiplying double digit numbers was standardised in the practices of many schools. Mathematics, if viewed as a language, necessarily comprises some system of signification. Mathematical phenomena, as I have suggested, do not have a real existence, there are no referents, and any meaning is derived purely through relations perceived between these phenomena.

Walkerdine (1988, p. 3) has pointed out that Piaget's reading of Saussure resulted in him using the terms "signifier" and "signified" but in a very different way to Saussure himself. She suggests that Piaget sees the relationship of signifier to signified as one of representation and quotes him as saying the semiotic function:

> consists in the ability to represent something (a signified something: object, event, conceptual scheme, etc.) by means of a signifier which is differentiated and which serves only a representative purpose. Piaget (Gruber and Voneche, 1977, p. 489)

Such a view is more akin to the older form of structuralism, and Piaget's (1971, p. 12) reading of it, with its implied fixed relations between object and meaning, independent of any individual observer. In post-structuralism, words are not mere labels, but rather the tools by which reality is constructed and held in place.

Even in Saussure's much earlier work, language was both the *process* of articulating meaning and the *product,* namely communication (Groden and Kreiswirth, 1994, pp. 651-654).

Any field of mathematical symbolisation is a consequence of the mathematical qualities of the world being perceived in some way and then being classified within the categories of a language. Pursuing an earlier example: "Square" is a socially conventional signifier associated with the concept "square" but we never actually have a real square as understood in pure mathematics. Consider the commands:

> Draw a four sided regular polygon in pencil.
> Write REPEAT 4 [FD 100 RT 90] in LOGO.

The signified "square" is evoked by each of these but the referred to "square", as defined in geometry, is not physically present and never can be. It can only be imagined. Squareness is a quality that may be seen as being within the physical world but it only comes into being retroactively as part of a human naming (signifying) process. The name "square" is merely an arbitrary label given to a repeatable idea seen as being worthy of having a name to itself. However, whilst the notion may not have a physical reality, pursuit of it can govern actions. Althusser's (1971) suggests that something is real if people say it is and act as if it is. "Square" is a human construct that shapes our way of describing the world and our acting within it. The referent here might be seen as the Lacanian "objet petit a"; that is, the object of desire that becomes the "lack" encountered after stripping away the various layers of description. Such a post-structuralist position would emphasise the play of meaning held in the various descriptions and would not see meaning produced outside of these descriptions. (The "objet petit a" is described by, for example, Lacan, 1979; Benvenuto et al. 1986, pp. 176-181; Zizek, 1989, p. 95; Bowie, 1991, pp. 165-178; Brown et al., 1993. For an introduction to Lacan's complex work see Benvenuto et al., Bowie, or Sturrock, 1979, or Lemaire, 1977.)

Fixed notation with evolving meaning

Within Saussure's model, meaning cannot be seen as being associated with individual signs within the system. It only emerges as signs are combined in “stories” generated within activity. Meaning is produced in the process of signifying. The language and symbols used shape developing understanding and provide the components within this. The ontological qualities of the mathematical phenomena located are not specified outside of this frame. The symbolic expressions themselves are formative of meaning.

Whilst Saussure saw the sign relation as holding signifier and signified together, Lacan has offered a new reading where the signifier has primacy over the signified resulting in a more fundamental instability. Lacan (1977) seems to suggest that notation as printed on a page, or held in a spoken word, or even held as notation in the mind, has more stability than that to which it refers. In perceiving some phenomena I can capture this in a description employing symbols. This symbolisation however, once introduced, and held in a material form as notation on a page, affects the way in which the phenomena is dealt with subsequently and also mediates any subsequent change in this symbolisation. Rather than speaking of conception it is more appropriate to talk of conceiving since we are referring to a time and context dependent activity. (This distinction is discussed more fully by Brookes, 1978.) Within mathematics we can imagine expressions being used as “holding devices”, in some way fixed as the student considers what is their meaning in a particular instance.

The signifier has a holding effect on the signified resulting in a stable notation being associated with a conceptualised phenomena, subject to contextual and chronological changes. As an example I shall refer to a lesson where ten year old children were investigating the interior angles of polygons (Brown, 1994 a f). Having worked through a few examples they had concluded that for three sided shapes the total was 180°, for four sided shapes, 360° and for six sided shapes, 720°. On this basis they made the hypothesis: *“For five sided shapes the total degrees is 540°”.* This expression was being used to hold a hypothesised relationship. They then proceeded to make five sided shapes out of wooden “pattern blocks”. Each time an appropriate shape was made they attempted to sum the angles, employing a variety of techniques for measuring individual angles. However, they made numerous slips in measuring angles and in counting the total, reaching many totals that were not 540. The holding effect of the hypothesis was such that they were prepared to reject other results and recheck by other methods. After an hour they had three examples of five sided

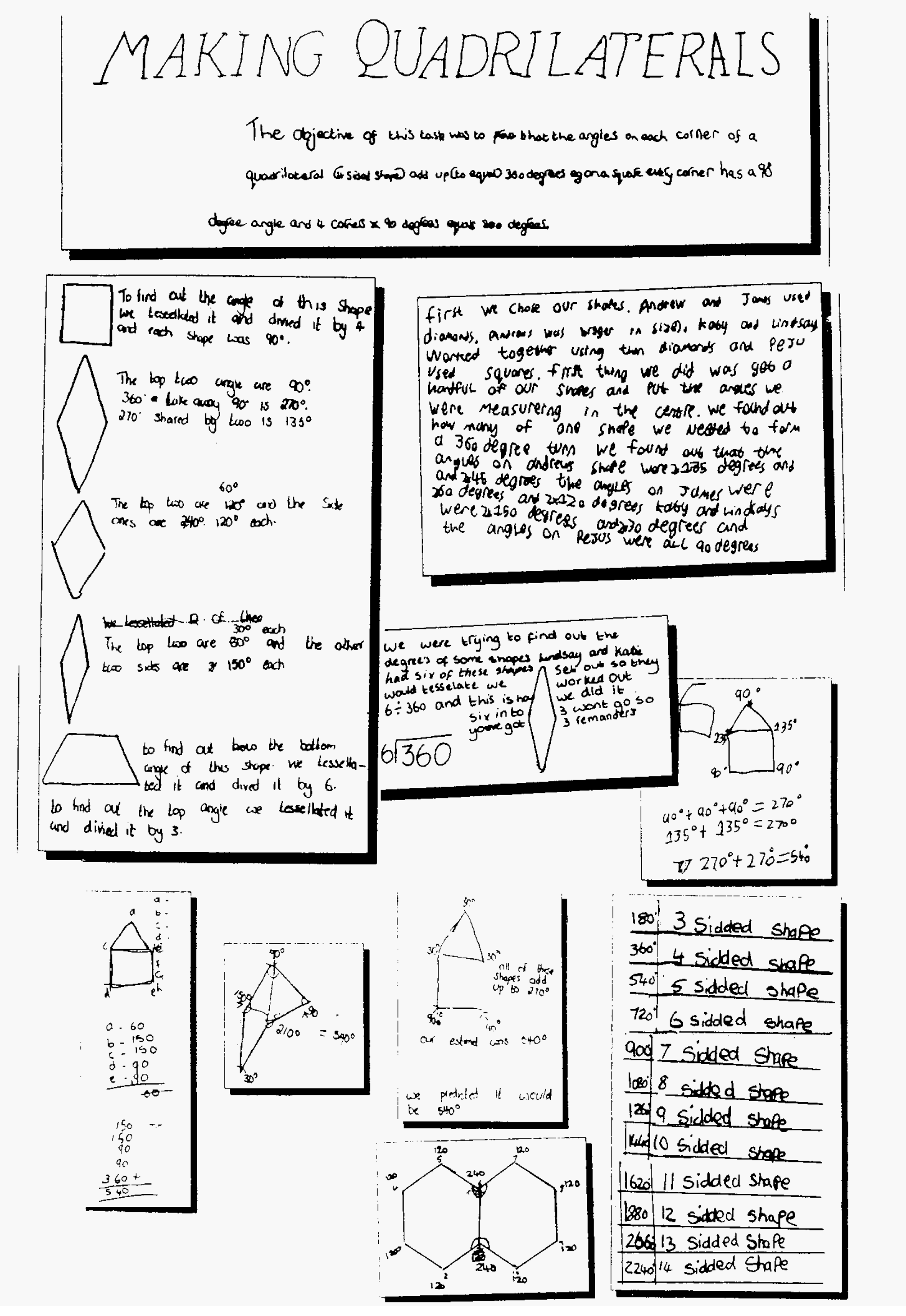

MAKING QUADRILATERALS
The objective of this task was to find that the angles on each corner of a quadrilateral (4 sided shape) add up (to equal) 360 degrees eg on a square every corner has a 90 degree angle and 4 corners x 90 degrees equals 360 degrees.
To find out the angle of this shape we tessellated it and dived it by 4 and each shape was 90°.
The top two angle are 90°. 360° take away 90° is 270°. 270° shared by two is 135°
The top two are 60° and the side ones are 120° each.
The top two are 30° each and the other two sides are 150° each
to find out how the bottom angle of this shape we tessellated it and dived it by 6.
to find out the top angle we tessellated it and divided it by 3.
first we chose our shapes. Andrew and James used diamonds, Andrews was bigger in size, katy and Lindsay worked together using thin diamonds and Peju used squares. first thing we did was get a handful of our shapes and put the angles we were measuring in the centre. we found out how many of one shape we needed to form a 360 degree turn we found out that the angles on andrews shape were 2x135 degrees and 2x45 degrees the angles on James were 2x160 degrees and 2x20 degrees katy and lindsays were 2x150 degrees and 2x30 degrees and the angles on Pejus were all 90 degrees
we were trying to find out the degree's of some shapes lindsay and katie had six of these shapes set out so they would tesselate we worked out we did it 6÷360 and this is how six into 3 wont go so 3 remanders youve got
6)360
90° 135° 135° 90° 90°
90°+90°+90° = 270°
135°+135° = 270°
270°+270°=540
a - 60
b - 150
c - 150
d - 90
e - 90
150 150 90 90
360 +
540
90° 150° 30° 2100 = 540°
all of these shapes add up to 270°
our estimate was 540°
we predicted it would be 540°
120 240 120 120 120 240 120 120
180° 3 Sidded Shape
360° 4 Sidded shape
540° 5 Sidded Shape
720° 6 Sidded Shape
900 7 Sidded Shape
1080 8 Sidded Shape
1260 9 Sidded Shape
1440 10 Sidded Shape
1620 11 Sidded Shape
1880 12 Sidded Shape
2060 13 Sidded Shape
2240 14 Sidded Shape

Figure 2

shapes where they felt the total was 540°. They saw this as adequate proof of the hypothesis and when asked the total for ten sided shapes were quickly able to give a convincing figure. This was followed by the production of a ten sided shape (two hexagons with an edge coinciding) with the angles marked and adding to the expected total.

I would suggest that the expression, whilst "materially stable", was associated with an idea whose meaning shifted during the course of this activity. The expression was initially simply the sentence that filled the gap in the sequence 180, 360, ..., 720. It then proceeded to be the story associated with the three examples of five sided shapes. In the light of this it was seen as sufficient confirmation of the pattern to justify projecting forward to more sophisticated shapes.

Langue and Parole

Saussure (1966) highlighted an important distinction between *"langue",* the system and structure of language common to all speakers, and *"parole"* (speaking) the manifestation of this in everyday speech and writing. Traditionally, in the study of language, primacy was given to langue. "Structuralist" writers (for example, Barthes), however, were seen to become more "post-structuralist" as they shifted their emphasis from langue to parole. Lyotard (1979, p. 37), a prominent post-modernist writer, went further when he asserted the central theme of post-modernism; that "the grand narrative has lost its credibility", individuals can only express themselves within a particular "story", in language conditioned by their particular circumstances. That is, there is no universally agreed langue, but rather we live in a world governed by acts of parole. No two people will perceive langue in the same way; each will make use of different regions, see different scopes, use "it" in different ways. In this scenario, we cannot appeal to some ultimate authority about what any statement might mean. Any utterance by an individual derives its meaning both through its relationship to the langue and through its relationship to other things said by that individual. In particular, it relates to their personal history. Whilst personal acts of parole draw on langue, they also display a certain amount of reflexivity, revealing something of the person speaking and the society from which she comes. As we have seen, spoken words are always part of an action. Such communication transcends a purely literal message, a perlocutionary dimension is introduced. Tone of voice or the context in which the speaking is taking place can modify any literal meaning.

I suggest that a similar distinction can be drawn between the system and structure of "mathematics", as it might be imagined holistically, and "mathematical activity", that is, the way it is externalised in everyday human usage in school, jobs of work etc. In mathematics education we are principally concerned with the performance of mathematics by people, whether or not we explicitly seek to connect it with other cultural practices. To suppose some externally defined expert view of mathematics results in an immediate distancing from the activity of novices. Novices are less able to discern the limits seen by the expert and so are governed by other frames. They cannot see the limits of the expert's mathematical idea until they are no longer a novice. The system and structure of mathematics can never be seen in its totality and it is always accessed through specific action. Nevertheless, people, individually or collectively, may act as if it exists as some sort of body of knowledge. Mathematics as practised in the social world of work necessarily has an interpretive dimension which cannot be removed. The popular image of mathematics as a meta-language, as an overarching langue, can result in the cultural packaging, within which it is transmitted, being disregarded. Mathematics in schools is highly dependent on how it is presented. As teachers and learners we are concerned with initiation into the culture of mathematics and, in particular, with the culture of school mathematics. The media through which we view mathematics condition what we see and the stories we tell about it. Such media include the published teaching scheme being used, the teaching philosophy of the teacher, the technical facilities available, the curriculum being followed, all of which change through time. A post-modernist account of mathematics might be seen as moving us away from seeing a universal "correct" view of mathematics anchored around the "facts" of mathematics. As we shall see later, mathematics can be viewed as being oriented around personal awarenesses engendered through specific practices. Certain styles of school mathematics activity, for example, investigation work often results in students exercising their interpretive skills in an overt way (for example, ATM, 1977). Other styles, perhaps more focused on responses to specific questions, still require the student to articulate some sense of their understanding in respect of the questions. In emphasising the interpretive aspects of mathematics the student needs to bring mathematical statements to their experience and there is a need to decide on their appropriateness in particular situations. Seen in this way mathematical learning is less about finding correct answers to specific questions arising from a supposed universal view of mathematics than about an on-going fitting of language to mathematical activity. This idea will be developed through an example in the next chapter.

In any act of parole the observer is always describing something of which she is part, since the mathematics we describe depends on our perspective. It is this self-reflexivity to which I now turn.

SELF-REFLEXIVITY IN MATHEMATICAL ENGAGEMENT

In using expressions such as *"For five sided shapes the total degrees is 540^o''* the students are selecting the expressions which for them capture the situation they are observing. They are mapping out their enquiry in a very personal way. Their quest may be to fit an ever more exact form but the very attempt materially affects the context of which the phenomena is part, thus affecting the position and perspective of the subject doing this. In this way other expressions may emerge as holding the essence of their work. The introduction of any expression is dependent in the first place on the position and perspective of the subject making an association. The subject's attempts to fit symbolic forms to particular phenomena are also deeply rooted in her perspective of how the phenomena relates to her. The child learns to position herself in the world before she has learnt to speak. However, there remains a residue of this pre-linguistic state as the child captures ever more of her world in language (Lacan, 1977). In this way the subject perceives part of herself in combining any symbolic form with phenomena which are necessarily seen from an individual perspective. The act of introducing any form has a reflexive dimension where, simultaneously, the form signifies both the perspective of the subject and the meaning of the phenomena to her. In this way the quest to find the most appropriate form is in part concerned with self description. However, the final description of this subject-in-context and her perspective is always in the future since any attempt at closure affects that described. Zizek (1989, pp. 201-232) argues that this thwarted attempt at reaching some concluding position always goes hand in hand with a failure on the part of the subject to constitute herself (in language). The story, told by the students about their work on angles is never finished, since as their work proceeds the meaning of the expressions they use forever shift as the perceived context for them changes.

In emphasising that mathematics only ever comes to life in human exchanges we highlight this self-reflexive dimension. For Derrida, meaning is always in the future, always "deferred", there is never a closure to a story because this story can always be extended (for example, 1992). Any story seen as complete can then be contextualised alongside other stories. This is not dissimilar to Mason's (1989 a) work on mathematical learning introduced in the last chapter where a quest to find the key result becomes transformed when, after the result is found, it is recognised as merely yet another result alongside others. We can always explore further and revise the meanings we have created. The meaning we derive is always contingent. Our understanding is the sense we make of what we have done so far. We are unable to perceive mathematics except through our acts of engagement in it. Whilst there might be some over arching system of mathematics (analogous to langue), understood collectively by the community of mathematicians, we can never survey this holistically in a neutral way. Our performance in mathematics can only ever be judged through our acts of parole. Meaning then cannot be seen as being associated with individual signs within the system. Meaning is only created as signs are combined in stories that arise within the activities performed. These stories are unique since they are necessarily from an individual perspective and are, as a consequence, time dependent.

As an example, at a conference of the Association of Teachers of Mathematics (ATM) in England, I was presented with an intriguing task by Deliah Pawluch. I had to imagine I was a spider positioned somewhere in the room. Other people then had to request information about the shape of things I saw. For example, from perspective I had a table top looked like a trapezium. Gathering together such information my questioners needed to decide where I was. The whole task was about positioning myself in relation to the world around me and it was through this process that I became familiar with aspects of myself through describing my relation with the world of which I was part.

The above incident took place in a "workshop" comprising a room full of practical mathematics equipment. All tasks entered into in this room could be understood in this way; not so much a case of exploring the properties of the things available but rather I could see them as opportunities to find out about myself and how I saw things. Clearly, this is true not only of practical mathematics but also of any representation of any mathematical phenomena. An act of mathematics can be seen as an act of construction where I simultaneously construct in language mathematical notions and the world around me. Meaning is produced as I get to know my relationships to these things. This process is the source of the post-structuralist notion of the human subject being constructed through being positioned in discourse. In this instance, the individual subject constructs himself in language through describing his relations to the world around. Such a view asserts an essential instability in both subject and situation so that there is a need to analyse both, which can be seen as part of each other, as processes. The subject, and the structure in which he acts, is asserted, in the ways that they are represented in language, through time. This is always subject to change as more things can always be said. These representations are not mere labellings but are instrumental in the construction of subject and structure. Language here does not just describe the performance but it is also part of the performance. It is the very process of signifying in language that brings into being the notions described and these notions then serve in shaping subsequent actions.

I am reminded here of a seminar I attended where Caleb Gattegno discussed algebra. He spoke of a young child pointing to a fly on the ceiling. This arm movement meant the fly was by the window, this one meant it was by the light. The child's bodily movements were substitutions for the fly walking between two positions. Likewise, I can imagine myself in the ATM workshop making all sorts of other bodily substitutions as I get to know myself in relation to the objects on offer. What can these Dienes blocks tell me about myself, and this tray of polyhedral models? Mathematical education folklore is that these bodily substitutions gradually are replaced by mental substitutions or by movements of the fingers in getting a pen to produce symbols on paper. As a consequence of maturation, I become initiated into an inherited language of symbols with culturally derived rules for combining these symbols and inherited social practices within which these linguistic practices arise.

I cannot disentangle things independently of my history. My intellectual engagements depend on where I have been before. If I am presented with a new piece of mathematics I bring to it a whole history of myself. Any construction I make in respect of this new task cannot be independent of this history. Nor can it ignore the circuit of exchange through which I will present any account of this work in the social forum. If I describe a piece of mathematics I am involved in an act of describing a situation of which I am part. I choose the bits that seem important to me. I am describing an extract from my own history, an experience, a process now gathered together in words and symbols. The story becomes in some way fixed as that which it describes carries on moving as the story is told. The story becomes more powerful than that to which it refers. Reality can thus be seen as being asserted through the "stories" told about it. Nevertheless, there is always something remaining outside these stories, outside theory, resisting over-arching narratives that account for everything (cf. Zizek 1991, p. 99, who in discussing post-modernism identifies this as the Lacanian Real).

In exploring these issues I would like to offer three examples of activities I employ in my own teaching (two of which were previously reported in Brown, 1994 d) that seek to give primacy to this linguistic production of meaning:

Example 1

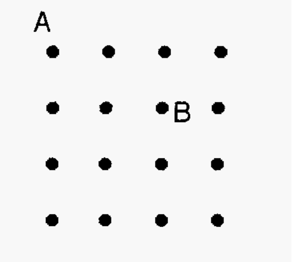
As a teacher I have shown Nicolet's animated geometry films to a variety of groups from 11 year old to adult. (For details of the films, refer to Beeney et al. 1982, p. 134.) After showing a brief section of film I ask pairs of people to share what they saw using spoken words only. After a few minutes I ask each pair to combine with another pair. Each group of four is then asked to produce a verbal account of what happened on the screen in as much detail as possible. The reality of the event becomes held in this string of words that itself becomes more solid than the memory of an image on the screen.

In an extension of this activity with trainee teachers I ask each group of four to select a "teacher" who is then asked to leave the room. A second short sequence of film is then shown to the "pupils" who remain. The teachers are then invited back into the room and are given the task of finding out what the pupils saw, again only through the use of spoken word. Each "teacher" is then required to give a "lesson" based on what happened on the screen.

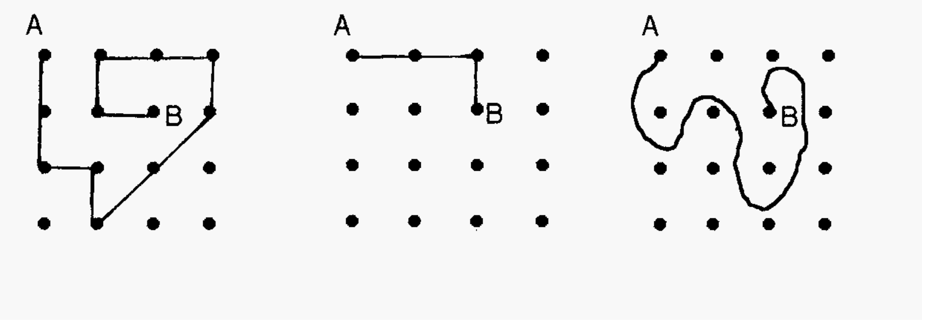
Example 2
In another lesson for trainee teachers I give each student a different model made of five centimetre cubes fitted together and say that they have been given a “Pentoid”. However, I hand it to them behind their backs so they are unable to see it. They have to rely on what they can feel with their fingers. One person is asked to describe their model to everyone else as someone records this description on the blackboard. Other students are then asked to say in which ways their own model is similar or different.

The students are then paired and given extra cubes. They are requested to describe their own model (with their eyes closed) so that their partner can build a replica. When each pair has made their replicas the original models are collected, without having been seen. The replicas are then gathered together for all to see. The students are then requested to imagine that they are about to telephone a friend in order to describe what a Pentoid is. How would they do this in a one minute call?

Example 3
Students are asked to draw a number of identical 4x4 dot lattices with A and B marked on each in the positions shown:



On each they are asked to draw a “path” which connects A to B. “Any path that you like”.



Each person then draws their "favourite" one on to the blackboard contributing to a collection of around 12 different paths. Selecting two at a time I ask, "In which ways are these two the same?", "In which ways are they different?". This produces a variety of approaches to categorising, for example, only straight lines, that one has diagonals while the other doesn't, both less than four units long. Following this I ask each group of four students to devise a set of rules which all paths they draw subsequently must obey, for example, paths must stay within grid, no more than four segments, less than eight units long. With these rules; which is the longest?.. do you have any missing? ..why are these lengths not possible? Finally, I ask whether they can "fiddle" the rules to ensure you get between 10 and 20 possibilities. This then leads to a variety of possible follow on activities (Banwell, Saunders and Tahta, 1972, pp. 130-131; ATM, 1980, pp. 2-9)

In the first activity each participant is able to describe the film using the terms they have available to them. Anyone can describe what they see although such descriptions presuppose an intention to be understood by others. The task is very much to do with classifying perception in language. Meaning is brought to the film in the descriptions people make. The very complexity of the film would thwart any attempt to claim an intrinsic meaning. The way of structuring the image of the film will necessarily be individual but likely to be offered in words from an inherited and (in some ways) shared language. The films may suggest familiar concepts to learned mathematicians but these will not hamper the vision of more naive viewers. At some level ideas are imposed by the film but the viewer can stress and ignore as she chooses according to what she sees. The task is more a question of finding the best way of communicating with others. It is difficult to assert a threshold level of achievement on such a task. Much of this also applies to the second activity where there is a more obvious move towards defining a concept. Whilst one may or may not gather what a Pentoid is, the emphasis of the activity is on bringing language to experience, a task of describing one's perspective. The reality of a "Pentoid" is held in the words that participants use to describe it. There is little need for an external authority validating work done, since the quest for the correct answer is not the point. With the third activity there is an emphasis on moving between creating rules and following rules. It requires considerable negotiation between students in deciding upon sensible sets of rules and whether or not particular paths obey them. I have conducted this activity on numerous occasions with both children and teachers. I am usually quite amused by how it is interpreted as an activity with too much freedom, yet the chief consequence of this is that it is executed as conservatively as possible with most participants electing to study rectilinear arrangements joining the dots, with no diagonals or curves.

The essential task in all three activities is for the students to introduce structures and through this produce meanings in the act of signifying. In stressing the linguistic dimension of mathematics in this way we shift away from emphasising mathematical activity as being to do with converging to pre-defined and well known concepts.

CREATING AND INHERITING MATHEMATICS

Mathematics is only ever manifested in culturally derived systems of exchange and, as such, can be seen as being subject to linguistic analysis. Saussure's work has taught us that meaning is not derived from individual terms but is consequential to the play of differences between successive terms in a particular discourse. When applied to mathematical terms we abandon the notion of individual terms having intrinsic meaning but rather see meaning as dependent on the individual construction of mathematical expressions. Ultimate meaning, however, is always deferred since we cannot speak of meaning in any absolute sense. We always await the final word (Derrida).

Whilst post-structuralism is principally concerned with parole, the performance of language by individuals, it takes away some of the responsibility for construction away from the individual by highlighting that this always happens in an inherited language, amidst conventional ways of doing things. Whilst students may be constructing, they ordinarily do this in the dominant language of the culture. Constructions are always already partly constructed by virtue of the language used in their construction. A specific style of structuring is implicit in the conventional ways of describing the world used by teacher and student in speaking with each other, This is not only true of the phenomena seen as being within mathematics but also of the procedures applied to these.

I have argued that the cultural derivation of mathematics is essentially a consequence of certain notions being captured in language and being passed down. For example, the base ten system, Euclidian geometry, algebraic structures are all inventions from the past that have become absorbed in our culture and are among the frameworks we employ when we speak of our world in a mathematical way. They all offer overlays for partitioning what we see but in turn condition what we see. There is nothing natural about them and so do not lend themselves to being discovered as such. They can only be "discovered", as it were, after a fairly comprehensive initiation into the cultural ways of describing the world in language, of which mathematics is a necessary part. Described in this way, "discovery" methods might be seen as being associated with pre-defined products, preexisting mathematical notions and as such may be seen as being more akin to the older form of structuralism and Piaget's representational view of mathematics. The relative stability, in the way in which the mathematical field is partitioned for the purposes of describing it, underplay any linguistic negotiation. The task of such discovery methods is to discover the way in which such notions are conventionally described with a particular view of their meaning. Clearly, much mathematical learning is to do with becoming initiated into conventional cultural usage. It seems to me that this sort of work is important in supporting the teaching of certain basic skills which need to form part of any curriculum. As such this echoes the constitutedness of post-structuralism.

Meanwhile, investigational styles, of the sort that were initiated in schools in the United Kingdom in the seventies and eighties (for example, ATM, 1977), seem different in one important respect. They are not so much about discovery but rather invite the student to introduce and develop structure. The emphasis is not on understanding a particular concept but is more as a result of conceiving. They have more of a quality of a game. If we set these conditions what can we say? By focussing of the fitting of structures they do offer more scope for linguistic invention (albeit within a borrowed language) than in tasks that are about introducing culturally conventional ways of gathering ideas. They have a creative component which permits the student to assert a more personal identity in the output of this work. "Investigations" are more concerned with production of both mathematical structures and the linguistic categories associated with them; the more creative side of post-structuralism. (cf. Yackel et al., 1996, who draw a similar conclusion and implicitly suggest that this production of structures is an outcome of teacher-student exchanges within any "inquiry" focused mathematical learning which gives student the opportunity of verbally expressing themselves.)

The generation of statements within mathematical activity is necessarily part of a human classifying process concerned with the selection and combination of signs - "discursive practices", which build categories and "produce" meaning (Walkerdine). In this way, mathematical writing can be seen as comprising statements, associated by the writer with what they see as circumscribed mathematical ideas. Mathematics is generally performed in a social sphere and there are definite requirements for participants. So there is a dual task of enabling the student to be conventional in her language usage but at the same time inventive in buildings structures and meaning for themselves. Both discovery methods and investigations incorporate both product and process but the former emphasise product whilst the latter emphasise process. The mathematics teacher, on the one hand, must enable her students need to talk about mathematics in conventional ways to be able to partake of the society's systems of exchange. This is to do with learning the language and the conventional ways of using it. But, on the other hand, the teacher needs to enable the students to gain experience of linguistic invention towards producing structures and meaning. This latter task is to do with students describing situations of which they are part; understanding their relation to the things they describe, learning to signify by attaching linguistic and symbolic forms to experience (and vice versa).

CHAPTER 3

SHARING MATHEMATICAL PERSPECTIVES

As I pointed out in the last chapter Saussure saw language as having the dual function of articulating meaning (process) and communication(product). However, Saussure and also his modern day advocates in post-structuralism see both process and product as contingent on each other. As educators we are often concerned not so much with the learning itself as with giving an account of what learning has taken place (cf. Garfinkel, 1967, pp. 24-31, who discusses the difficulties of moving between what is said and what is being spoken about). We need to decide what has been learnt by a student and how this has been demonstrated through tangible product, whether this be the application of a method, a reproduction of a famous result, or some verbal explanation of work completed, etc. Learning by the student, however, continuously evolves, oscillating between understanding and explanation; between an on-going learning process and statements generated within that, which become frozen in time. Further, this learning encompasses concerns beyond the frame anticipated by the teacher.

Questions were raised in earlier chapters concerned with the problematic relationship between mathematical ideas and the symbols which represent them. Such ideas, it has been suggested, cannot be transferred "ready-made" but, rather, are susceptible to interpretive modification as they move between people and through time. Hermeneutical views of mathematical learning emphasise the individuality and time dependency in our understanding of specific mathematical ideas. In assessing mathematical work we are thus faced with a task of finding an adequate way of locating mathematical knowledge - with seeing how it is held between the perspectives of teacher and student. Further, if we see mathematical work as linguistic activity, our view of developing understanding is dependent on the way in which we see language functioning in relation to reality and also on the way in which we see mathematics relating to its symbolic and physical embodiments. This chapter commences by briefly reviewing how the range of views of language offered by the various hermeneutical schools provide a framework for understanding mathematical learning and how they condition what mathematics is. This is followed by an examination of the task teachers and students face in sharing mathematical perspectives. The chapter concludes with an example of a lesson featuring students capturing their mathematical understanding in words, diagrams and pictures.

DISCOURSE OR REALITY?

In his book, *"Post-Modernism, or the Cultural Logic of Late Capitalism",* Jameson (1971, pp. 6-10) makes an interesting distinction between Modernism and Post-Modernism. He associates hermeneutical depth interpretation with the former and the textuality of post-structuralism with the latter. He illustrates this by contrasting two paintings; "A Pair of Boots" by van Gogh and "Diamond Dust Shoes" by Warhol. He suggests that van Gogh's painting of a pair of peasants boots gives rise to the possibility of various interpretations. He offers the magnificence of the bucolic landscapes we might normally associate with the paintings of van Gogh or alternatively, the stark peasant lifestyle suggested by such clothing. Either view can be developed as a fairly full account of what the painting might be seen as evoking. Warhol's effort, however, a dark, sparse, shadowy affair that may have been produced with the help of an X-Ray machine seems to defy any such generation of stories. It seems to be *all in the surface* - it begins and ends with the painting.

With this as our analogy in mind how might we distinguish between Modernist and Post-Modernist presentation of mathematics? Interpretation of mathematics might be seen as a relatively new idea, unless you are talking about statistics or mechanics. As suggested in previous chapters there needs to be an emphasis on the activity of mathematics before interpretation seems tenable. It is only when we consider people choosing to use some piece of mathematics or other that alternatives present themselves. This choosing however, has only recently been reintroduced into the vocabulary of school mathematics. The emphasis has generally been on mathematics where the choices have already been made. The students have typically been delivered to the already formed ideas and told to work with them.

In another reflection on post-modernism, Zizek (1989, p. 96) explores the Coca Cola advert which declares "This is it". What is "it"? he asks. He suggests that "it" is nothing other than America itself and the associated glossy lifestyle of which Coke is supposedly part. I suggest that "mathematics" has pulled off a similar *coup d'etat.* Mathematics is ordered, it is logical, obeys strict methods, is fully decidable... or so they say! After carrying out mathematics for years we have looked back on it and claimed certain features as being "it". These features however, seem completely devoid of the humans and their struggle that brought them into existence. Mathematics which has been derived from the activity that has given rise to it, is, it might be claimed, *all in the surface.*

Another feature which Jameson sees as distinguishing Modernism from Post-Modernism is that styles of the former have become icons of the latter- for example, a process of iteration becomes a button on a calculator. Things shrink as the field they are in expands into ever greater complexity. Interpretation comes firmly into play as we are forced to choose between an ever increasing number of things. The activity of choosing at all levels forces a permanent oscillation between interpretable mathematical activity and making statements *as if* free from the situation which gave rise to them. It may be that this manifestation of the hermeneutic circle asserts the dynamics which prevent mathematics standing still long enough to be defined in either way.

This echoes a distinction drawn earlier between the hermeneutics of Gadamer and Habermas. In defining the scope of language we oscillate between seeing it as something of which we are part to seeing it as something we can operate on. I suggested Gadamer uses the former as his home base while Habermas uses the latter. Different breeds of hermeneuticians position themselves at varying points around this spectrum.

Post-structuralists analyse texts as individual performances of language (parole) but without any detailed investigation of any external reality to which they may refer. Their focus is exclusively on the the usage of language rather than on any externally defined meaning of language. Meanwhile, Gadamer prefers to emphasise how text resonates with the reader's experience. Language is seen as being in a dialectical relationship with reality, albeit a reality conditioned by language. Habermas prefers to assume a critical distance from language, seeking to understand how it functions in relation to its creators' intentions. A conservative view would neutralise language to the more limited function of labelling reality as in the work of Russell (1914, pp. 63-97) or the early Wittgenstein (1961). Nevertheless in all of these various positions we are concerned with how experience gets mapped into language and vice versa. If We take mathematics as a language we similarly move between seeing it as a dimension of human activity and as something *as if* free of human intervention - between seeing mathematics as discourse and seeing it as transcending human experience.

LOCATING MATHEMATICAL KNOWLEDGE

In assessing mathematics we seem at first to be caught between on the one hand, working with a style of mathematics where we assert a field of symbols *as if* devoid of humans and, on the other, speaking of mathematics as a depth interpretation of a certain style of human activity. This however, is not a satisfactory dichotomy, if only because we never have a choice of one over-arching symbolic framework. Mathematics can assume a multitude of linguistic styles, which can sometimes meet and intersect, but which very often conflict, or get confused. The choice is perhaps more accurately between on the one hand, depth interpretation of activity and on the other, composing fragments from alternative discourses. Both of these have an implicit interpretive dimension since choice is central in each.

If we emphasise mathematics as generative discourse we downplay the supposed permanence of its attributes. In the absence of hard mathematical knowledge which can be transported around intact we are faced with a difficulty in assessing the results of mathematical activity. If mathematics is seen as being interpretable it becomes more malleable and susceptible to variation according to who is presenting it. This causes problems for assessment of mathematical achievement. It might be suggested that the supposed transferability of mathematical topics influences the prominence they are given in the school curriculum. That is, those areas of mathematics more easily describable in clearly defined linguistic categories are more robust since they are more easily accounted for. Gattegno (1988, pp. 118-119), for example, argues that school education in general is mainly verbal and that many areas of mathematics sit uneasily in such a curriculum. As an example, he sees this as having led to a widespread deficiency in geometrical intuition - an area which needs to be taught yet does not lend itself to easy description. In Gattegno's view, school geometry is generally algebraic in nature and is mainly about categorising geometrical phenomena into discrete notions, a partitioning which frustrates intuition. Geometrical understanding is not fully classifiable in language and as soon as it becomes framed in language it is reduced into an algebraic style of thinking. Such an emphasis allows it to be mechanised, made repeatable, so that it is more manageable in a school setting. Geometrical intuition is harder to account for since any attempt to share it with others requires translation into algebra with the cost to "geometrical" experience that entails. Intuition thus becomes a "spin-off" of teaching rather than something easily targeted in didactic presentations, or described in curricula.

We are then faced with a question of how much of any mathematical experience can be held in the language which describes it. We are also concerned with how we might witness others attempting to capture their experience in language. Clearly, whatever view you take, mathematical expressions themselves do not mean the same to all people; individuals see expressions in the context of their own experience, cultural perspective and current intentions. Their intended meaning depends both on the way in which the individual perceives their task and on their familiarity with such expressions. Nevertheless, there remains an issue of how far we see such activity gravitating around "correct" or culturally specific meanings. In the teaching of mathematics it is the norm to assume that the teacher's task has something to do with drawing the student to a particular point of view. The teacher needs to find ways of enabling his students to share a perspective. Insofar as developing understanding is seen hermeneutically, however, learning is not necessarily about the reproduction of the teacher's knowledge in the mind of the student, but rather can be seen as a transforming of both positions. Knowledge is not a fully constituted object being confronted by a fully constituted student, rather, both change through a time-dependent process. Learning is not just about adding to knowledge, rather knowledge, or at least our state of knowing, can be transformed in many ways; one subtracts from it as well as adds to it, forgetting as well as remembering, one reorganises so that known "things" get new meanings - and knowing is not just about things.

In his early days I sometimes tried pointing out things to my baby son Elliot, but he just looked at my pointing hand. For pupils in classrooms there is a frequent conflict between attending to the teacher's understanding and attending to the object of that understanding. Do you pay attention to the pointing hand or to the thing being pointed at? For the traditionalist anchored by reality, the object itself arbitrates. However, the further we move away towards views of language that see it constructing reality we are faced with more complex decisions as to the location of knowledge.

This problem might be characterised as a “double-bind” - a statement that seems to contain two conflicting messages (Mellin-Olsen, 1991). One might imagine a teacher’s plaintive assertion - *Look at what I see - but it’s what you see that's important.*

Brousseau and Otte (1991) outline the problems endemic in what they call the “didactical trap”, where a teacher finds himself forced into giving the student knowledge rather than allowing the student to “reconstruct the knowledge” for themselves. The student, in paying attention to what the teacher wants, and thereby making this the focus of their learning task, is drawn away from the “thing” itself. Brousseau and Otte resolve this potential dichotomy by rejecting notions of knowledge seen as a found object preferring to assert its essential “fragility”, something that can not be held as fixed. The “fragile” learner they describe has much in common with the human subject described variously within post-structuralism and hermeneutic phenomenology.

For example, Walkerdine’s (1982, 1988) psycho-semiotic account of developing mathematical understanding offers a framework which takes us away from more individualist notions of psychology rooted in cognition based around an abstract epistemic subject, who is because he thinks. Rather, her focus is on how subjects are created through their insertion into social practices and on how they evolve through the stories which position them. Solomon (1989) adopts a similar socio-linguistic stance. In such perspectives we meet both a softening of subject positions and a softening of that shared between them. Whilst, the activity of mathematics is often characterised as being oriented around the certainties it contains, stories describing “mathematical” experience and do not provide a stable orientation. As we have seen, if we emphasise individual perspectives we move away from seeing a universal “correct” view of mathematics anchored around the “facts” of mathematics. Students come to know mathematics through participation in mathematical activity with others, including the teacher, and in this way the students learn to talk about and engage in mathematics. There are socially conventional ways of going about the business of doing mathematics in classrooms and the more abstract notions of mathematics are met within such conventions. Walkerdine’s emphasis is on the process of students learning to signify, i.e. to capture their experience in language (ibid). She has questioned the strong faith many teachers have in “doing” practical tasks as a sufficient requirement for understanding mathematical phenomena. She sees practical engagement with physical materials as a cover for this socio-linguistic initiation. Initially she sees learning being tied to familiar situations based around conventional social practices. This she sees as being not so much concerned with ideas developing in the mind but rather the students are learning how to talk about and engage in classroom mathematical activity. However, she suggests that for students to be capable of abstract thought they need to be able to suppress their attention to everyday associations and instead concentrate on the internal relations of the mathematics. Nevertheless, she sees this as resulting in some cost to the subject seeking to connect mathematical discourses with their own everyday discourses (for example, gardening and cooking), which, in part, depend on their metaphoric association with mathematical discourses.

ASSESSING MATHEMATICAL ACTIVITY

By seeing the assessment of mathematics as being directed towards understanding the student making sense of his mathematical activity, we overtly move in to the realm of interpretations. A two-tiered interpretation is implied; the student capturing his experience in symbolic form, and the teacher assessing this symbolic product as an index of understanding (cf. Garfinkel, 1967). This suggests that a possible reorientation of the teaching relation. Whilst the teacher may have selected the work she can nevertheless ask the student to describe it in his own terms and then enter into a dialogue oriented around these terms. This enables the student to articulate aspects of his thinking which helps clarify this thinking. It also enables the teacher to gain some insight into the student's view and the language he uses. The resulting dialogue might be seen as an attempt to communicate in a shared language. However, the teacher might see part of her task as guiding the student towards conventional usage of certain expressions. What is not implied here is any notion of a universal meaning to which both teacher and student converge, but rather "...objectivity is achieved through the coincidence of interpreting, that is, agreeing" (Brookes, 1977).

Nevertheless, one might legitimately protest that there is a certain power relation here that creates a somewhat asymmetrical sort of agreeing, where the teacher, as representative of the conventional way of talking about things, sees her task as introducing this. Whilst the student may have the opportunity of offering some account of their understanding, within their own mode of signifying, the teacher, in entering any discussion, may be introducing a more conventional mode. The communication being sought in such an exchange brings into play some symbolic medium, comprising symbols, actions and words. But such is the power of the conventional mode of discourse that the quest for the learner may be to believe that he is joining the teacher in using the inherited language. This highlights a particular aspect of the teacher's power, consequential to the linguistic overlay she brings to the situation. The teacher's style of looking is accustomed to spotting concepts which are, after all, merely culturally conventional labellings. In this way the teacher's way of making sense of a student's work involves classifying this work as if looking to tick off categories on a curriculum checklist. The student's access to any notional transcendental mathematics is always mediated by a social pressure to capture this in the categories introduced by our ancestors.

Traditionally, assessments have been concerned with assessing the student's production of "correct" mathematical statements as evidence of a broader mathematical understanding. An alternative to this places emphasis on the "story" told about the event of a mathematical activity (cf. Mason, 1989 b) . Such a story might be no more than the set of statements offered by the students under the label of "symmetry". Here assessment is not so much based on the proportion of correct statements but rather, on the quality of understanding demonstrated in giving an account of the activity. Thus the assessment might be more like one normally associated with a piece of writing. Here, the "content" of the mathematical activity might be seen as the outcome of the "process" as described by the individual learner. Whilst modifying notions of mathematics which underlie syllabi constructed from a content-oriented point of view, traditional mathematical content still has a home here. However, the syllabus cannot be seen as remaining intact as the student progresses through it since the content of such a syllabus is flavoured by the activities that give rise to it. A residue remains of the experience in any identification of content covered which will be present in statements made by the student. Assessing the student's understanding of his mathematical work through the statements he makes in respect of it necessarily requires personal interpretation from his teachers in deciding how these statements signify the student's understanding. This does not rely solely on the student's production of correct mathematical statements. Commentary on the sense the student's make of the experience cannot necessarily be reduced to such a form.

COMBINING LANGUAGE AND EXPERIENCE

Previously, I have suggested that investigational work can permit the student to develop their style of signification more fully, prior to interception by the teacher introducing more conventional ways of describing the product, than might be possible in more traditional approaches, or even "discovery", "inquiry" or "constructivist" approaches targeted at specific concepts. In this section I provide a more developed example of such a style of work.

Example: Investigating gardens

I offer a description of an investigational task I initiated with a class of ten year olds, which lasted several lessons (Brown, 1990 c). This involved exploring the areas of rectangular "gardens" comprising a metre wide path around the perimeter of a lawn. At the beginning of the session each child was provided with plastic interlocking "Polydron" squares. I began by holding up a "lawn" I had made and then showed the class how it looked after I had surrounded it with squares which made a "path". I followed this by making other lawns myself and asking the students to show me what it would look like after being surrounded by a path. I had arranged the students on tables of four and had only provided enough pieces for a group effort and so some sort of negotiation between them was necessary. After a few examples I became fairly confident that each group were able to respond to a "Polydron" lawn made by me with a similar sized lawn surrounded by a path. At this point I asked them to make different ones.

In describing such gardens to the students and then having them represent these in plastic models they are perhaps making a connection between the image of such gardens in their minds and the plastic models before them. However, they were in some sense suppressing this connection as they focused on exploring different sorts of plastic models obeying the given rules. Here they are focussing on the internal rules and relations pertaining to plastic models. There is also another sort of representation evident in this activity. It is in the way they talk about the model. In saying something like "This is the lawn and it's got a path around it" they are associating the image in their mind or a plastic model with some words.

When it seemed to me that the students had made a number of plastic models obeying the rules I suggested they found as many as they could where the lawn was only one square wide. I also handed out centimetre square paper and suggested that they draw all the ones they found after first making them out of Polydron. The various groups proceeded at different rates and I did not suggest drawing until I felt students were beginning to find the Polydron pieces cumbersome. Many students stayed with the plastic models throughout the first lesson whilst others confidently moved into scale drawing representations of them. Many of these students had drawn the gardens with lawn of width one in order of size. For those who had found them more randomly I suggested that they be redrawn in order of size.

In initiating this shift into scale drawing representation of gardens the students are encountering a new association between domains. After imagining gardens and then making models of them, they are now employing the representation of scale drawings. As the students explore different sorts of scale drawings attention is moved away from the plastic models.

For students who had completed a number of scale drawings showing gardens with lawns of width one and in order of size, I suggested that they tabulated their results in the following format.

Number	1	2	3	4	5	6
Area of Lawn	1	2	3	4	5	6
Area of Path	8	10	12	14	16	18

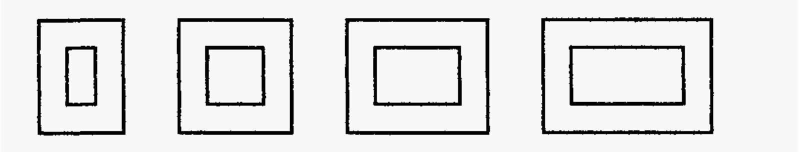
Most seemed to find this relatively easy although for some it involved going back to the Polydron models or scale drawings to check results. On completing such a table I intervened and asked them what they could say about the table. I received comments such as "The lawn is always the same number as the garden" and "The path goes up in twos". I suggested that they write about all of the things they could notice. For most students I suggested that they now try to construct a table of results for gardens with lawns of width two. I suggested that they could use either Polydron or scale drawings if they wanted to, to help them find the results. Most students seemed to do this but after a few drawings they were normally able to carry on the table without using other representations.

The questions I asked of more able students were suitably taxing. For example, after completing the first table they had to hypothesise about the next, then the tenth garden and so on. Similarly, the students were asked that given they knew the area of the lawn how would they work out the area of the path. What could they say about the hundredth garden? What could you say about the nth garden? I attempted to match my level of questioning to the sort of responses I was getting from individual students but in all cases I insisted they wrote everything down in prose.

Here I see two new representations being introduced and related by the students to those they had employed already. Firstly, tabulation provides a valuable, perhaps less cumbersome, alternative to plastic models or scale drawings. The students were normally, after some work, able to see the internal workings of these tables without reference to other representations, that is they could continue the tables without reference to models. Secondly, the students were representing their work in writing. To provide the flavour of this I see this having I shall reproduce exactly, in its entirety, the summative written work of Stephen. It becomes especially interesting as patterns are described. The work is taken from a poster created jointly with three other boys, Peter, Andrew and Simon. Work by these other boys is included in figures 3-5.

GARDENS

We started off with a border with a 4x3 area. The path was 1 metre wide. We had to work out the area of the grass in the middle of the square. Peter has done a series of diagrams to show the shape of the garden.



	3x4	*4x4*	*5x4*	*6x4*	*7x4*	*8x4*	*9x4*	*10x4*
Path	*10*	*12*	*14*	*16*	*18*	*20*	*22*	*24*
Garden	*2*	*4*	*6*	*8*	*10*	*12*	*14*	*16*

(Peter also included diagrams and tables for lawns 3 and 4 metres wide)

We also had to work out the area of the path. Here is the answer for the 4x3 square. The answer for the grass is 2. The answer for the path is 10. The next thing we had to do was the 4x4 square. The answer for the grass is 4 and the answer for the path is 12. The 4x5 square: The answer for the path is 6 , and the answer for the path is 14. The grass area for 4x6 is 8 and the area of the path is 16. The grass area for 4x7 is 10, and the path area 18. 4x8's grass area is 12 and the path area is 20. 4x9's grass area is 14 and the path is 22. 4x10s grass area is 16 and the path area is 24. 4x11 's grass area is 18 and the area of the path 26.

It goes up in two's, 2,4,6,8,10. Also if you look at Peter's diagram for the drawing of the 4X, take away 2 squares and you get the starting number for the grass area. The area of the path also goes up in two's. On the 4x3 square, the area of the grass and the area of the path, if you take away the grass from the path, you get 8. Do that on the rest of the 4X, and you always get 8. Here is a chart to make it clearer.

Dimension	*4x3*	*4x4*	*4x5*	*4x6*	*4x7*	*4x8*	*4x9*
Area Grass	*2*	*4*	*6*	*8*	*10*	*12*	*14*
Area Path	*10*	*12*	*14*	*16*	*18*	*20*	*22*

Between 2 and 10 is 8, between 4 and 12, 8 between 6 and 14, 8, and so on. The grass area goes up in two's and so does the path area. But in 5 wide, or 5X it does not go in 2 and 2. The area of the path stays the same, but the area of the grass goes up in 3's. Between the grass area and the path area for the 5x3 is 9 and for the 5x4 it is 8, for the 5x5 it is 7, so 5x6 would be 6. You can still get the starting number by -2 off the path.

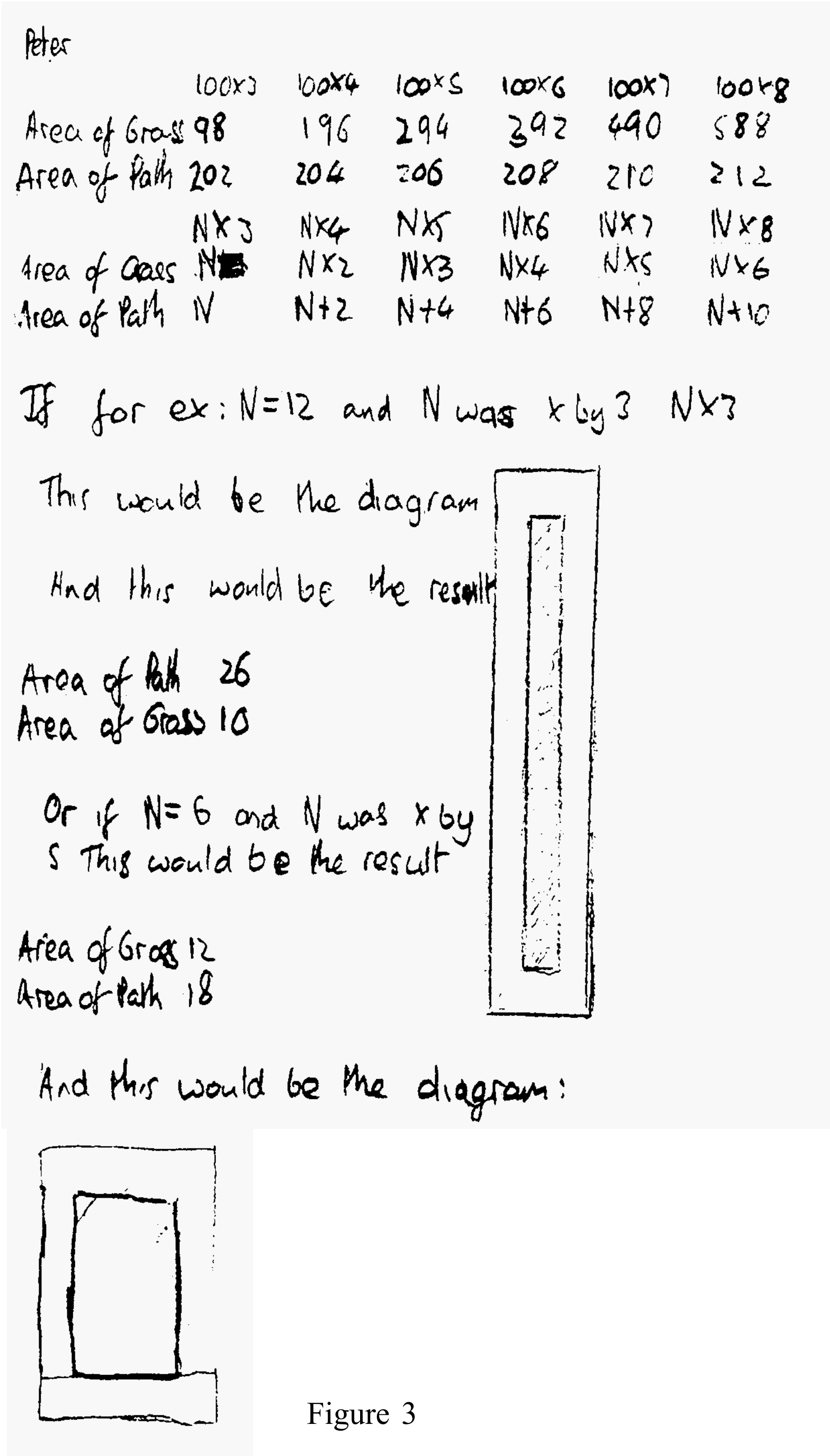
Now onto the 6X. The area of the grass is 4 for a 6x3 square. To get 4 you do what I have been writing all the time, you -2 off one of the sides of the path. The 6x4 grass area is 8. 6x5's area of the grass is 12. 6x6's area of the grass is 16. Between those number is 4. On the 4X, it was 2 between the numbers off the grass area, then on the 5X it was 3. The 6X it was 4. The area of the path starts with 14. To get that number and 12 for 5X and 10 for 4X. Between the grass and the path area on 4X, it is 8. on the 5X the number between is 9. On the 6X the number between is 10. So on the 7X which I am not going to do it would be 11.Next we had to do 10X. The grass area started with started with 8 for the 10x3. The

10x4 square started with 16. The grass area that is. The grass area for the the 10x5 is 26. The 10x6 grass area is 32. The 10x7 is 40. The 10x8 grass area is 48. The 10x9's grass area is 56. So the 10X10's grass area is 56. Between those numbers is 8. The path area starts with 22. It still goes up in two's. Between the grass number and the path number is 14 for the 10x3. For the 10x4, the number between is 8. For the 10x5 the number between is 2. It is 6 between.

Now onto the 100's. The area of the grass on the 100x3 square is 98. The area for the 100x4 on the grass is 196. I don't have to go any higher because you can work it out from these two numbers. The number between them is 98. So, all you've got to do is work out your 98X table! The path area is still in the 2's. It starts with 202.

Now for the NX. The NX3 grass area is N So is the path area. The NX4 grass area is NX2. The path area is N+2. So the path area for the rest of the N's would be plussing 2.

Although most students did not proceed beyond here, some, including those mentioned, were able to extend their results. Many students were able to say how they would go about working out the areas in a garden of a given width but for any length (see Figure 3). For example, they could indicate something like "you add the path at the top to the path at the bottom and then the two sides and then add 4 for the corners". The final stage tackled by some students was concerned with recognising they had only observed one sort of garden. A few went on to explore the possibility of imagining any garden. Andrew was delegated to write the description of the work that resulted. I include an extract (see Figure 4). The diagrams drawn by Simon provide an illustrative summary (see Figure 5). The new representation of gardens here extends beyond those that are rectangular with a metre wide path to an exploration within the symbolic notation, together with diagrams that no longer display the discrete properties inherent in the Polydron models. However, it seems to me that for the students involved this notation is supported by the imagery they have explored and developed in earlier work. These new representations are no more than a shorthand for all the talking and writing that has gone on about the gardens obeying these rules but become a powerful statement of generality with a whole story around them.



Peter

	100x3	100x4	100x5	100x6	100x7	100x8
Area of Grass	98	196	294	392	490	588
Area of Path	202	204	206	208	210	212
	NX3	NX4	NX5	NX6	NX7	NX8
Area of Grass	N	NX2	NX3	NX4	NX5	NX6
Area of Path	N	N+2	N+4	N+6	N+8	N+10

If for ex: N=12 and N was x by 3 NX3

This would be the diagram

And this would be the result

Area of Path 26
Area of Grass 10

Or if N=6 and N was x by 5 This would be the result

Area of Grass 12
Area of Path 18

And this would be the diagram:

Figure 3

Andrew 1st March

Gardens

We had to work out the area of a M by N garden. The area we had work out was the area of the path and of the garden. N could equal any number while M could any number N doesn't The path round the garden was always a meter wide no mater how big the garden is. Simon have drawn some diagrams of the gardens. The shape he as drawn first is a rectangle shape and this is the one that we started with using N and M. We found out the area of the grass is M-2xN-2 and the area of the path is MxN-M-2xN-2 and this would be the out line. Next we had to use a L shaped garden and path. The L shaped garden also had a meter wide path around the grass (as shown is Simon diagram) Next we had to do the same as the first garden but with a two meter wide path round it. the area of the grass is M-4xN-4 and the area of the path is MxN-M-4xN-4 and this shows the out line of the path and garden. After this we had to do a L shaped garden with a 2 meter path round it. This grass is M-4xN-4÷4 x3 and the path is MxN-M-4xN-4 to get the area of the path and garden of the L shaped garden with a one meter path round it you must take the L shaped garden with 2 meter path around it you change the fours to two then you have the L shaped garden with 1 meter path

Figure 4

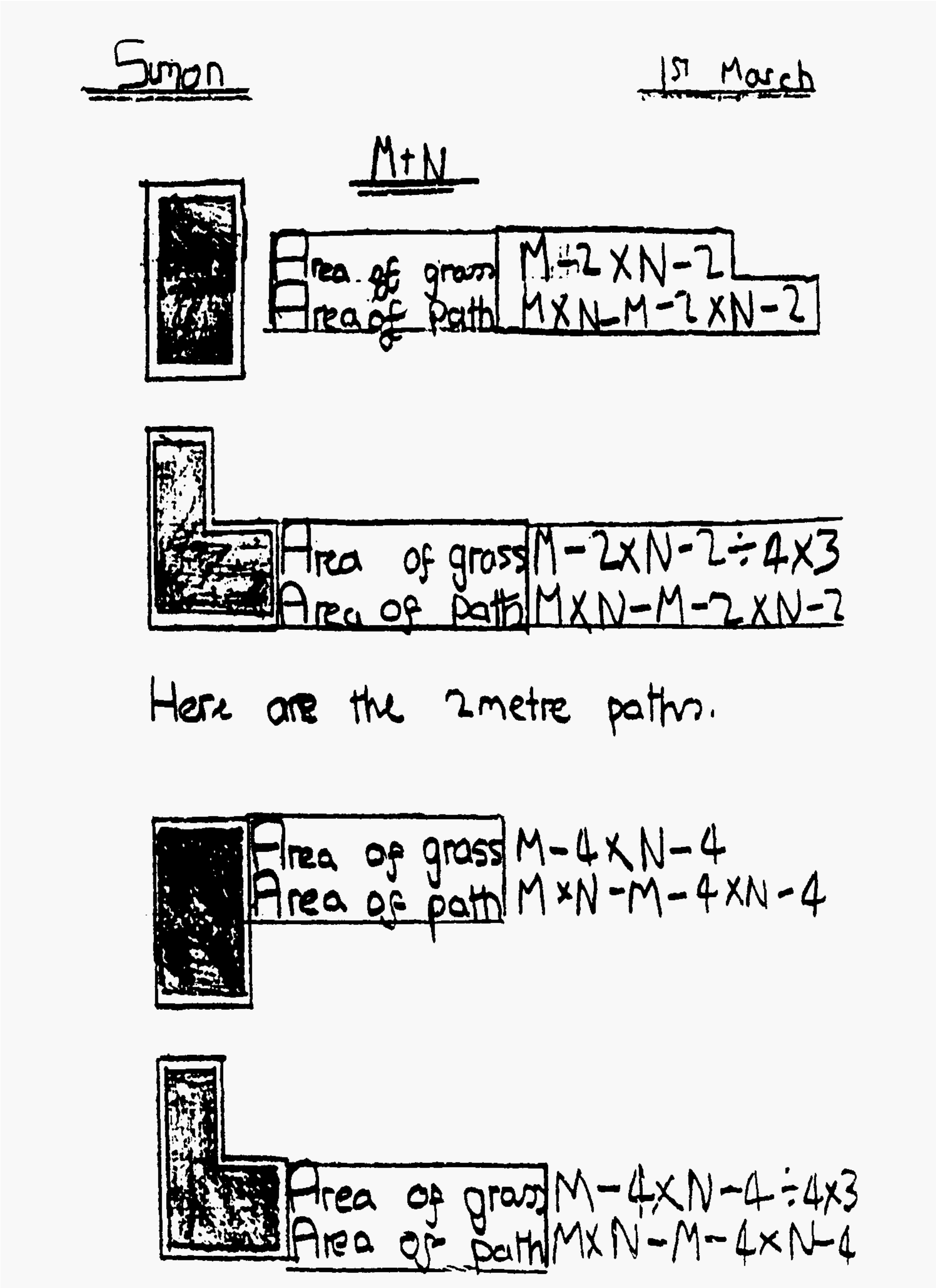

Simon
1st March
M+N
Area of grass M-2 X N-2
Area of Path M X N-M-2 X N-2
Area of grass M-2 X N-2 ÷ 4 X 3
Area of path M X N-M-2 X N-2
Here are the 2metre paths.
Area of grass M-4 X N-4
Area of path M X N-M-4 X N-4
Area of grass M-4 X N-4 ÷ 4 X 3
Area of path M X N-M-4 X N-4

Figure 5

Whilst the work described here was the most articulate I received, the mathematical work described in it was tackled by many students in the class. The extended writing is a fairly unusual and perhaps cumbersome way of presenting mathematical work but I suggest that there is great value in the students being able to do this. In translating the work as represented in Polydron models, scale drawings, tables and spoken description into written description the student will reflect on the rules and relations inherent in each of these specific fields. The student is actively moving within a field in attending to one representation but in the act of writing the student is actively placing different representations alongside each other (see Figure 6). I feel this active movement between fields underlies students developing an active relationship to the mathematics they are doing.

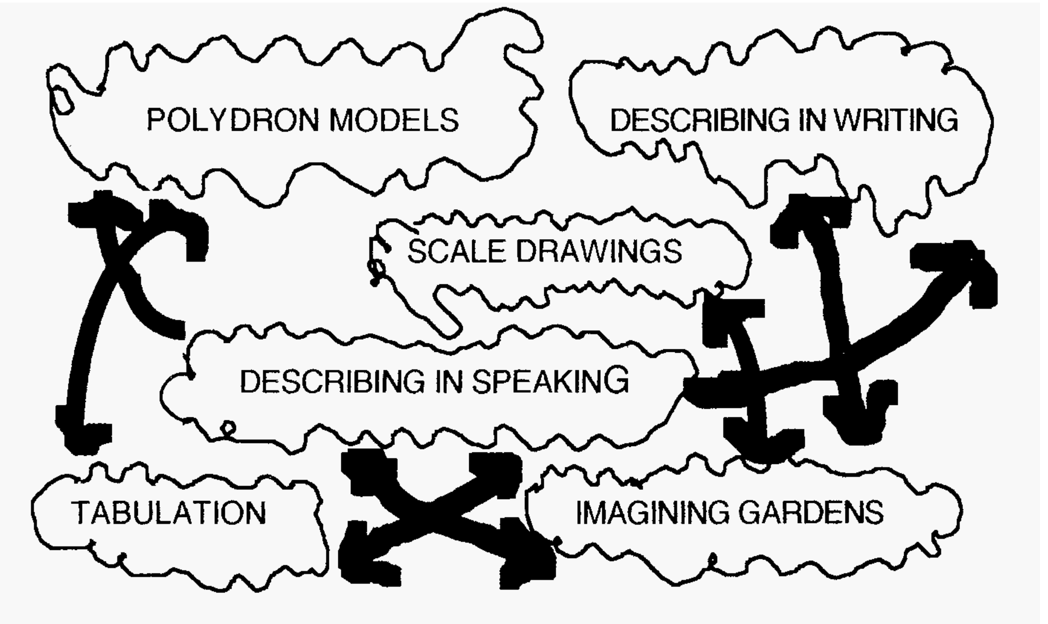


Figure 6

I suggest that the students' work in my lesson was following various routines and ways of working which had been established. Tied initially to work with tangible objects such as Polydron they gain experience of exploring and comparing different representations. However, through such experiences they become able to dispense with these relatively concrete representations and work increasingly with more abstract notions. Summarising the lesson: Initially, the students made models of the gardens out of plastic squares and counted the squares to find the appropriate areas each time. The statements about "gardens" referred to the plastic models. However, in due course the construction of the plastic models became cumbersome and the students readily transferred their work on to squared paper which permitted a more efficient way of producing representations of "gardens". For a while this proved successful but as bigger drawings were produced the limitations of the paper became evident. The possibility of tabulating the areas then seemed an appropriate way of gathering together the data that had been generated. In doing this, number patterns were suggested which allowed new data to be produced without the need for making or drawing new models. As more tables, were generated, more general statements could be made about them. For some students these statements were translated to a shorthand in the form of a more conventional algebraic symbolisation. During this work verbal and written statements from the students included:

> *"You add the top to the bottom and then add on the two sides"*
> *"The area of the path goes up in twos"*
> *"The path is always bigger than the lawn"*
> *"Area of Path = MXN-M-2XN-2"*

Whilst some of these statements may lack the precision of formal mathematical statements they suggest real attempts by the students to represent the mathematical phenomena they are dealing with and function in guiding subsequent action. It is evident that the student's work in respect of tasks developed in this way does not fit comfortably into the categories conventionally found in content-oriented syllabi. There clearly is "content", however, and it seems inappropriate to classify this task as solely "process-oriented". A more complementary relation between process and content seems necessary. The "content" of mathematics is only ever seen by humans engaged in the "process" of mathematical activity.

So viewed the outcomes of mathematical activity are in terms of organising and making sense of the experience of doing mathematics. This presupposes a selection from the different aspects of a given mathematical activity and a softening of approaches to mathematics that see the subject as being about exercises comprising strings of questions with singular correct responses. By teaching the students to characterise particular aspects of their experience they develop a means of orientation. Dockar-Drysdale (1990, pp. 98-111), who in her work as a teacher and psychotherapist with emotionally deprived adolescents, calls this "symbolization" and sees this as a basic human trait, albeit one that sometimes needs to be encouraged in those who have not benefited from normal socialisation processes. I suggest similarly for students engaged in mathematical activity structure does not emerge naturally, but rather, is constructed, making use of, to a greater or lesser extent, components borrowed from the broader community. In the "gardens" activity the students are attempting to group the results of their work in different ways so as to be able to talk about what they have done. However, there is great scope for them generating and running with their own structures prior to any closing down by the teacher. I saw my task as a teacher utilising investigational style activities as being to enable students to function and generate structure within the space they inhabited, not merely help the students locate the structure I, the teacher saw myself. Within the task I clearly retained my hold over the activity, perhaps too much, certainly my nudges concealed many conventions. Nevertheless, on many occasions, during the course of the lessons featuring this activity, students were placing emphasis on how they saw things and my task as teacher was to enable them to stay with their particular understanding and develop it. My emphasis was as much on developing their ability to articulate as on what they articulated.

As we have seen, Ricoeur's formulation of the hermeneutic circle features an oscillation between understanding which continuously evolves and explanations (statements) which are relatively fixed in time. Here, explanation and understanding can be seen as a complementary duality under the umbrella of interpretation. As Elliott (1987, p. 160) puts it, in discussing the hermeneutics of Gadamer: "Interpretation constitutes a moment within the process of understanding". Students' understanding evolves continuously but is represented through tangible product, capturing the moment, such as pieces of writing, which, in some way, offer insight into their learning. Teachers are concerned with interpreting how students' thinking is captured in evidence of it. These attempts to describe experience, however, are unable to capture everything. There is always something left outside of the language used. Opinions differ between the various schools of hermeneutics as to how language performs in this task. Such views variously imply roles of the teacher. Traditional notions of teaching may have supported the view that the language of the teacher held the knowledge to be communicated. The student's task was merely to understand what the teacher intended through reconciling the things the teacher said and did with their overall intention. However, all but the most conservative writers in modern hermeneutics now tell us that teaching is not about transferring the teacher's intention. Rather, teaching is more about cultivating the significance the learner gives to the teacher's input, which necessarily entails a reconciliation of personal experience with social coding.

The teacher of mathematics needs to ensure that she nurtures the mathematical powers of the student whilst, at the same time, initiating the student into the ways of the community. The Gadamer-Habermas debate about the scope of language helps us to highlight some of the issues involved in codifying personal mathematical experience in notation. The demand in schools to work with socially specific notions of mathematics brings into play particular ideological views of mathematics. An initiation into these necessarily involves a submission to an authority governed by culturally derived notations which can subsume more personal notions. However, this initiation is an essential component of learning where children interpret their learning experience and channel it in to an increasingly conventional linguistic form. It is through such a process that students become inserted into conventional social practices, as they frame their personal activity in the language of their community. Focusing on the need to initiate the student in to conventions can, however, often lead to an emphasis in schools on a mechanistic view of mathematics which stresses the repeatable features so as to make the task of teaching more manageable. Whilst, inevitably, in coping with the world, we need to group things in order to store and communicate them (cf. Dockar-Drysdale, ibid), such an emphasis can go hand in hand with a down-playing of mathematics which does not lend itself to categorisation or easy transference in language. As a consequence, the development of the personal powers of the student is in some ways under-stressed in the school learning process, in attempting to make the mathematics more communicable and accountable.

If we were to follow Habermas in defining more "emancipatory" forms of educational practice we would need to differentiate more clearly between *teacher's intention* and *significance for the student* and stress the developing critical powers of the individual student. Such moves towards emphasising interpretive aspects of mathematical activity, however, inevitably result in placing less stress on the conventional categories of mathematics, as may be represented in the teacher's input or school curriculums. This suggests that in order to broaden the nature of mathematical experience for the student we need to loosen those categories, conventionally associated with mathematics, if we are to enable to the student to develop intuition and other ideas not easily captured in verbal or written description. For the teacher this may entail extending the period prior to closing down the student's personal activity into conventional notation. In doing this we may hope to achieve a style of teaching which enables students to critically examine the purpose and scope of the mathematics they meet, whilst at the same time recognising its grounding in their personal experience. In reflecting on a mathematical investigation or on some piece of research, for example, I may offer some comment, which for the time being, encapsulates my understanding. However, as work proceeds refinements can be made. Nevertheless, in order to engage with a mathematical task it is necessary for me to make occasional tentative statements towards providing markers which help me in finding my bearings. In this way I describe, in a personal way, the mathematics in terms of my perceived relationship with it. However, I do this in the language of the society of which I am part and so assert my allegiance.

By moving between the notion of an agreed and common mathematical language (langue) and seeing mathematics as being manifested and shared through individual acts (parole) we emphasise the interpretive dimension of mathematics, albeit gravitating around certain social norms. In meeting mathematics in educational settings and in the world of work an interpretive dimension is endemic in the mathematics itself which cannot be extracted. Hermeneutical views of language do not see communication as passing around the ready made objects of the traditionalists, nor as reconstructing knowledge, as suggested by the social constructivists. Rather, communicating is about operating on someone's knowing. Mathematical statements made during the course of activity provide snapshots along the way and serve to orientate thinking which continues to evolve.

PART TWO

THE CLASSROOM ENVIRONMENT

Let me begin with an anecdote about two ten year old boys demonstrating various skills in a game of “Multiplication Snap” (Brown, 1994 c). Each had a share of playing cards and took turns to place a card on a central pile. My understanding of the teacher’s intention was that if the product of two successive cards was in the range twenty to forty the first person to say “snap” collected the central pile. The game finished when one player ran out of cards.

It soon became clear that if I placed too much concern about what the teacher had in mind I would be distracted from what was really going on. To suggest that adherence to the teacher’s rules would optimise the use of skill would be to ignore a considerable range of talents on offer. A variety of strategies were being employed by the boys towards winning the game. For it was the appearance of winning which governed much of what followed.

One of the key strategies was to slap a hand on the table each time a new card was placed on the central pile. Invariably, the boy placing the new card had most success with this since it arose prior to any detailed concern about whether or not the product of the two numbers was in the required range. There was no shame in asserting an incorrect pair, rather it displayed confidence and engagement. This ritual persisted throughout the game although as time progressed more obvious pairings, such as two picture cards which scored 10x10, escaped the slap. Whilst the initial slap often appeared as a demonstration of absolute certainty both players saw through this. On each occasion the successful slapper reluctantly withdrew his hand if confirmation was not forthcoming in the following few seconds, to make way for “reflection”.

The ensuing period of relative calm permitted the search for some sort of justification - calculators appeared, jottings took place, friends were asked and searching looks were directed towards me (as the absolute authority!). This process of validation was full of tension and any half baked notion was worthy of an airing if only as a holding device. After all, it would appear from the play that any concern for your partner’s ideas distracted you from coming up with your own. Each new declaration was preceded by a renewed slap of the hand on the central pile with varying degrees of decisiveness. Degrees of certainty seemed to displace any sort of right/wrong dichotomy. Uncertainties were often resolved by the loudest granting themselves the benefit of the doubt.

On closer inspection quite a few other strategies were being employed. These included; slapping a hand down as soon as it appeared the opponent was about to slap, offering new interpretations of the original rules, proceeding rapidly through controversial decisions, bluffing, claiming ownership of arguments offered by the opponent, blatant cheating such as changing the order of cards on the central pile. The quest was to convince others rather than to be correct and it was important to present a good case regardless of whether or not you had grounds for actually thinking you were right. The pressure was on to offer convincing arguments and there was no absolute authority available to offer any final confirmation.

The mathematics was inseparable from the social activity which generated it. In social situations generally, negotiation skills and the ability to appear correct are as important as actually being correct. One might suggest that modern day economics has less to do with statistical facts than with assertions of particular interpretations. A recent finance minister in the United Kingdom was sacked for lacking the required political aptitude to supplement his economic skill, (For a classroom example of an economics activity with young children discussed from this perspective, see Brown and Mears, 1994.) However, in classroom mathematics there are many ways of concealing ineptitude (cf. Holt, 1969). But, of course, these strategies should not detain us here since we are concerned with the teaching of mathematics!

In my analysis so far I have introduced some perspectives in which mathematical objects and the perception of them is softened, as the objects and perception of them, evolve together through time. It is this approach which will be taken further in this section. Our scope of interest however, will be a little broader. Rather than restricting ourselves to the individual's experiencing of mathematics per se, we step back a little further towards examining more closely how ideas emerge for the individual in the broader context of the mathematics classroom. In this, the classroom will be understood as an environment of signs, comprising things and people, which impinge on the reality of the individual student and influence the way in which ideas are identified and experienced. Conventional views of mathematical phenomena will not be presupposed, nor will physical embodiments of mathematical ideas be seen as transparent. Also, in line with radical constructivist philosophy I will not be relying on an expert overview of mathematics motivating this task, since such an overview is not available to the learner. I will seek to avoid assumptions about an independent, preexisting world outside the mind of the knower. Consequently, mathematical ideas will be seen as being held in the minds of teacher and students, without the anchoring of "actual" ideas. In addressing these issues I shall lean on the seminal work on social phenomenology by Alfred Schütz (1962, 1967).

A particular focus will be on how physical apparatus and language intervene in the process of developing mathematical thinking. I follow Kaput (1991) in seeing physical instructional apparatus as contributing to the "architectural" environment within which students build their own constructions, where physical apparatus guides thinking in much the same way as furniture guides movement around a room. Similarly, following on from earlier discussion, I see language as guiding rather than holding thinking. I will suggest that the characteristics and relative importances of phenomena perceived by the student in the classroom, evolve through time, and, in due course, some of these phenomena may be treated as "mathematical" as they are seen to be displaying particular qualities. However, even in work presented as "mathematical" to students by teachers, the mathematical qualities may not necessarily be immediately apparent for the student. For this reason, I focus on the initiation for the individual which takes place prior to becoming part of a (mathematical) *consensual domain* (cf. Kaput, 1991). That is, before fully formed ideas have been derived from the complexity of classroom engagement and been understood as being mathematical. I will consider how such ideas develop in the mind of a student, through time, in relation to that seen in immediate perception. This activates a concern that will span the next four chapters where I question both how mathematical thinking develops in time and space to produce language, but also how language is produced to create notions of time and space.

Later on I shall introduce a framework through which mathematical thinking is seen as taking place in the imagined world through the filter of the world in immediate perception, with reference to the work of Schütz and Goffman.. I will suggest that mathematical ideas are contained and shaped by the student's personal phenomenology, which evolves through time. In particular, I will question how students become aware of mathematical ideas in the complex environment of the classroom. Further, I will argue these ideas are never encountered directly, but rather, are met through a circular hermeneutic process of reconciling expectation with experience. A theoretical framework will be offered which accommodates the time-dependency implicit in this. But for now, by way of introduction, I offer some anecdotes from a number of lessons towards identifying and outlining some aspects the composition of the spaces in which teacher and student see themselves working.

CHAPTER 4

SOME LESSONS

In this chapter, I offer some classroom examples of students doing mathematics as a prelude to examining how the student reads the situation they are in and how the significance of the teacher's input shows itself in their activity. I also consider how a teacher's intention is framed in the instructions she gives to students and in the physical instructional apparatus employed. We shall see that whilst the teacher's instructions may appear to be associated by them with very specific actions to be carried out by students, the students' reading and related actions may not be so precise before achieving any ultimate sharing of the teacher's way of seeing things.

Whilst working with trainee teachers in Dominica, I became involved in supervising a project investigating the role teacher's speech has in the management of a primary mathematics lessons. It was addressing the specific issue of finding alternatives to the "chalk and talk strategies prevalent in a country where school based apprenticeship models of teacher training resulted in many teachers adopting styles similar to how they were themselves taught at school. At the beginning of the project it was common among these teachers to have a heavy reliance on their speech, both in their teaching and as a management technique, yet it seemed that much of this speech was ignored by the students, perhaps exacerbated by many children speaking a French Patois as their mother tongue (Brown, 1984, 1987 d). There often seemed to be a considerable gulf between the teachers' intentions as represented in their speech and the actual work being carried out by the students. The teachers on the project worked on the task of exploring the consequences of reducing their own speech in the classroom towards expressing themselves more economically and relying more on other strategies. By observing each other teach and through taking time out to look at their own teaching they became more aware of the way in which speech was used, but also, of the other factors governing the management of the class. This project, and in particular its concern with how classroom participants understand the space they are in, later became the basis of my doctoral dissertation (Brown, 1987 b). Lessons given by some of the teachers involved in this project will provide examples for the discussion which follows. My intention is to trace out some of the facets of the filter which translates teacher intention into responses by students.

Initially, I explore some situations where the teacher's verbal instructions combine with physical materials to influence the activity of the students. My focus here is on how the environment is read as functioning in shaping ideas in the activity of the students. I offer some anecdotes, produced at the time of the research, in an attempt to capture some of the issues that concerned us during the enquiry. They offer brief accounts of interludes within lessons, followed by some discussion of how the activity of students was being guided. These may be viewed as preliminary notes for the more rigorous treatment in the subsequent chapter. The first example features students representing and adding numbers using base ten strips (a home-made 2-D version of Dienes materials). The teacher's requests for students to make specific arrangements are greeted by substantial delays and much deliberation. Precise requests failed to receive precise responses. In the second example students are producing the sequence of square numbers. A precise course of action was prescribed by the teacher, yet the pupils interpretation of this or their inability to arrange the pieces as requested, stood in the way of smooth responses. The activity is shaped by the materials and the teacher's verbal requests but considerable scope for manoeuvre results from both intellectual and technical difficulties. The third example focuses on a counting exercise where students become immersed in the space created through their own actions, as the teacher's intentions become ever more distant. In particular, I focus on how the individual's perceived space is shaped by the actions of others. In the fourth example, a group of students, confronted with a task of ordering sticks by length, are governed as much by learnt rituals as by physical constraints. The final two examples describe college sessions led by me and attended by the teachers involved in the project. The focus in these two sessions was on how the teacher's intention is captured, transmitted and received in language.

Example 1: Addition using base ten strips

A group of six year old students were working on some problems set by their teacher. These involve using "base ten strips" in tackling double digit addition. The teacher's speech was very brief and sparse, consisting, almost entirely, of requests such as "Make 34", "Now make 21" and "Now put them together". The students, it seemed, were expected to make the appropriate arrangements with the strips and then write the sum into their books. It was quite noticeable that few of the teacher's requests were carried out immediately, but rather, arrangements of strips were made after much deliberation. Most striking to me, however, was a situation where the teacher explicitly described the intended procedure to a girl who, after half an hour of moving pieces around, had not succeeded in producing the required arrangements. After the teacher's intervention the girl continued arranging the pieces for a further four minutes before completing three calculations in a matter of seconds. It was unclear what was happening during those four minutes. However, after this time she seemed certain. In response to the teacher saying "Now make 34" we might expect the girl to arrange three strips and four squares. The girl did this and with apparent certainty but four minutes after some explicit direction and a good half hour after the initial request. During this time she moved the strips continuously and, even after the teacher's more explicit assistance, checked that there were ten squares in a strip. All of the activity seemed to be in the vicinity of the teacher's apparent expected response and, in due course, her work finally converged to this expectation.

In the format being followed the teacher made a succession of definite statements each of which seemed to suggest a particular response from the students. "Now make 34" could be seen as being associated with the students making a certain arrangement of strips. The activity observed may have finally included this but it often existed in the context of many other movements and arrangements that also seemed to occur between the teacher's statement and the completion of the expected response to it. The activity was clearly influenced by the teacher's statement. However, the students seemed unable to respond immediately, rather they had to work around it before they could venture to offer the response with any certainty. Meanwhile, the teacher's attempt at explicitness seemed to achieve more than mere mechanical instruction.

Example 2: Building the sequence of square numbers

In the following transcription of a lesson with ten year olds, the teacher is again, intentionally, saying as little as she feels she can (Brown, 1988). I shall focus on the way in which the activity by

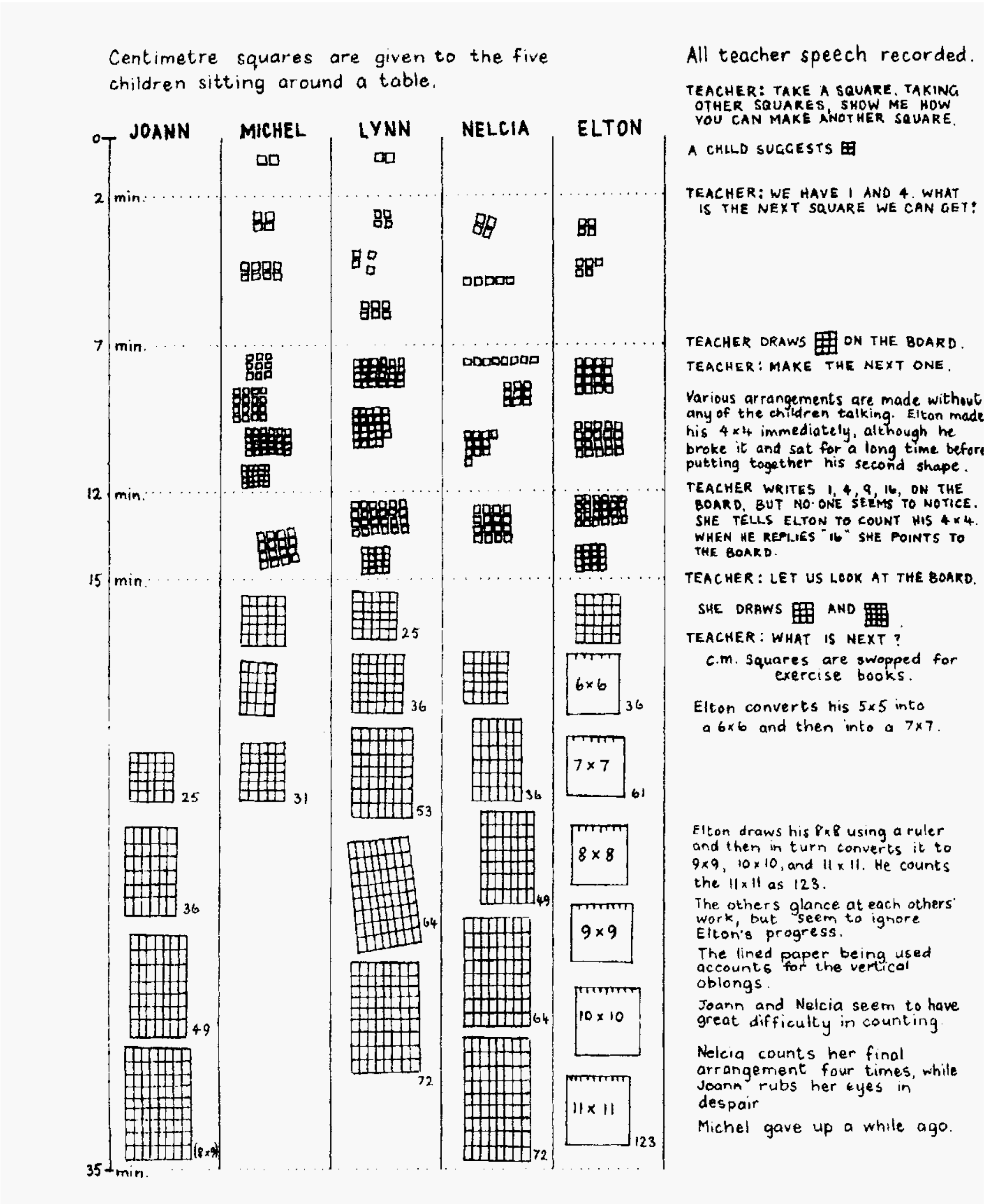
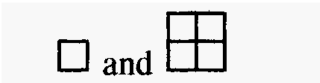

Centimetre squares are given to the five children sitting around a table.
All teacher speech recorded.
JOANN
MICHEL
LYNN
NELCIA
ELTON
0
2 min.
7 min.
12 min.
15 min.
35 min.
TEACHER: TAKE A SQUARE. TAKING OTHER SQUARES, SHOW ME HOW YOU CAN MAKE ANOTHER SQUARE.
A CHILD SUGGESTS
TEACHER: WE HAVE 1 AND 4. WHAT IS THE NEXT SQUARE WE CAN GET?
TEACHER DRAWS ON THE BOARD.
TEACHER: MAKE THE NEXT ONE.
Various arrangements are made without any of the children talking. Elton made his 4×4 immediately, although he broke it and sat for a long time before putting together his second shape.
TEACHER WRITES 1, 4, 9, 16, ON THE BOARD, BUT NO-ONE SEEMS TO NOTICE. SHE TELLS ELTON TO COUNT HIS 4×4. WHEN HE REPLIES "16" SHE POINTS TO THE BOARD.
TEACHER: LET US LOOK AT THE BOARD.
SHE DRAWS AND
TEACHER: WHAT IS NEXT?
c.m. Squares are swopped for exercise books.
Elton converts his 5×5 into a 6×6 and then into a 7×7.
Elton draws his 8×8 using a ruler and then in turn converts it to 9×9, 10×10, and 11×11. He counts the 11×11 as 123.
The others glance at each others' work, but seem to ignore Elton's progress.
The lined paper being used accounts for the vertical oblongs.
Joann and Nelcia seem to have great difficulty in counting.
Nelcia counts her final arrangement four times, while Joann rubs her eyes in despair.
Michel gave up a while ago.
25
36
49
(8×9)
31
53
64
72
49
6×6
7×7
8×8
9×9
10×10
11×11
61
123

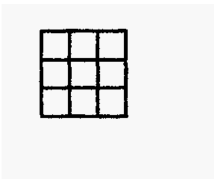
Figure 7

the students forms around her speech (see Figure 7). When the teacher drew attention to

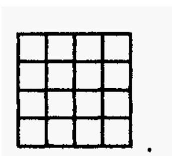


and asked for the next one, all four students simply copied these configurations and then followed this with more manipulation of the pieces producing various non-square arrangements.

The teacher then drew

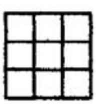
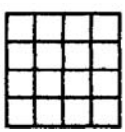


and asked for the next one. Michel and then later Nelcia simply copied this. Only Elton immediately drew



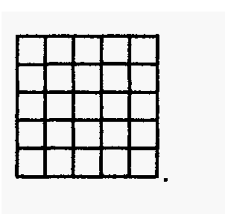
He continues adding squares to this but then remade the 4x4 arrangement.

The teacher wrote 1,4,9,16, on the board but this seemed to go unnoticed. Later on she drew

 and 

and asks “what is next?”.

In due course, all of the students eventually produced

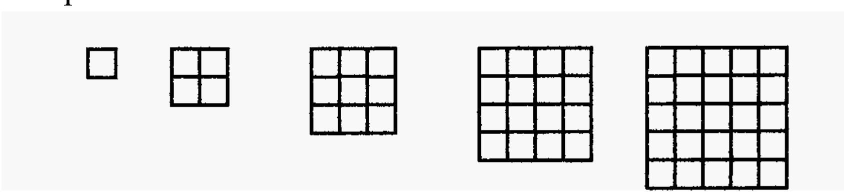


From here there seemed to be some alignment between the teacher's expectations and the students' work. The only remaining deviations from the expected appeared to be caused by difficulty in counting the pieces of the bigger squares and with drawing the squares on paper which only had horizontal lines. The students now seemed able to classify square numbers but the technical problems of counting the squares and drawing them remained.

What we have then, is a sequence of sentences that were spoken by the teacher together with some diagrams that were written on the board. For each student we have some models which, in some ways, track their own sequence of actions. There is an occasional one-to-one association between some of the teacher's sentences and some of the drawings and arrangements made by the students. However, there is quite often a significant time delay between the teacher's sentence and its associated response by the student. Also, I am very conscious that in my own observations I am trying to use my supposed view of the teacher's intentions as my own yardstick in assessing the students' constructions and have not included many that I chose not to see as relevant.

There are two sequences to which the teacher's sentences seem to refer.

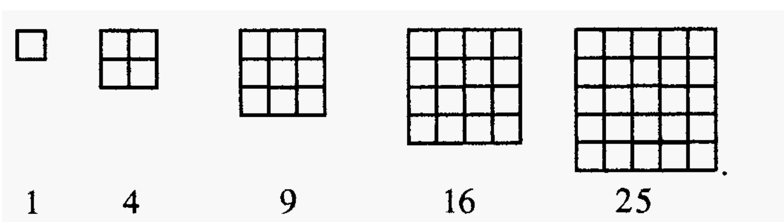
Sequence A:



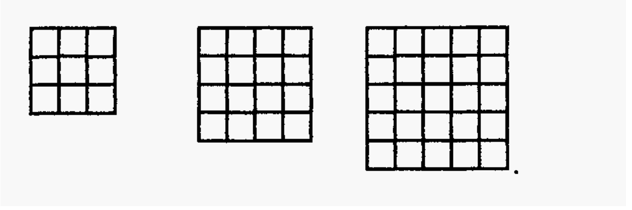
and the sequence B

1, 4, 9, 16, 25, 36, 49,..... .

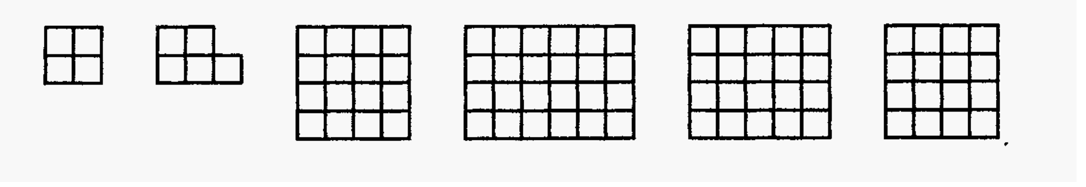
Combining these we have



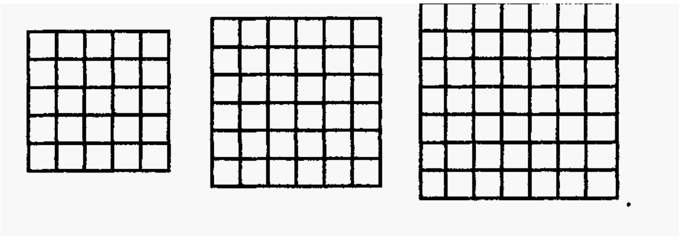
It is this composite sequence that ultimately seems to guide the students' actions in the lesson. The teacher's approach is based on her drawing an element from the composite sequence on to the board and saying “Make the next one”. After a certain point in the lesson it seems that the students are able to continue on their own. The amount of peripheral activity by the student reduces as the task is more clearly understood and as activity focuses around later terms in the sequence. As an example, with Elton at the beginning of the lesson, the teacher sets up the task so that, if her directions are followed, she would expect him to make:



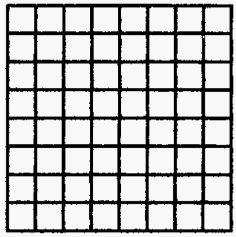
However, Elton makes the sequence:



There are some similarities between the two but these are not especially clear. Elton has made arrangements other than those expected and his attention does not appear to have been focused on to the sequence. He seems to engage in much peripheral activity. However, when the teacher said “5 across and 5 down” Elton does in fact produce:

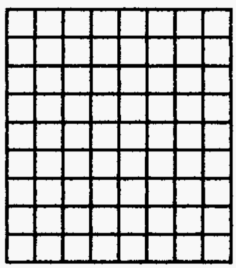


And so it seems, at this later stage of the lesson, with Elton’s responses, at least, the drawings and the teacher’s expectations of them are much the same, with little peripheral activity taking place “off the ball”. Whilst this also becomes true for the other students they also encounter a new sort of difficulty. For example, Joann, Nelcia and Lynn it seems they can imagine



(8x8)

but draw



(8x9).

They do not appear to be confident in drawing this image from their imagination. So although their mental images may coincide with the composite sequence, their drawings do not, or, at least, not without some distortion. However, one has a sense that they are near and that they may be getting nearer to the required sequence.

Example 3: Counting sequences

1, 2, -, 4, -, 6,7   5, -, 7, -,9, 10,-   -, 4,-, 6, 7,- 9, -

-, 11, 12, 13, -, -,

The teacher writes the above on the chalkboard and asks the children what they think they have to do.

Teacher: Find the numbers that are missing.

Roger, Clifford and Chester (all aged six) are given a sheet of paper between them and are asked to work together with only one person writing. Chester takes the paper and copies the numbers from the board. When he has finished writing they all look at the first set.

Roger: Three.......then five

Chester writes these numbers in and they all turn to the second set.

Roger: Do a six. *(Chester writes this in)*

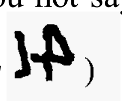
Clifford: Let's count our hands - five add seven
(He is ignored)

Roger: A eight ...eight, nine a ten...

Chester writes the eight and then the eleven. They look at the third set. Chester quickly fills it in himself with the correct numbers. They look at the last set and Chester quickly fills in the ten.

Clifford: Twelve that there.

Roger: A one and a four *(pointing to the space after 13)*

Chester: Clifford you not saying it - a one and a four
(*He writes* )

Clifford: Whey! That wrong side ; is so a four going? Miss!
Chester writing a four like that - the wrong side.
Chester erases it and rewrites it property.

In this extract the teacher does little to guide the students beyond the initial setting up (Brown, 1986). Rather, the lesson seems to take on a momentum of its own as the students engage with the perceived activity. The teacher, by writing on the board, by distributing the sheet of paper and by saying one sentence of direction, has suggested a course of action. The boys show different levels of initiative in responding. Chester chooses to copy the sequence from the board on to the paper and then Roger follows this by suggesting numbers to be put into the spaces. At this point there is no apparent evidence that Chester could proceed alone; he continues writing while Roger suggests entries until Roger hesitates and Chester fills in the eleven. Chester then fills in several more without any comment from the others until Roger comes back with "a one and a four", suggesting an entry in the way he had suggested earlier ones. Throughout this however, Clifford has said very little. Chester and Roger have done various bits and pieces but Clifford seems unable to contribute anything more. His first statement which is ignored does not fit the flow of work as seen by the other two. His second attempt, “twelve that there”, appears to be an incorrect attempt to fill the space, It is only when Chester writes the four back to front that Clifford is able to assert himself - and he makes a meal of it! It is also interesting to note the general lack of overt acknowledgement of each other’s actions as such. The students seem to focus on the immediate situation without any obvious concern for the overall social context or work plan. It is as if they are seeing no further than their own next step but rather acting according to their immediate situations, being aware of the last situation as they left it and of the next as they entered it. Also, since the students were working on a common focus, an action by one of them affected the context that was subsequently worked in. As more actions occur, the introduction by the teacher becomes ever more distant. The students’ current reading of her original actions is through the filter of all the intervening actions in the group since. In general, however, the sequence of situations described here seemed to be consistent with what the teacher had in mind. The following example is rather different, the framing of the activity is proving less successful in keeping the boys on track.

Here, the same students are counting up in twos, having started with 2, 4, 6, 8,....(Brown, 1987 c). Their progress has not been without controversy. The following sequence occurs several minutes into the activity.

Richardson:	27 now. A 2 and a 7. *(Chester writes it.)*
Richardson:	29 then 30.
Chester:	No, is not 30. Is not 10, after 9 is not 10.
Clifford:	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Is 12. 12 more.
	Chester writes 30 since Richardson insists it is 30.
Richardson:	32 now.
	Chester writes it as 23 - a common mistakefor him
Richardson:	Whey! *Chester writes 25.*
Richardson:	Whey! Is not that. All what that there already, you putting!
	They begin to count again but get stuck where they stopped previously.
	Chester writes 45.
Richardson:	70 now.
Chester:	How 70 going?
Clifford:	A 4 and a 6.

Richardson: A 4 and a 6 *(sarcastically)*. 70 that should go there.
Chester writes 16.
Clifford: A 6 and a 1.
Richardson: 17 you should put there, the other one is 19.

Here it seems clear that the boys are affecting the contexts for each other's actions. They are responding to each other's contributions and so affecting each other's contributions. For example, Chester's actions are usually a response to Richardson. They seem, however, to have completely lost touch with the direction anticipated by the teacher and their progress now has a life of its own; a dreamlike momentum that has dissociated itself from the ties of "common" sense. They are playing and they probably know they are, but their contributions are not random. Actions that seemed appropriate were made into the immediately perceived situation. They started with the sequence 2, 4, 6, 8,... but their sequence of actions has taken them well away from it. Their memories of their starting point seem vague, seen, as they are, through the sequence of activities that led them to where they are now. The context to which they now bring their memories of the teacher's introduction has shifted radically, undermining her intended structure.

Example 4: Ordering sticks

Eight bamboo sticks of varying lengths between 8-14 inches are distributed to each group. We rejoin the same group of six year olds.

Chester: See the bigger one.
Teacher: Put all the sticks together.
Chester: Put the largest one.
He seems to try to put them in order.

Clifford takes one away.
Richardson: Put it down. Take the smaller one.
Chester: I've got five Richardson.

Chester puts three in order of size.
Chester : You have plenty. Give Roger. (to Clifford)

After a few minutes the teacher arrives.

Teacher:	All the sticks must lie this way.

This seems to be followed by a lull in activity.

Teacher:	Start with the longest first.

Immediately hands become entwined in apparent activity but this only leads to disarray. Chester picks up the sticks and holds all of them in his hands.

Teacher:	Put your sticks down. Chester. Fix these three children.

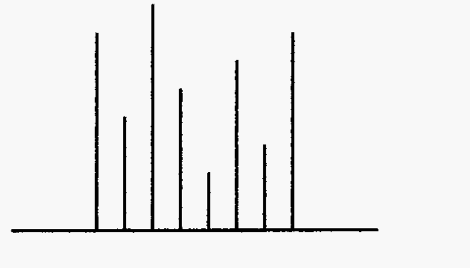
Chester arranges three children in order of size.

Teacher:	How did he fix them?

Some children seem to indicate that the tallest was put first.

Teacher:	Yes he placed the tallest first. See if you can fix your sticks according to height.

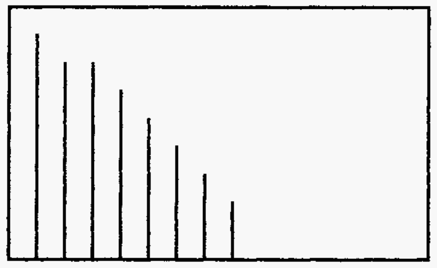
The teacher draws a chalk base line for this group. Richardson and Chester put sticks next to it.



Teacher:	Look at your sticks very well. Did you arrange them according to height?
Richardson:	That one taller than that one and that one taller than that one.

Four sticks are arranged in order of height, apparently by Richardson and Chester, but then there is disarray again. The other groups seem to be having similar success. The teacher holds up a card on which she has drawn eight parallel lines in order of length.

Teacher: Look at this. See if you can arrange your sticks like this.



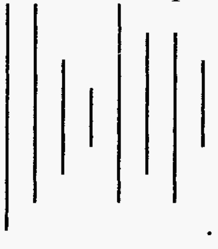
The three biggest are put in order.

Richardson: Put the bigger one.

Chester: You should put that there. *(Disarray follows again)*

Richardson starts concentrating again and studies the teacher's card. Chester and Clifford start chatting. Roger stares around the classroom which has been his main activity all lesson. Clifford pushes all the sticks together.

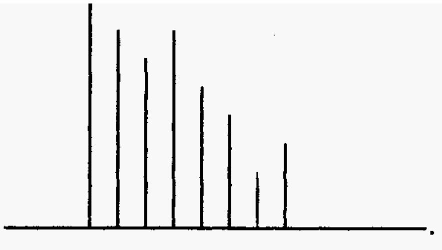
Richardson places



Teacher: Let's stop. *(Chester pushes all the sticks together)*

Richardson: Don't do that.

Richardson with some assistance from Chester places



The teacher seeing this brings over her card.

Chester then places

Chester: That one longer. That one the shorter.

Teacher: OK. We'll try again tomorrow. *(They do!)*

Here, the teacher did very little to introduce the lesson to get it started. She distributed the sticks and waited a few minutes until some sort of activity had emerged before she went over to the students. It was quite some time before she herself did anything to clarify her intention in setting the task, by which time the students had already started work on what she apparently had in mind. The distribution of the sticks seemed to be enough in itself to get the activity started. Meanwhile, the students were in a lesson which they knew to be concerned with mathematics. "Mathematics" was written on the timetable for that time of day. The teacher finished the previous activity and announced that it was time for maths and she had written "Mathematics" on to the blackboard. I was present and it was known that I only attended maths lessons. With their experience of mathematics lessons they had certain expectations of the sort of things they were expected to do. The teacher in distributing the sticks was suggesting that they were going to be doing some activity with them. The teacher was ordinarily fairly quiet, leaving the students to get on on their own, so her quietness in this instance was not in any way unusual. The boys were together in their usual work groups for mathematics. Furthermore, in this group of students, it was usually assumed that Chester would start. So although the teacher had not set a specific activity, the space that each student perceived around them was fairly well structured, such that their behaviour was guided within fairly tight boundaries. They were looking for signs that suggested the task and in this case all they needed to know was what to do with the sticks.

Now it might seem there are only so many things you can do with eight bamboo sticks of varying lengths, especially in a mathematics lesson. The teacher had started the momentum by giving out the sticks. The boys responded to this momentum by moving the sticks themselves; each new arrangement initiated by one affected the context for the others to act in. The possibilities that the boys seemed to consider were - to share the sticks out between them; to compare the number held by each boy; and to put them in order of length. In this general activity the boys seemed to become aware of various properties of the sticks. After a few minutes the teacher said that all of the sticks were to lie in a certain way starting with the longest. This seemed to be enough to have the students working towards ordering the sticks. The teacher's words seemed to make sense to them in the context of their sorting even though she had not given directions that clearly defined the task. She was still being vague when she asked Chester to "fix" the children. But by then most of the students seemed to be aware of what the intended task was. It is only after this that the teacher first says a sentence which describes the task. It was as if the teacher allowed the activity to gain momentum before she intervened in any firm way. The teacher's words assumed the existence of the activity rather than defined it, which served to present the activity without distorting it in description. The context into which her words were issued seemed to complement them and made them powerful in confirming the nature of the activity. Her words seemed consistent with expectations.

The task itself had a very precise solution but this work being done in respect of it took place in a field of research rather than on a linear course towards the solution. In this respect it could be seen as open. In the subsequent lesson, however, although the boys managed collectively to get the sticks in order, in an arrangement on the floor, and recognised it as the required arrangement, they were unable to remake it once it was broken by a passing boy who accidentally kicked it. So, although in a sense, they saw the closure of the activity, the task was by no means finished. This seems to suggest a sequence of overlapping sub-tasks:

1. They explore the situation and look for signs that suggest a possible task.
2. They formulate a problem and imagine its solution.
3. They look for techniques to achieve this solution.
4. The solution is confirmed and given this confirmation they continue to look for a method of achieving it,

In this lesson, it seems that both the teacher and the students were acting according to how they perceived the task, even though the teacher had not been explicit about what she intended. Rather, having distributing the sticks, she waited before engaging in the activity pursued by the pupils. It was as if the task was assumed to be proceeding, with everyone acting accordingly, and with the nature of the task emerging as the actions suggested its form. The momentum of the lesson was indicated by the suggestion of boundaries being passed through (Leach, 1976), and everyone seemed to be acting on the assumption that this change in the situation, that is, the change in their context for action, was occurring. Various things orchestrated the transition from non-maths to maths. Boundary rituals that had become institutionalised by their familiarity served to indicate the nature of the forthcoming thirty minutes, Expectations and motives would have developed in the light of this transition. The distribution of the sticks to each group of four students further restricted these expectations suggesting to each individual more about the type of mathematical activity and also that it was to be done with three other students. After this boundary at the commencement of the lesson had been passed, the concerns of each individual would have undergone a change since their motives would now be in respect of this new context for action. Further, their chosen action in respect of this new situation, was restricted, or at least influenced, by the suggestion that it was to be based around an activity with bamboo sticks. So although the teacher's actions concerned with managing the activity had been minimal, I conjecture the space perceived by the students was fairly restrictive, and only a limited field would have been suggested to them for possible action. Assuming that the actions undertaken by the students fell within this field, the teacher would be in a strong position to negotiate with them over the nature of the work to be done. Her actions in line with her assumption that the students were doing that intended by her were, it seems, enough to confirm the nature of the activity to the students.

Example 5: Imaginings

"You are standing at a comer in a square room, facing into the room Slowly, along the wall to your left you start walking until you are halfway to the next corner. Youstop. Youface right, and then walk until you reach the wall. Then stop. Face left and walk in that direction untilyou hump into a wall. Nowface left and walk until you hump into another wall. Now face the corner furthest

from you and walk towards it."

I constructed the above set of sentences for a group of teachers following an in-service course in Dominica (reported in Brown, in press a). My intention was to explore the nature of giving directions. I was not seeking to catch the students out, rather I hoped to choose a reasonably unambiguous set of instructions that would allow us to agree on the path I had described. I anticipated this leading to a discussion about how we communicate mathematical ideas in words. I asked the students to close their eyes and I read out my script very slowly. After a second reading students were asked to draw the path they imagined on the board. Their drawings are reproduced Figure 8.

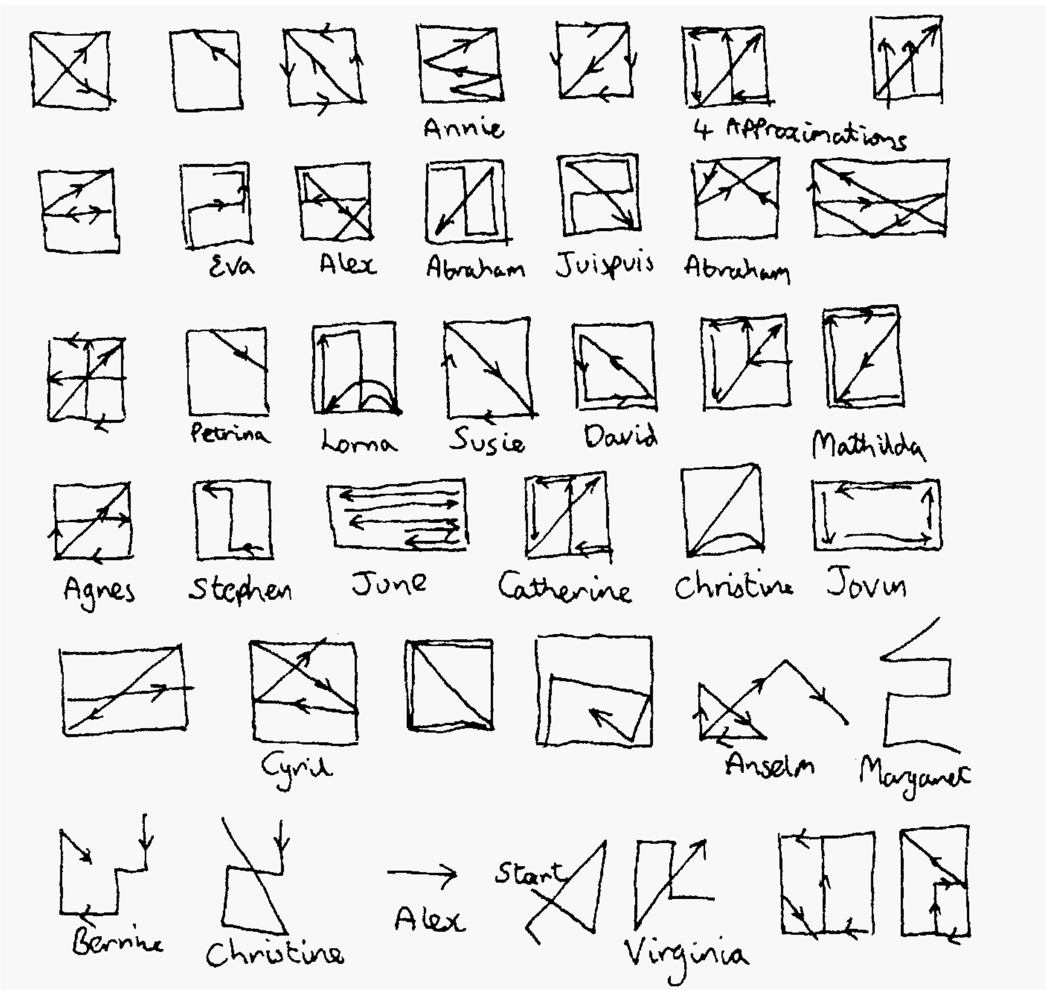


Figure 8

In another example I read the following :

"Imagine a large black circular screen. At the top of the screen is a white dot. Slowly, clockwise, the dot starts to move around the edge of the screen. 3 O'clock, 6 O'clock, 9 O'clock and back to 12 O'clock. The dot stops. It now carries on in the same direction until it reaches the far right hand side of the screen. It stops. To the left, it moves until it reaches the edge of the screen again. It stops. It now moves South-East until it reaches the edge of the screen. It stops. It now moves vertically upwards until it reaches the edge of the screen."

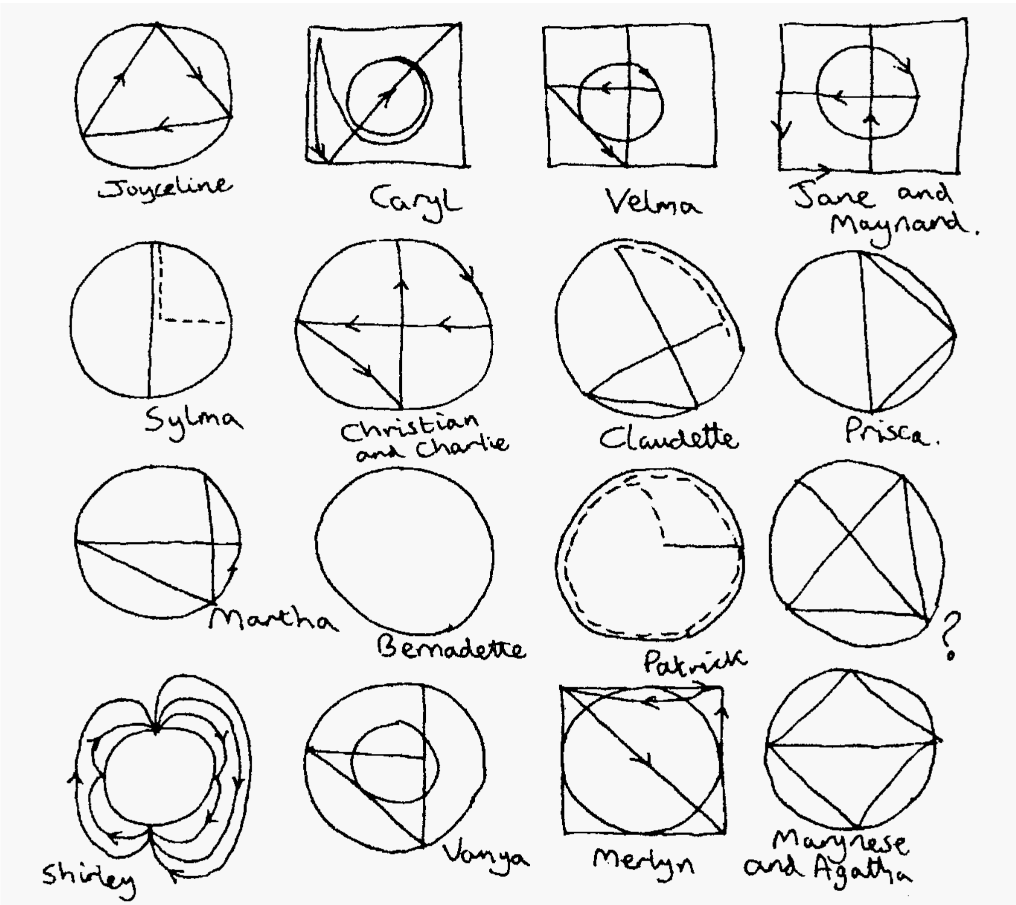


Figure 9

I am not sure whether I am surprised about the variety among the results. I came up with all sorts of possible explanations - the instructions were unclear or ambiguous, the teachers had difficulty with my London accent, their background in geometry was fairly weak. I can carry on speculating but somehow this seems to be missing the point. There are bigger issues in the background; issues not far from those being addressed in my original intentions behind the activity, namely the capturing, transmitting and receiving of mathematics in language.

When reading out this “Imagining” I was attempting to use language that I felt would be comfortably within the vocabulary of my students. I was perhaps insufficiently aware of the obvious ambiguities. However, there were numerous deviations from what I had in mind in the teachers’ responses (Figure 9). I discussed with the students the type of difficulties they experienced and also attempted to identify the roots of alternative interpretations among the majority who felt they had drawn the “correct” image. At no point did I reveal my image, nor did anyone actually ask me to.

When quizzed the teachers reported the various difficulties:

1) “South-East’’ was seen as being any direction between South and East, which assumed that the term was more tolerant than its strict navigational definition of being 135 from North.

2) For “East”, some students went “West” thereby assigning a completely different meaning to the term.

3) While still thinking about a particular sentence, the next sentence was being read. In this way some sentences were missed causing disjointed sentences to be articulated. This resulted in the complete paragraph having a different meaning.

4) The whole reading was a blur and snippets were taken at random and articulated. This again resulted in the paragraph having a different meaning.

5) Some students expressed a difficulty in seeing anything at all. Here they experienced an inability to engage with the words.

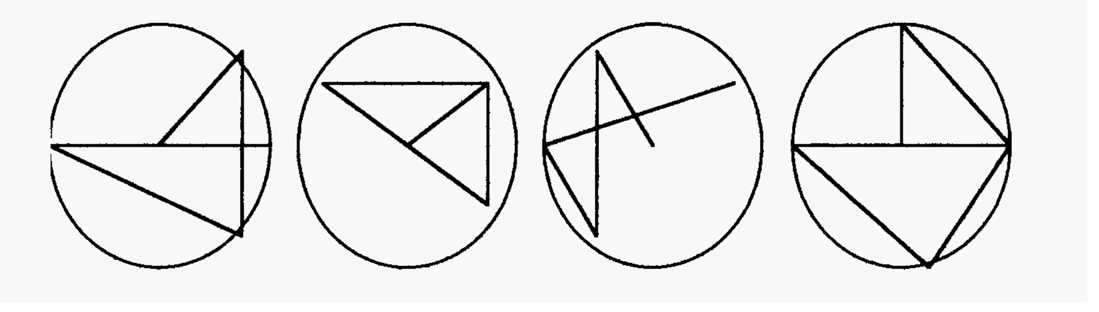
6) Some sentences were forgotten causing a similar result.

7) Others experienced confusion over their perspective. (In

"Logo-speak" - Am I the turtle or do I have an aerial view of the turtle?) Thus by taking a different perspective the words were seen in another context which changed their meaning.

I was careful not to say that my image was right. It was simply the one that I connected to my words. If anything, my words were wrong in that they failed to communicate my image. I was made very aware of this when the students read out some of their own Imaginings. Here is one that was read by, Helen, one of the teachers.

"Imagine a large circular screen. There is a dot at the centre of the screen. It moves North-East to the edge of the screen. It goes down to the edge of the screen. It goes North-West to the edge of the screen. It goes right to the edge of the screen."



Veronica + Helen   Myself + Huguette   Marcella   Angelina

I protested vigorously about the correctness of my version only to have the teacher quite rightly assert "It's my Imagining so I know what I'm talking about, so that's your problem." Whilst the Imaginings were designed to be comprehensible, not one of us succeeded in creating one that everyone agreed on.

Example 6: Giving directions: "Leading the blind"

Twelve teachers hold hands and form a circle. Another is blindfolded and stands in the middle. I ask the twelve to give instructions to the blindfolded person so that she described a figure four with her feet. The only directions to be allowed are "Clockwise", "Anti clockwise", "Forward" and "Stop". This requires the teachers to agree and rehearse a set of instructions to be given to their blindfolded colleague.

After a moment's thought the teachers start pointing to possible routes within the circle. They discussed whether it should be a 4 with an open top or a closed top.

Everyone was obliged to remain in the circle as they described proposed routes with words and their hands.

"She start here and goes so."
"She goes clockwise then to Cleo. Then so."
"Where she go then? To Joycelyn."
"Too wide. She come to me, then anti clockwise."
"Then to Rosanne. So there. To she. Clockwise."
"Over there. To me then anti clockwise."

This sort of talk goes on for a few minutes until they decide they are ready to start. The four instructions are henceforth the most common words, but despite protestations from me, occasional discussions broke out to decide the next command.

For the letter "M".

"She start here. Then so. To you."
"But that's a W."
"A W to us but an M to them."
"Then to...." etc etc.

Finally, the directions were changed. "Come to me" is now the only command allowed. Not even "Stop" is permitted.

"Draw a curved figure 3"

"He face this way. If Mr. Brown say it."
"Then Joycelyn, then Aurelia, then Gloria."
"Then me, then so to Veronica."
"That back to front."
"To us but not to them."
"That too sharp. She say it first. Then he."
"Right. Let's go."
"Come to me", "Come to me", "Come to me", ...
(amidst frantic pointing!). See Figure 10.

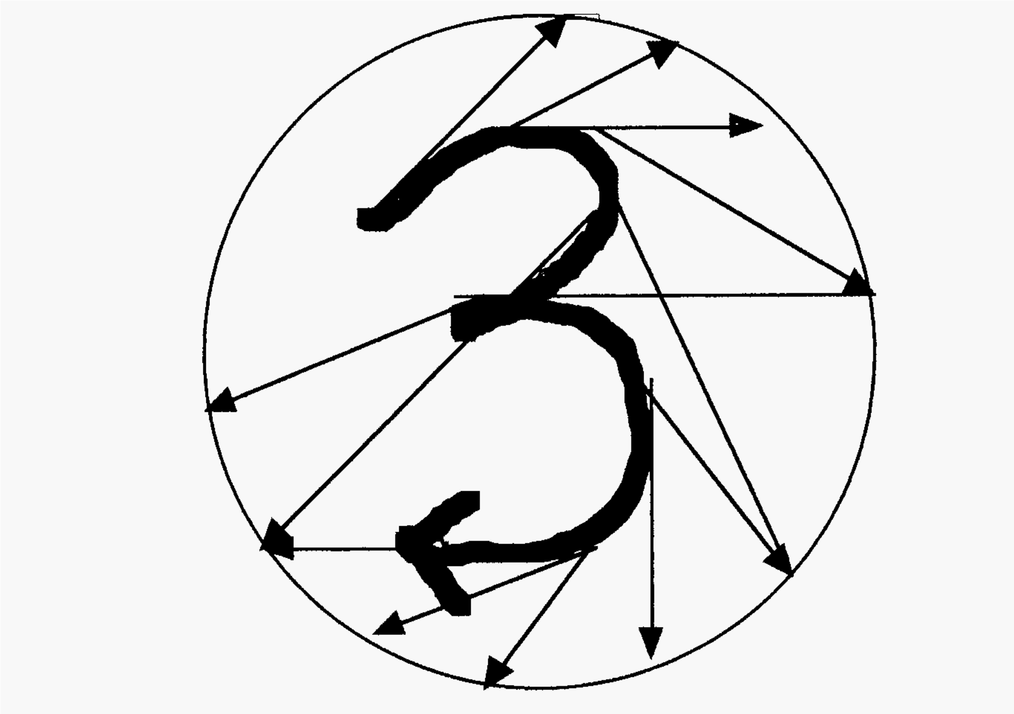


Figure 10

In all of these exercises it was interesting to note how the exercise was initially regarded as a personal problem but slowly thoughts were shared, first with one or two others, but then with the whole group as the communal nature of the problem revealed itself. Throughout the blindfolded person relied entirely on what she heard. In all three of the cases described we are dealing with precise statements that have little tolerance in that any deviation from the expected response results in the person performing the activity being incorrectly positioned for subsequent directions.

In this activity one could classify the sentences spoken according to whether they should be heard by the blindfolded person or not. Since this person was unable to see, the only sentences of any use were the legitimate commands. However, for those in the circle there were different levels to the speech. There was an exact course to be imagined which the blind person would follow. However, this had to be jointly conceived by twelve people and then communicated to another by means of specific commands. Initially it appeared that the twelve people in the circle were all imagining a figure four but some were open at the top whilst some were closed and most people seemed to think that they themselves would see it as upright.

Eventually it was realised that it could not be solved by individuals and so they started talking with each other. At first they spoke quietly with the person next to them in the circle. I was not allowing them to move. With this person next to them, it was fairly easy to agree since they were facing the same way. However, all twelve needed to agree but they were all facing in different directions.

SUMMARY

These examples have highlighted aspects of the physical and social environment in which students and teachers face their respective tasks. I have sought to outline briefly how both social and physical factors can function in framing the specific spaces students encounter. The "architectural space" and the way others work within it govern the way I act myself. Nevertheless, whilst linguistic, physical and imagined phenomena function in restricting the possibilities open to the individual student, they also function in framing and highlighting possibilities. I hope to develop a framework for analysing these more rigorously in the next two chapters. However, before concluding this chapter I will summarise and develop a few of the points made.

Working in space shaped by words

I am reminded of Wittgenstein's exposition of language games (1958, pp. 2-7). In an example he describes a micro-world comprising people each with a limited vocabulary of just a few words. One of the characters is a builder whose vocabulary includes words such as "brick", "block" and "slab" which he uses in giving instructions to an assistant to pass him bricks, blocks etc. If he says "brick", his assistant gives him a brick. As such the request is unproblematic and apparently results in an immediate response, rather like a composer writing a musical note which requires a specific response from an experienced musician. When however, in our first example, the teacher says "Now put them together" the response is not immediate. There is a need for the student to spend a little more time deciding what is required. I conjecture that in this particular example more time is needed for the teacher's words to connect with the students' experience of the physical pieces. The student needs to go to and fro a little before she can take a stand. The statements made by the teacher herself suggested actions and presupposed students who would perform them. The student was required to interpret the statement and then perform the actions consistent with it. However, both the interpretation and performance may require skills outside the person's experience. So although the statement and expected response to it might both seem unambiguous, the actual response may be obscured through these interpretive and performative layers.

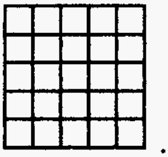
Similarly, in the imaging exercise, my attempts as teacher to share a mental image with my students through the medium of words was frustrated through the inevitable differences in the way people associate words with physical and mental phenomena. Whilst words focus my thoughts in certain ways they are not fully deterministic. I always need to engage in an hermeneutic process of reconciling the words with my experience before reaching some sort of closure which enables me to act. Understandings of particular terms are constantly modified as the contexts within which they are seen increase, and it is unlikely that the contexts for any two people are the same. As the complexity of a statement increases, misinterpretations of the types described above will increase. In attempting to achieve clarity there is a temptation to suggest moving towards a perfect description each time. However, as Polanyi (1962, p. 119)cautions,

> ..the gain in exactitude, (when the complete formalisation of a proof is attempted in mathematics) resulting from a stricter elimination of ambiguities, is accompanied by a loss of clarity and intelligibility.

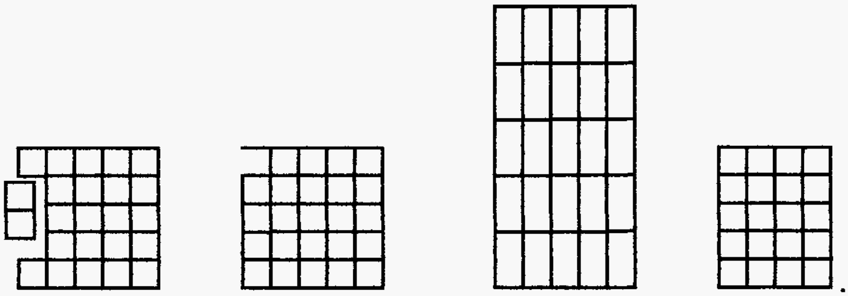
Also, the clarity of a statement can only be checked by consulting those to whom it is addressed. The production of meaning always requires negotiation between the speaker and interlocutor and the speaker cannot assume, in advance, the outcome of that negotiation. The speaker has to declare himself before the negotiation can begin. In giving instructions in a stressful class one always relies on saying things again, in a different way, responding to questions, emphasising difficult bits; so that one might suggest that there is never any such thing as a "clear instruction".

Working in space shaped by physical materials

In the square number sequence exercise the physical pieces guide but do not determine the response. The activity is based around the generation of a sequence of numbers 1,4,9,16,25,.. which is being associated with a sequence of patterns made out of centimetre squares. The students' responses seem to "hold" loosely around this sequence as anticipated by the teacher. The teacher's task seemed to be concerned with enabling students to create this sequence. She attempted to achieve this by focusing on the construction of successive terms. The activity by the students however, wavered around the generation of these sequences, serving to disrupt any direct association, in the student's mind, between the teacher's statement and the student's performance of it. There were often long time delays, sometimes of at last five minutes, between the teacher's statements and the responses to them by the students. This very wavering surely influences any subsequent idea held about the nature of the sequence of square numbers. It will only be seen as a sequence comprising stable terms after it has been experienced as a wavering set of possibilities. The response is not already constructed as is the "brick" in Wittgenstein's example. The search for the term after the 4x4 square, for example, involves surveying a field of close possibilities. The specific qualities of the 5x5 term may only be appreciated when seen in relation to other arrangements with some similarities. The activity seems to be "held" in a field around the teacher's speech, in the context of other constraints perceived by the students. It looks as though the work being done by the students depends on how they interpret the teacher's directions and on their technical ability to carry them out. Activity does occur in response to the teacher's statement but this activity need not be as well defined or as specific as the teacher might suppose but rather the activity occurs, or polarises, around the directions. It is as if an exact directive statement from the teacher does not so much prescribe a specific action by the student but rather frames a field for scrutiny to be investigated with the aid of the materials. For example, at one point the teacher seemed to be focusing on



However, among the students' drawings around the classroom as a whole I noticed a variety of approximations, for example,



These may or may not themselves fix the correct configuration in the student's mind but, in any case, may help guide towards it, once cognitive and technical difficulties have been reconciled.

Working with imagined constraints.

In most mathematics lessons there are conventional ways of behaving and conventional ways of using words. In certain situations the teacher is expected to do certain things. Forexample, the traditional piece of advice on classroom discipline, given to new teachers of "starting off firm and easing off later" supposes that by creating the expectation of the teacher being severe, this is sufficient to maintain order in the long run. As another example, if, as a mathematics teacher, I say "square" I might suppose certain conventions about the sort of square pictured by my students, the view we have of it, the sort of uses to which it might be put etc. If my students pictured the most outrageous possibility for every word I say, little communication would take place. Repetition is an essential requisite of knowing. We know things through characteristics that reoccur - we may choose to assume the most conventional unless we are guided elsewhere.

In the lesson with bamboo sticks the students were, to a large extent, guided by their expectations rather than by tangible constraints. They had learned to read the classroom situation in way not unlike how one might read verbal instructions. The classroom environment had become coded so that actions by the students became triggered by certain sequences of events. The lesson took place about six months into the project and the character of this teacher's lessons seemed to have changed during this time. In my first observation of this teacher there was no speech between the studentds whatsoever and the teacher emphasised her own speech as her main strategy both in her teaching and her classroom management. In considering successive lessons of hers in the reports, it seems her reliance on speech decreased through time, as her awareness of other techniques developed and also, as the speech of the students developed. As elements of her lessons became routine, they could be assumed without being stated. In this way, the overt characteristics of interactions, such as those pertaining to the rituals announcing the lesson, changed through time. The tangible markers became no longer necessary, or rather, the physical markers became "metaphysical", in Collingwood's sense (1940, pp. 58-77). For Collingwood the term is understood historically, where current action with "metaphysical" phenomena is understood in relation to past action with corresponding physical phenomena, such as a verbal instruction. The students, for example, are dealing with the expectation of the teacher saying things rather than with the teacher <u>actually</u> saying things. The nature of pupil responses change as they become governed, to a greater extent, by such metaphysical phenomena. In the past the boys may have responded to certain words from the teacher in a particular way. However, as they became familiar with ways of working their actions occurred without the teacher speaking so many words as before. From the point of view of the boys, their context for action, or the space they perceived themselves to be acting in, developed in the light of the teacher failing to continue certain contributions that she had offered in the past. In a sense the actions they are to take are still suggested, but somehow this suggestion seems softer and the implied response seems less rigidly delineated. In such a situation, it seems that the specific tasks within the activity (including those ordinarily assigned to the teacher) become less clearly defined. Such a withdrawal by the teacher provides more space for the students to take more responsibility in the tasks they perform. Their spaces, despite having fewer tangible constraints, remain complex. This very complexity, however, might be seen as offering more freedom to the students in their choice of action, but in a space conditioned as if a palimpsest flavoured by explicit structures from the past.

Working in the space left by others.

A teacher might intentionally frame a particular task for a student that governs the space the students find themselves working in. Within joint work with other students however, each individual will carve out or get landed with a particular situation to work with. To some extent, each student works in the space left by his peers. As new actions occur, situations arise which suggest subsequent actions. This both constrains and enables certain possibilities. For example, in the group of boys described in the third example, Chester generally takes the lead in responding to tasks set by the teacher. This results in the others following and thus experiencing a different sort of task to the one tackled by Chester. Poor Clifford seems to get totally marginalised as he attempts to squeeze in his own contributions among the more forthright assertions of his colleagues.

Working in space constrained by the point of view of another.

It is possible to work in the space left by others without being too concerned about why they left the situation in such a state. Sometimes however, we are also concerned with the intention of others. In the “Leading the Blind” exercise there was a need to understand how others were using words and a need to know how oneself was being understood. The commands given to the blind person are precise statements suggesting a well defined response that has only one “correct” interpretation. However, this correct interpretation comes into existence only after the twelve in the circle have finished their negotiations. Furthermore, the correct interpretation is subject to reexamination after each time the blindfolded person follows a command. For example, the blindfolded person cannot be made to turn precisely enough, or her head and feet face in different directions, or she does not respond to the commands quickly enough, or those giving commands cannot estimate with sufficient accuracy the speed at which their commands will be responded to. All sorts of things could result in the blindfolded person not being where she is meant to be with consequent adjustments to the plans being made necessary as the exercise proceeded. The imagined exact path exists in a field of negotiation. The twelve must agree on where it is but this agreement will have some tolerance. They must then agree on the words to describe it and on a timing schedule for these words that is tied in with the perceived response of the blindfolded person. That is, the twelve people need to agree on how the blindfolded person will interpret them and how she will act as a consequence. This can only be achieved by each person de-centring to understand the perspective of others involved.

CHAPTER FIVE

THE PHENOMENOLOGY OF THE MATHEMATICS CLASSROOM

This chapter's primary purpose is to offer some preliminary work in theoretising the individual learner's perspective in mathematics lessons within a model derived from Schütz's seminal work in social phenomenology (for example, 1962, 1967). Here, the mathematics classroom is seen as an environment of signs, comprising things and people, which impinge on the reality of the individual student (cf. Brown 1996 c). The chapter introduces a framework through which mathematical work is seen as taking place in the imagined world through the filter of the world in immediate perception. This provides an approach to structuring evolving mathematical understanding. It is suggested that mathematical ideas are contained and shaped by the student's personal phenomenology, which evolves through time. Further, I argue these ideas are never encountered directly but rather are met through a circular process of reconciling expectation with experience.

In particular, I examine Schütz's framework used in describing how an individual experiences their world, as an approach to understanding how the student experiences the mathematics classroom. The focus in this paper is on the socio-cognitive aspects of learning mathematics seen from the individual learner's perspective as he builds an understanding of mathematics. Seeing a student as an insider of a particular way of life, I employ this perspective as a basis for offering a description of the process through which he develops mathematical ideas. It is this perspective that will be used as a home base in this enquiry rather than any sort of mathematical framework. That is, we shall concern ourselves with the task of the novice as he sees it, moving from a state of relative naivete, without the benefit of the expert mathematician pinpointing for us the mathematical objective governing the teacher's intention. From the outset this chapter should be understood as a one-sided enterprise, focusing on the insider point of view. In line with radical constructivist philosophy I will not be relying on such an expert overview of mathematics overseeing the students' work, since this is not available to the learner. I will be proceeding as if there is no "independent, preexisting world outside the mind of the knower" (Lerman, 1989, p. 211), where mathematics can only ever be perceived from particular positions and perspectives by observers with individual interests, from specific historically and culturally determined backgrounds. Whilst certain perspectives presuppose a social plane capable of producing a social view, social phenomenology focuses on the individual's experience of this social plane.

In this chapter I examine an approach to describing how individual students create mathematics in the physical and social situation they inhabit. Mathematical activity is seen as mediating access by the individual to any supposed externally defined objective mathematics. Extending an earlier metaphor, I suggest that their task is to identify (or even build) the furniture as well as find their way around it. Conventional views of mathematical phenomena are not presupposed, nor are physical embodiments of mathematical ideas seen as transparent (cf. Voigt, 1994, pp. 172-176). Rather, I build a framework for describing how these phenomena develop in the mind of a student, through time, in relation to that seen in immediate perception. I suggest that the student faces a whole variety of things and people which hold his attention in different ways. The characteristics and relative importances of these things, as perceived by the student, evolve through time and, in due course, some of these may be treated as "mathematical" as they are seen to be displaying particular qualities. However, even in work presented as "mathematical" to students by teachers, the mathematical qualities are not necessarily immediately apparent for the student. This chapter focuses on a theoretical framework for describing mathematics which accommodates the shifts in form and meaning that mathematical notions undergo in the mind of the individual. The influence of the work of Goffman (1975) and Schon (1983, 1987) and their notions of *frame* and *re-framing* will be evident in many parts of my discussion.

In the first part I introduce the notion of "personal space"; the space in which an individual sees himself acting. This is derived from Husserl's Cartesian phenomenology and developed in relation to Schütz's extension of this work. I show how it can provide a model for describing how students proceed through the classroom environment of phenomena towards establishing mathematical sense. Mathematical ideas are seen as developing for the individual within activity, where activity is seen as being "held in" by various kinds of constraints; imagined or real, seen or unseen, some imposed by the teacher, some by other students, some by the physical environment and some by the student herself. As such, his world is captured in an evolving phenomenological frame, where there is a mutual dependency between the overarching frame and the components within it. The effect of these various constraints on an individual student depends on how he interprets and responds to them. The negotiation of these very constraints and the identification of the components of this space result in mathematical ideas being shaped in the mind of the individual. I seek to illustrate this process with an example of some students working on a mathematics task where the notion of "the line of symmetry" is embodied in some physical apparatus.

In the second part I introduce Schütz's theoretical structure. This model provides a framework for differentiating between the world as seen in immediate perception and the world as interpreted as a space for action (physical or mental). I also demonstrate how this provides a useful mechanism for structuring time and change. Following this model I suggest that the individual acts in the world he imagines to exist. I further suggest that mathematics resides in this imagined world and is in an interactive relation with the world of surface appearance. I develop this discussion in relation to the lesson on symmetry and show how the physical apparatus employed in this lesson can be seen as anchoring, although not determining, the students' mathematical constructions.

In the third part, I develop the discussion by proposing a mismatch between the individual's expectations and experience. Whilst the individual might (voluntarily) act in the world as they imagine it to exist, the world may resist these actions in an unexpected way (involuntary response) and so cause a shift in the way in which an individual perceives the world. This extends to the individual's use of a mathematical idea. In this process I suggest that the individual never reaches a final definitive version of any mathematical idea, but rather, is destined to be always working with his most recent version.

PERSONAL SPACE

In a classroom situation each person is acting according to how the world appears to him. In this section I focus on the student's insider view of his classroom situation. I wish to introduce a notion of *personal space,* an extension of that which Schütz (1962, p. 224) calls the *world within reach;*

the stratum of the world of working which the individual experiences as the kernel of his reality.... This world of his includes not only Mead's manipulatory area (which includes those objects which are both seen and handled) but also things within his view and the range of his hearing, moreover not only the realm of the world open to his actual but also the adjacent ones of his potential working. Of course, these realms have no rigid frontiers, they have their halos and open horizons and these are subject to modifications of interests and attentional attitudes. It is clear that this whole system of "world within my reach" undergoes changes by any of my locomotions; by displacing my body I shift the centre 0 of my system of coordinates, and this alone changes all the numbers (coordinates) pertaining to this system.

So viewed, the notion "personal space" lean's firmly on Cartesian notions as developed by Husserl. A unified subject is implied; a thinking subject who therefore is (Descartes' "cogito ergo sum"). This is an idea treated with a certain disdain by post-structuralist writers insofar as it supposes any "completed and finished identity, knowing always where it is going" (Coward and Ellis, 1977, pp. 108-109). As I have indicated Derrida would reject the binary opposition between individual and social perspectives. Lacan (1977, pp. 1-7), meanwhile, stresses the importance of Descartes' notion, but places much more emphasis on the formation of the thinking subject in the reflexivity of the thinking done. Nevertheless, the thinking subject may not be aware of this theoretical perspective on his actions and so assumes he has more control over his own destiny than may be supposed in post-structuralistformulations. It is this personal perspective I wish to examine now.

In my formulation I incorporate the accents and emphases the individual places on the elements he perceives to be forming in this space according to his particular phenomenological frame; that is, the way in which the individual carves up his own particular perceptual field. Such a frame is consequential to the "biographically determined position" and current motives of the individual, which taken together form what Schütz calls the individual's interest. (Schütz, 1962, pp. 76-77; Goffman, 1975, pp. 8-9). This notion is akin to someone having an "interest" in a business - an interest which governs that person's actions in respect of the business. It is through this interest that various associations give rise to phenomena not in immediate perception. This interest also motivates the individual's will to act. Habermas (1972) sees knowledge in general as being flavoured by the interests it serves -

a notion pertinent to what follows here. Such an interest may be, for example, a student's desire to solve a particular mathematical problem as quickly as possible so as to satisfy his teacher. This could be qualitatively different to the interest of someone wishing to solve a problem for its own sake and seeking to understand the experience of being "inside" a problem (cf. Mason, 1992).

The personal space of any individual also incorporates some concern about other people sharing the social situation and how these people contribute to the perceived constraints. This concern may be about the way in which they impinge on the physical space, or be more directly about social interactions. This is discussed fully by Schütz (1967, pp. 97-207, 1962, pp. 312-329). Schütz's analysis is based on an individual society member "guided by the system of typical relevances prevailing within our social environment", who assumes he uses language in much the same way as everyone else (1962, pp. 327-328). He is cautious about the objective character of the reality of which he speaks. Goffman (1975, pp. 4-5) pinpoints this:

> We speak of provinces of meaning and not of sub-universes because it is the meaning of our experience and not the ontological structure of the objects which constitute reality (Schütz, 1962, p. 230),

attributing its priority to ourselves, not the world:

> For we will find that the world of everyday life, the common sense world, has a paramount position among the various provinces of reality, since, only within it does communication with our fellow men become possible. But the common sense world is from the outset a socio-cultural world, and the many questions connected with the intersubjectivity of the symbolic relations originate within it , and find their solution within it (Schütz, 1962, p. 294)

Similarly, here I work from the premise that it is the individual's experience of the world, of mathematics and of social interaction which govern his actions rather than externally defined notion of mathematics itself.

I wish to offer some notes from my classroom based research to assist me in demonstrating the character of this notion of personal space as it might be for a student in a mathematics classroom. In the lesson described below some students are working together on a mathematical activity. I will discuss an extract from a transcript as a prelude to introducing Schütz's theoretical framework for describing the individual's perspective. I will attempt to differentiate between the world of surface appearances and the world which the students see as their space for action. The example is taken from data produced for my doctoral dissertation which examined how students interact in lessons featuring mathematical investigations, especially in situations with low teacher input (Brown, 1987 b, see also 1987 d, 1996 c). This project was specifically concerned with examining how students made sense of their classroom environment and how this influenced their actions within it. My purpose here however, is to focus on the theoretical structure employed, in exploring the applicability of Schütz's work within mathematics education research. I am not seeking to make empirical claims of my own beyond a few speculative comments offering a provisional illustration of this structure. Inevitably, the reported lesson is being described through the filter of my own observations as a researcher attending the actual lesson.

A lesson on symmetry

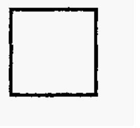
Four 12 year old girls in a London school in 1985 are tackling a task on the topic of symmetry. The lesson began with the following being written on the blackboard.

Symmetry Investigation.

You have been given three shapes; a square, a rectangle and an L-shape.

Try to make as many symmetrical shapes as you can.
Please mark on the line of symmetry with a dotted line.

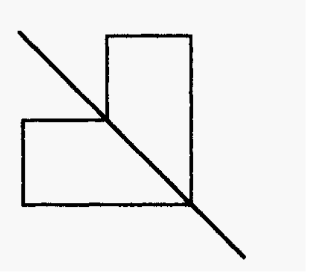
The teacher then gave a short demonstration of a few examples and suggested the possibility of cutting out the shape and folding it as a check. The following section occurs about fifteen minutes in to the lesson and follows the group of four girls who, between them, have already made and drawn about ten "correct" arrangements.

Klavanti: "We could do that  A

and just do that"  B

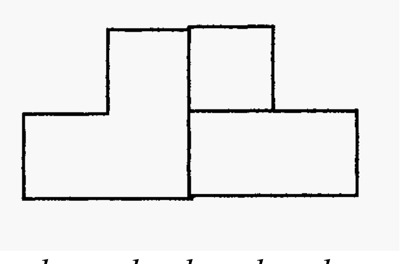
She points out the line of symmetry in the square and rectangle and then indicates the line of symmetry in the L-shape to Meerah.

She then draws it.



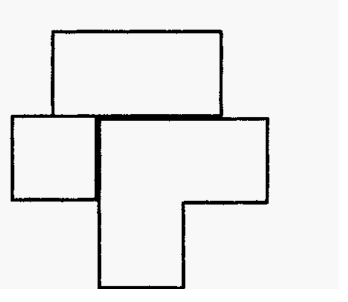
C

Gitar starts to draw



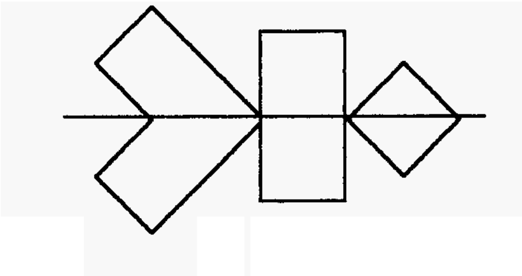
This shape had previously been drawn by the other three girls.

Meanwhile Meerah makes



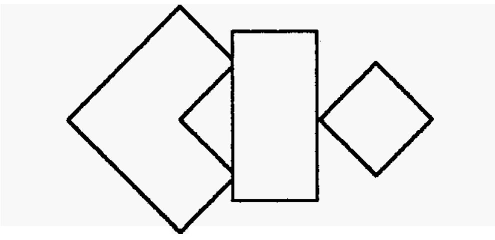
This had previously been made by Michelle.

Michelle: "It's boring you keep using all three shapes. Hey! I've got one"



D i

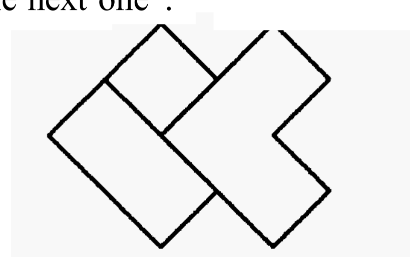
Michelle: "Just go down the middle".
Michelle: "Or you can do it my way".



D ii

Michelle: "It works".

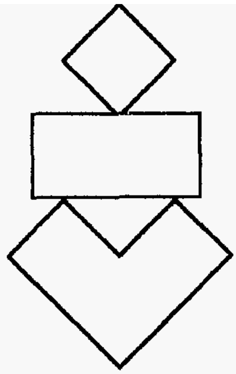
Michelle: "I made the next one".



E

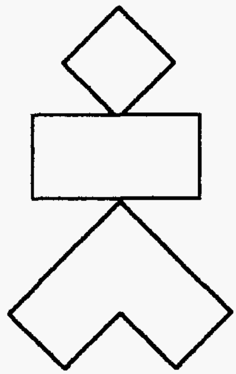
Meerah disagrees. Michelle then shows it to Klavanti and Gitar and tries to explain it without any apparent success. This is followed by a silent period during which they all draw some of the earlier shapes into their exercise books.

Michelle and Meerah draw



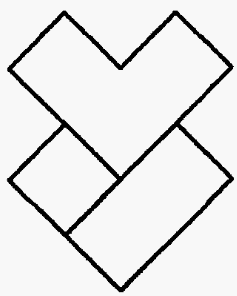
Fi

Gitar and Kluvanti draw



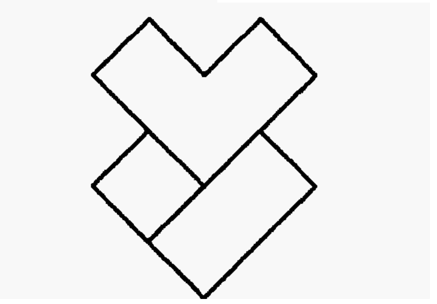
Fii

Michelle remakes



and shows it to Klavanti.

The pieces are then rearranged into various other arrangements by Michelle, Klavanti and Gitar until Michelle again remakes

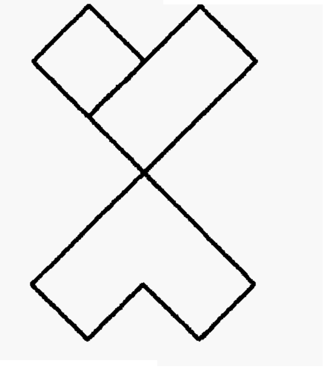


During the subsequent heated discussion

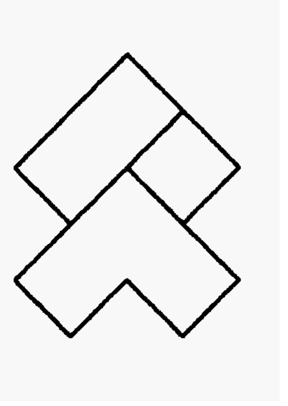
Michelle: "I'm talking about straight up the middle".
Meerah: "Don't call Miss. She'll tell you to cut it out".

The teacher comes over after she is called by Michelle and Klavanti and she suggests that they draw around it and cut it out. Another discussion follows but it is not cut out.

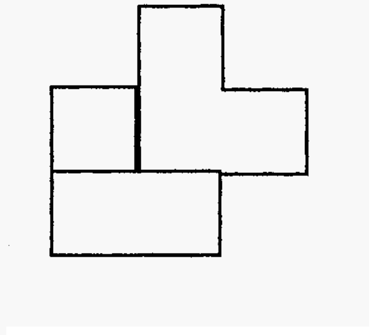
Michelle makes



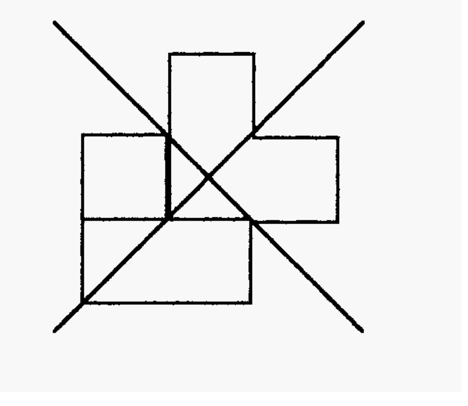
Klavanti and Gitar make



(the inversion of the shape probably being explained by them facing Michelle and seeing her shape upside down).
They all use squared paper to draw



Klavanti adds two "lines of symmetry"



G

Before introducing the theoretical structure I wish to outline aspects of the perspectives I see being represented in these notes of the lesson. In particular, I focus on the evidence of the girls recognising the symmetry of the L-shape, on route to using this information in the construction of composite shapes using all three pieces. Firstly, I will sketch an outline of one the students' perspectives. Secondly, I will focus on the teacher's point of view.

Thirdly, I will consider how these perspectives might meet.

The student's perception of her own space

As an example of a student's perspective, I will put myself in the shoes of Klavanti and imagine how she see things. My purpose is to illustrate the notion of *personal space* and not make claims about what Klavanti actually sees. I follow Schütz in mapping out the Cartesian perspective of an individual understanding the situation they are in.

Klavanti's perceiving is continuously changing during the course of the lesson. As she proceeds she has a variety of things before her, for example:

i) The drawings of previous shapes seen now.
ii) The teacher's presence seen now.
iii) The cardboard pieces manipulated and touched now.
iv) The table arrangement and the way her friends face her.
v) Particular arrangements of the pieces seen now.
vi) The apparent attitude of the other girls towards her.

She will also have experienced, for example:

i) The teacher's lesson introduction.
ii) Arrangements previously made with the cardboard pieces.
iii) Drawings of the various arrangements.
iv) Past situations restraining personal behaviour.
v) Previous work on geometry and line symmetry.
vi) Past social situations in general and with Meerah, Gitar and Michelle in particular.

Taken together such categories, which evolve through time, form aspects of Klavanti's perceived world. Any mathematical component present in her situation is embedded (or embodied) in the variety of phenomena attended to. The cardboard shapes, for example, are, I suggest, being used by the girls in symbolising *their* developing mathematical activity vis a vis the "line of symmetry" (cf. Cobb, Yackel and Wood, 1992). The meaning Klavanti gives to any shape is dependent on its location *vis a vis* other shapes and the way and order in which these other shapes have been placed and described by other students and by the teacher. The mathematical notion of symmetry for Klavanti is "held in" by a variety of accounts, particular examples, physical manifestations and the power of particular individuals to convince others - elements of her perceived world now assessed in terms of their potential referral to a sought after view of symmetry. Klavanti does not reach a final view on the concept of symmetry - rather, her view of symmetry develops with each new insight (cf. Brousseau 1986). She can never do more than believe her use of the expression "the line of symmetry" is shared with her teacher.

Also, things seen in the present offers clues about what is to follow. Klavanti may have an arrangement before her now and a memory, with a partial written record, of arrangements already made. But her motive is to find yet more symmetrical arrangements or perhaps all of those possible. Attention to the three pieces varies continuously as the pieces pass through successive arrangements.

The teacher's perception of a student's space

The teacher probably has other ways of seeing things. She may for example, be interested in understanding the student's perspective since it provides part of the context for her own actions in the teaching situation. Although, she does have some control over the things Klavanti sees, she will, inevitably, have certain difficulties in guiding her thoughts. The teacher needs to attend to a variety of things. For example:

a) She interprets what the student is doing now.
b) She can imagine what she herself will do.
c) She can imagine what she could do with certain changes in the situation.
d) She can act to initiate changes consistent with these possibilities.

All of these require interpretations of varying reliability. Ordinarily, however, the teacher will be concerned with a whole class of students and her attention to individual students will inevitably be limited. Her actions might be made in respect of some supposed fantasy student in given situation. Berger and Luckmann (1967, p. 45) suggest that our social interactions are patterned by the way in which we typify those around us.

> I apprehend the other as "a man", "a European", "a buyer", "a jovial type", and so on.

For example, in the lesson the teacher was characterising some pupils as "in need of help in checking the symmetry of particular shapes" and was employing the tactic of suggesting the cutting out and folding of any shape where its symmetry was in doubt. She could do this without making any further investigation of the individual case.

As an another example, from the same original project, I described some teachers who were attempting to encourage more mathematical discussion among the students (Brown, 1987 b). This required some de-centring where the teachers speculated on the spaces the students working In and how these might be modified towards effecting change in the students' behaviour. In doing this strategies were devised for making discussion more likely. For example:

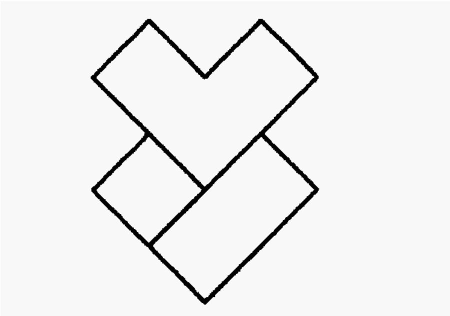
i) Arranging students in groups of four facing each other.
ii) Restricting materials (for example, counters), so that the students were obliged to share them and negotiate their use.
iii) Having one student act as secretary for the whole group.
iv) The teacher restricting her speech so that there was space for student discussion to develop.
v) Setting tasks that allowed delegation of responsibilities.
vi) Setting tasks that allowed the students to develop a mathematical situation for themselves.

These strategies were aimed at the class as a whole, without investigation of the needs of individual students. However, they did serve to alter the working space of each individual student.

The meeting of perspectives

The four girls and the teacher will all have some awareness of the succession of shapes which have been made on the table but clearly the nature of this awareness will be different for each of the five people. In meeting a new shape my way of seeing it is conditioned by the shapes I have already seen. I have learnt to highlight particular properties by studying earlier shapes.

For example, Michelle draws



after a sequence of other symmetrical shapes. The three other girls, however, meet this new composite shape directly without the experience of making for themselves the other composite shapes, previously put together by Michelle. Also, the teacher, when she comes over for the first time, has no awareness of the earlier shapes made by Michelle. The histories of the five involved are all different, as are the phenomenological frames they bring to the situation, Each of the five have motives in respect of the situation. Michelle wishes to defend her proposition that the shape is symmetrical. The teacher wishes to intervene or not, so as to influence a certain mode of activity. The other three wish to confirm or reject the shape as symmetrical as an element in their overall project that concerns finding arrangements of the three pieces that are symmetrical. Consequently, there are five distinct negotiating positions in the discussion and five distinct perceptions of the shape. There is no over-arching view of events. Each participant, including myself as an observer, experiences the social situation from their own perspective.

APPRESENTATIONAL ASSOCIATION

I shall work from the premise that neither mathematical ideas nor their physical embodiments are stable. Nevertheless, physical features of the world and, in particular, instructional apparatus, can function in anchoring mathematical thinking. My task in this section is to build a model that facilitates an understanding of evolving mathematical thinking in relation to an evolving understanding of how the physical world provides support. In doing this I wish to develop a way of describing how the individual's attention oscillates between physical phenomena in immediate perception and mathematics existing in a "referred to" world. Towards this end I introduce Schütz's theoretical framework to assist me in distinguishing between the worlds to which we attend. Firstly, I will outline the notion of "sign" underpinning Schütz's model. Secondly, I will introduce and exemplify Schütz's model which comprises systems of signs. Thirdly, I will show how this provides a structure for monitoring the passage of time in a lesson.

The sign

I understand my personal space through the signs that suggest it. For example, the drawing in my book I associate with some models I made out of plastic shapes earlier, the writing on the blackboard reminds me of the teacher's introductory talk or my rumbling tummy I associate with the lunch I will have in thirty minutes time. The use of the word "sign" here, however, is not Saussure's which associates a mental image of a word with a concept and there is a risk of confusion. In this present context I refer to the notion of "sign", as described by C. S. Peirce which is about an individual building meaning through pairing two associated phenomena, the sign and signified (see, for example, Groden and Kreiswirth, 1994, pp. 569-562; Hookway, 1985, p. 32; Kaput, 1991; pp. 59-61). In his "semiology", Saussure's sign comprises two mental phenomena, in a theory of language that does not extend beyond the linguistic frame. In his "semiotics", Peirce's sign is combined with a signified and allows for the possibility of bringing together physical and mental phenomena, understood through the "interpretant" (the human experience of creating intelligibility). Schütz's use of Husserl's notion of *appresentation* develops Peirce's framework for examining the way in which individuals associate pairs of elements. He employs it in building an over-arching framework combining groups of such pairings in associating surface appearance of the situation with the perceived field for potential action. I will suggest that mathematical phenomena are understood through signs, rather than as facts. That is, evolving mathematical ideas do not have stable embodiments - surface appearance (whether this be cardboard shapes, written symbols or the frame in which they are used) can be variously interpreted. It is through this route that this formulation avoids the trappings associated with a "representational" view of the human mind (Cobb, Yackel and Wood, 1992). In particular, by having the mathematical idea and its embodiment in an interactive hermeneutic relation we by-pass problems endemic in a stable dualism. Such a resolution is akin to the work of Mason (1987, 1989 a) where the evolutionary qualities of mathematical symbols representing mathematical ideas have been examined.

In introducing Husserl's notion of "appresentation", Schütz (1962, pp. 294-295) emphasises that it is a "form of passive synthesis which is commonly called association". In considering an associated pair of elements Schütz speaks of the *appresenting* element as that in immediate perception being paired with the *appresented* element; the perhaps invisible partner. For example, certain algebraic expressions written on a page *(appresenting)* may call to mind particular geometric configurations *(appresented)*.

A system of signs: appresentational situations

Schütz refers to situations where groups of such pairings arise as *appresentational situations.* The notion of personal space used above is an example. The space in which I actually see myself working depends on the reading I give to the things I see around me. Schütz identifies different facets to the individual's task of making sense of his immediate personal space. For example, the school bell may go at the end of the day and I associate this with putting my books in my desk and getting ready to go home. In the environment of the classroom there will be a multitude of "things" like the school bell impressing themselves on my immediate perception. Each person in the room will attend to different things according to their own personal phenomenological frame. This environment of things, however, can also be seen as an environment of signs. That is, my actions are governed by my individual reading of surface appearance. Schütz (1962, p. 299) tries to capture these varying ways of seeing things in identifying four orders he sees as being present in any *appresentational situation:* Schütz's account is rather brief and not transparently clear and elsewhere (Brown, 1996 c) I have sought to exemplify his framework in analysing this present data. Here I shall restrict myself to a briefer outline of this framework to specify the distinctions I will utilise shortly.

a) First order: apperceptual scheme
This comprises the world of surface appearances; objects seen as things in themselves devoid of any referral. For example, a young student might experience the expression $x2 + y2 = 1$ as a mixture of numbers and letters with no particular significance. A more experienced mathematician may perceive it as a circle. The first order is characterised by the former state.

b) Second order: appresentational scheme
Here the world is seen as an environment of signs; for example, the school bell is not seen as a thing in itself but rather as the signal that it is time to go home. In the mathematical example, the expression $x^2 + y^2 = 1$ would be seen as a <u>representation</u> of a circle.

c) Third order: referential scheme
This is the world in which I see myself acting - the world I imagine I am working in, given my reading of surface appearances. As an undergraduate mathematician I may use the shorthand of algebraic symbols, yet in my mind I picture geometric configurations. The *(appresenting)* symbols I write on the page track the *(appresented)* imagery in my mind. The third order comprises the domain of mental imagery where my thoughts function. $x2 + y^2 = 1$ would thus be "seen" as a circle.

d) Fourth order: interpretational scheme
This is the relationship I assume between the world of surface appearances and the world I imagine to exist. As an example, novice and proficient mathematicians would have different ways of bringing mental imagery to algebraic symbols. The fourth order could be seen as the ways in which algebraic symbols appresent mental imagery.

In summary, we have, very crudely, a) the object itself, b) the object seen as a sign, c) the thing signified and d) the connection between sign and signified. In the next section I wish to consider how these four items get disrupted as new understandings develop through time. Through this route I develop an account of how physical apparatus, or more generally the physical world, can be seen embodying mathematical phenomena for any individual in very transitory way (see Figure 11).

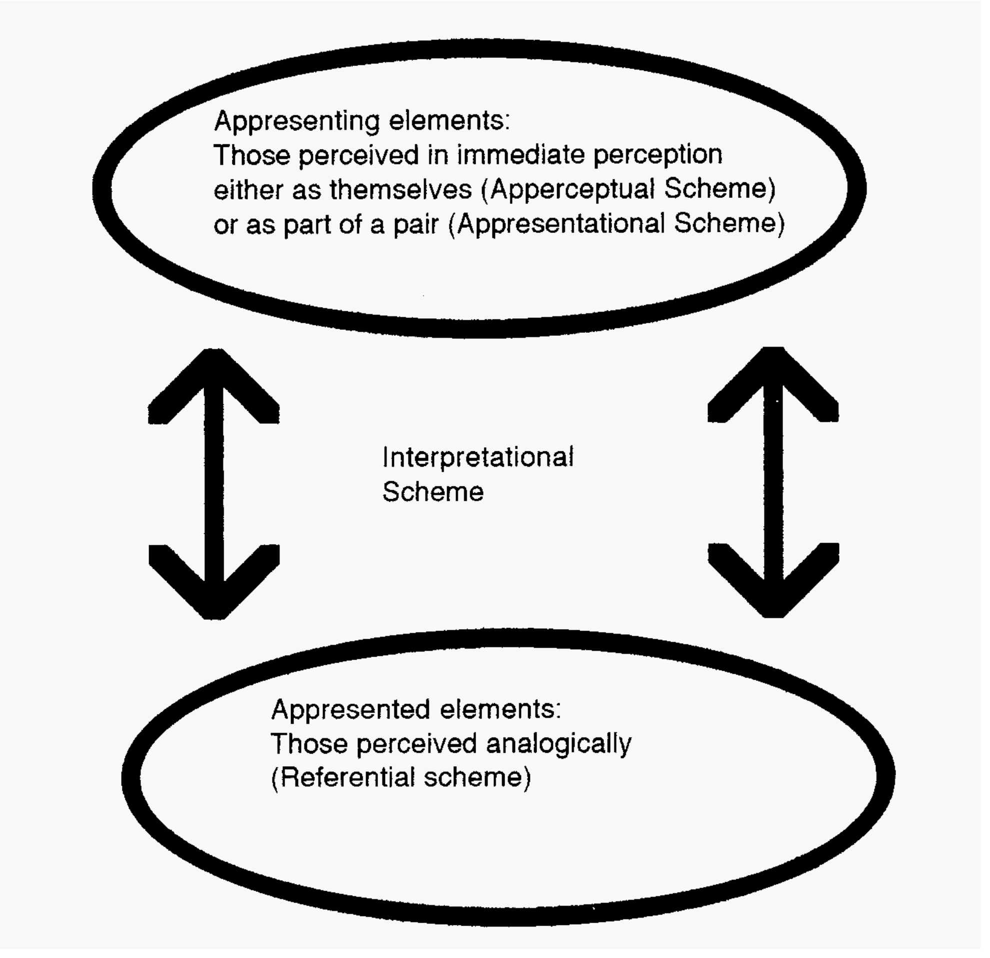
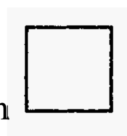


Figure 11

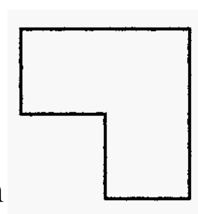
Conditioning time

I now wish to focus on how his system of distinctions might assist us when examining actions with uncertain outcomes. To do this I consider how an individual might get caught up in a process of revising her existing styles of association through a process of reflection. If for instance my actions have unexpected outcomes I need to question my current understanding of how surface appearance appresents the broader world I imagine to exist. For example, the sense I make of the physical world is not stable but rather evolves as I check out more things. Also, my experience of time is flavoured by my own specific interests. In acknowledging this Schütz (1962, p, 215) distinguishes between time in the *shared outer world,* as measured by clocks and *inner time,* which is shaped by the memories and anticipations affecting an individual pursuing his interests. It is these memories and anticipations which condition the individual's own experience. It is the individual's history as experienced by her which influences her perception of the task she now faces. It is also for the individual to decide how the thing she sees now is the result of previous things she has observed or advance notice of something about to happen (Mead, 1938, pp. 120-124). For example, a teacher may herself have said something to a student that produces in the teacher some expectations that this student might now offer a certain response. Here the teacher's statement would be connected with the anticipated or actual response by the student. The interpretation the teacher places on current actions by the student will accommodate this expectation. It is the composition of such expectations, derived from past perceptions, that form the basis for the teacher bringing structure to her own perception.

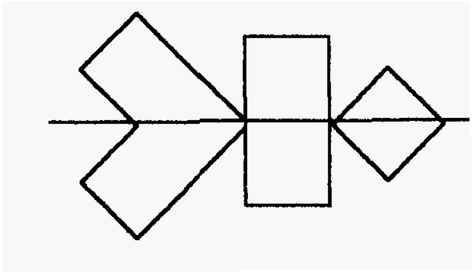
I wish to focus on how a student might project forward, on the basis of expectations, from a current state of mathematical knowing. I will develop this by considering Michelle's work in this sequence in the lesson. In the transcription there is some discussion concerning the symmetry of the double chevron placed by Michelle (E). Michelle made the arrangement after a sequence of small transformations using the symmetries of the individual pieces first discovered by Klavanti. There appear to be at least eight identifiable stages leading up to Michelle making the shape:

1) Identification of symmetry in  (Klavanti-A).

2) Identification of symmetry in  (Klavanti-B).

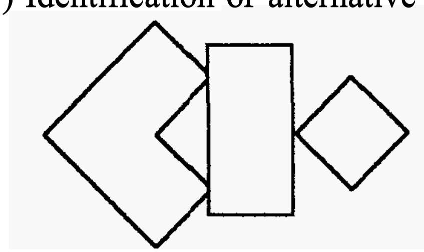
3) Identification of symmetry in  (Klavanti-C).

4) Identification of symmetry in a combination of three shapes



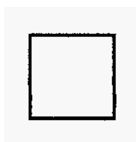
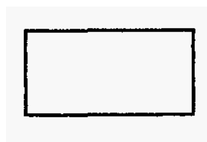
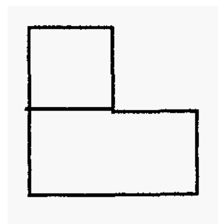
(Michelle -D i).

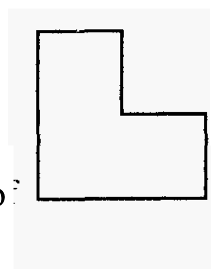
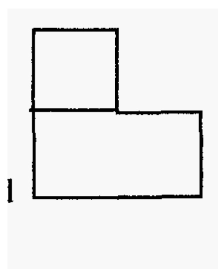
5) Identification of alternative arrangements



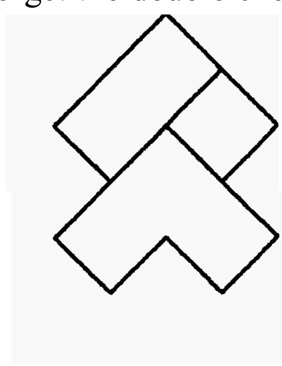
(Michelle -D ii).

And then three stages that seem to be embedded in the making of a single configuration.

6) Transforming  and  to  (Michelle-E).

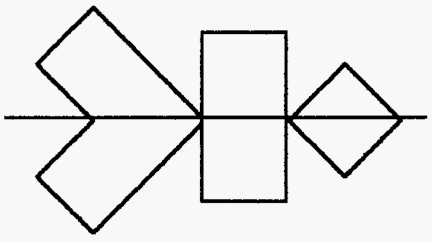
7) Recognition of equivalence o  l  (Michelle-E.

8) Combining these to shapes to get the double chevron

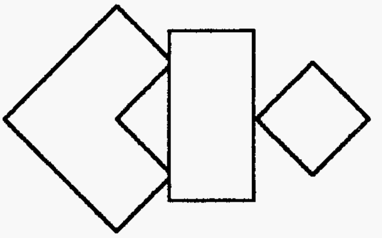


(Michelle-E)

Michelle makes an arrangement combining the three shapes with their lines of symmetry being co-linear (D i). She may have noticed Klavanti point out the symmetry of the three individual pieces shortly before (A-C). In any case she later displays this recognition herself in producing a number of shapes (D-E). Whilst Michelle can see only the shapes before her on the table she may consider these shapes as possible clues to other new possibilities since her *interest* is to find new shapes. As an example, Michelle makes two arrangements in quick succession:



D i



D ii

They are both possibilities requiring symmetry in each of the three components but when the first of these is placed both the selection of shapes in immediate perception and the way in which they are seen change. A new way of combining shapes suggests new possibilities for finding other shapes. The first shape can be assessed in terms of its potential for suggesting other composites with similar symmetrical properties. Consequently, the second shape arrives in a different context since the first shape has provided an additional clue. Similarly, the way in which clues are associated with new possibilities, accommodates each new connection made. By combining these newly recognised properties the gateway is opened to the double chevron being offered, built as it is out of already known constructions and recognitions. Through such a process, I suggest, the instinct for finding new shapes develops. (It is precisely this instinct that is addressed in the work of Mason (for example, 1992) who considers how one can develop the skills one needs "inside" a mathematical problem.) The way in which the physical materials are seen as embodying mathematical phenomena evolves. Whilst physical shapes and symbols provide anchorage, they do not fix the conceptions associated with them. In assessing any current situation, my perception is flavoured by my historical background and current interest. In the present, I see both the consolidation of the past and potentialities for the future. In the next section I wish to develop this further in suggesting that we always act in the world we imagine to exist and show how the tangible world provides anchorage. I suggest mathematical ideas can be seen as continuously evolving in relation to the individual's broader understanding of the world.

ACTING IN THE SUPPOSED WORLD

Each person acts in the world she imagines to exist. When she acts in this supposed world she might expect the world to resist her action in a way consistent with her structuring of it. If these resistances are not as expected, however, this suggests that either her structuring of the world was incorrect or her actions related to this structuring in an unexpected way. These two options may amount to being much the same. For example, when Klavanti draws the double chevron she includes two "lines of symmetry", one perpendicular to the other *(G)*. It is only after she has drawn the second that she realises it is incorrect. My reading of this is that she has experienced other shapes with perpendicular lines of symmetry and thought this might apply here. She can only see the error of this conjecture retroactively, after an attempt to realise it. In this on-going process of conjecturing and confirming the nature of the perceived task is modified. The structure the individual places on the phenomena seen in his immediate perception is always based on presuppositions made in the context of his experience. One comes to terms with the world through acting in it (cf. Piaget, 1972). The world is understood through the signs that suggest it and some of these signs can only be "felt" by instituting a disturbance by one's actions. Change, for example, is a particular feature of the world and to understand this change it may be necessary to see how it occurs in response to one's actions. The student can build her understanding of her space for action through learning how the world changes in response to her. The way in which she perceives the tangible world affects the way in which she describes it and so how she subsequently perceives it. The way in which she perceives and describes it affects her actions which serves to manipulate the tangible world which inevitably affects her subsequent perceptions and descriptions because the world has been changed by her. This locates a manifestation of the hermeneutic circle; it is only by acting in the world and thus changing the world that we learn how to act in the world. One is never at rest but rather continuously engaged in a dynamic between interpretation of the world and action in it. We are always exploring a situation that extends beyond our immediate perception and so can only focus on bits of our space as the whole of it changes. It may be that changes can only be perceived retroactively in a reflective mode (cf. Ryle, 1949, pp. 195-198).

I conjecture that mathematical notions, such as the line of symmetry, are understood through their perceived embodiment in the surface appearance of the world, yet reside in an imaginary field accessed through signs. That is, the world of mathematical objects runs in parallel with the world of material phenomena associated with them. Seen through this filter, mathematical notions, as experienced by the student, are not stable entities but rather, evolve as they are seen through successive framings. They are understood differently as their relations with their perceived physical embodiments evolve. We are always destined to deal with our current versions of mathematical ideas and definitive versions will remain, forever, elusive, whether or not we believe in an ultimate truth.

Within Schütz's model this can be understood as follows. The surface appearance I perceive *(apperceptual scheme)* is accented according to my current interest *(appresentational scheme)*. My actions are made into the supposed world, as created through my reading of this surface appearance (that is, according to my current *referential scheme)*. The resistances felt, however, may not be in line with expectations, which suggests that the world is perhaps different to expectations. This causes a reassessment of the relationship (the *interpretational scheme)* between the things immediately perceived and those things with which these are associated with. The system of associations is thus renewed.

Any action or statement we make is in a context that is never to be fully revealed and, at best, is suggested analogically. Such actions or statements can never be made with total certainty. This, however, is not to say that we cannot feel certain. Indeed, situations with which we are presented, require that we take a pragmatic approach. Usually it is not possible to obtain all the information we may find pertinent to proposed actions - we act on what we know, however, partial that might be. In, what Schütz calls, our "natural attitude" we may "reduce" our current perception to that which we now see as pertinent to our proposed action. Schütz (1962, p. 229) has suggested that we often place great faith in our particular reading of the situation in which we find ourselves:

> Phenomenology has taught us the concept of phenomenological epoche, the suspension of our belief in the reality of the world as a device to overcome the natural attitude... The suggestion may be ventured that man within the natural attitude also uses a specific epoche, of course quite another one than the phenomenologist. He does not suspend belief in the outer world and its objects, but on the contrary, he suspends doubt in its existence. What he puts in brackets is the doubt that the world and its objects might be otherwise than it appears to him.

Goffman (1975, pp. 247-299) who explores Schütz's notion of "multiple realities" speaks of realms of activity being so bracketed. In particular, he examines the way in which someone can switch between such realms (or frames) as in moving on to a stage to perform. One might similarly, see a student switching in and out of the rules presumed to apply within mathematical activity.

Since, the meaning of any mathematical idea evolves with the variety of situations in which we meet it, it is not possible to speak of a final, definitive understanding (cf. Brousseau and Otte, 1991). We are always obliged to work with the understanding we have, however inadequate this might be. In any case the student does not have an expert outside view of any limitations in her particular conception. We use mathematical ideas to the extent that they function for the purposes we have in mind. The precision often supposed for mathematics is necessarily undermined by the need for a human to decide where or when to use it. This sort of knowing is an essential dimension of enculturation, that conditions normative, as opposed to positive, dimensions of mathematical activity (cf. Bauersfeld, quoted by Cobb, 1994, p. 15). (N.B. An interesting theoretical analogy can be made with Habermas' current book, *"Between Facts and Norms"* which examines how individuals in societies experience legal constraint alongside normative behaviour (Habermas, 1996; Deflem, 1996)).

PART 3

THE TEACHER'S PERSPECTIVE

In chapter four we observed Chester and his chums engaged in an attempt to count up in twos. Their efforts, however, were not without a little confusion and mischief. Having started with 2, 4, 6, 8, they had somehow reached 27 to be followed by 29, 30, 32 (written as 23), 25, 45, 70, 46, 16, 17, 19. Whilst some seemed to be trying to throw the activity off course, others remained committed to adding a two to the number they were given, although occasional slips were made. I speculated that their actions were governed by the constraints of their immediate situation where they placed the required number next to the number they were presented with but without paying much attention to the overall scheme of things. They seemed to be "going with the flow" and were responding to each other in a lively interaction.

The teacher's perspective on what they were doing would, presumably, be somewhat different. Were she to have witnessed this diversion she may well have guided them back to her plan. For she clearly had a plan in her mind of some sort and this will have built certain expectations in her mind about what to look for in the boys' activity. In general, however, a typical classroom teacher will have maybe thirty students to attend to and she will only be able to observe brief extracts of the work being done by any particular group of students. In fact she may well see much of the lesson as a sequence of momentary glances with only a few extended interludes. At best she can regard any interaction as giving information about the work that has been done and what could be done. She might aim to do this for all the students in the class. The teacher might well observe extracts from the sequence of arrangements passed through by the students but her view will inevitably be partial and also coloured by her expectations in respect of her lesson plan. Much more for her than for the students, her lesson plan is likely to act as an anchor. I suggest that her perception of her lesson plan may have changed less than the perceptions of it held by the students working in a group who have passed through various arrangements since their teacher presented her plan to them. This plan conceals the way in which the teacher perceives her professional task and the way in which she intends to guide proceedings within a lesson. It would reveal much about the teacher's understanding of her practice, the constraints she feels on her time and her understanding of the mathematics being addressed.

It would flavour the teacher's own perception of the lesson as it proceeds, providing a framework for her understanding of the students' actions within it. However, as has been suggested, the link between teacher instruction and student response is often quite tenuous. The teacher needs to be cautious in assuming that children are carrying out her intentions in the way she herself understands them. Whilst she can always tell a "story" about what is going on in her classroom, new interpretations are always possible, with varying implications for modifying practice, both in the immediate situation and in the long term.

In this part of the book I focus on the issues governing the teacher's perception of their task. What motivates the teacher and how is her classroom practice guided by her underlying beliefs? How does she read the classroom situations she inhabits and how might she work on developing this understanding? In the first chapter I examine the constraints perceived in the immediate classroom situation with particular reference to interactions between teacher and students. I consider how such interactions function in keeping the teacher's plan on track (cf. Brown, 1987 a). In the second chapter I step back to consider the way in which the teacher conceptualises their practice within a broader framework of professional intent. I focus on teacher education courses where self-reflective professional writing provides an instrument for examining practice (as previously reported in Brown, 1994 e; Hanley and Brown, 1996 and Brown, 1996 b). As in Part 2, no empirical claims are being made. Rather, examples are offered in introducing theoretical perspectives.

CHAPTER 6

TEACHER-STUDENT INTERACTIONS

As a classroom teacher, I have often felt uneasy about the broad range of interpretations being brought by students to the words I offer. Insofar as I see my task as initiating students into certain conventional mathematical ideas I cannot fully predict the reading the students will give to the aurally and visually complex environment within which they encounter my contribution. Further, mathematical ideas, new to a student, cannot be seen as being static in relation to this environment, dependent as they are, on the frame within which they are seen. The flows of thinking associated with these "ideas" and "frames" are time-dependent, evolving as they absorb other "ideas" and, perhaps, dissolving as they recede in memory. Also, in a classroom situation where I am dealing with around thirty students, I only have minimal access to the understanding and intention of any individual student and often I am obliged to make sense of what the student is doing on the basis of very limited evidence. In the last chapter I started to explore how the various motives and concerns of the teacher and students interact. Here I examine a model for considering how a teacher might attempt to build an understanding of a student's activity through the limited time ordinarily available for interaction between a teacher and an individual pupil. The model is based on the idea of this being achieved by the teacher projecting backwards to characterise where the student has been and projecting forwards to create an outline plan of where the student might go, through an analysis of evidence in the present. The chapter considers a more detailed analysis of teacher-student interactions, as seen from various perspectives. In this I distinguish between interactions focusing on the individual work of a student and "stage-managed" interactions orchestrated by the teacher for the benefit of a wider audience.

Interactions concerned with an individual student's work

In earlier chapters I have raised the difficulties involved in locating and framing mathematical thinking between the perspectives of people attempting to share them. The teacher is seeking to understand how the student is making sense of work they have been given and how they encapsulate this in their verbal and written responses. However, generally, she also has some responsibility for ensuring that the student's Understanding bears some resemblance with a conventional understanding. Both teacher and student experience a lesson as a sequence of activity and learn continuously. There is no straightforward approach to framing mathematics in this dynamic scenario. In the space of a short interaction the teacher needs to interpret the student's understanding through a limited number of signs in the present which might enable her to say that the student has or has not "got the right idea". But of course, we are not trading in such ideas here, but rather, we are operating on knowing which forever evolves through time. We always remain with the difficulty of attempting to "see" an "idea" in a continuously unfolding situation.

In addressing the work of a student within the context of the classroom the teacher faces a variety of pressures on her time. Her decisions regarding the way in which she handles any exchange with a student will be influenced by the amount of time she feels able to allocate, given the other demands upon her. Her time allocation will in part relate to the plan she sees governing the structure of the lesson but also to the immediate demands that present themselves as the lesson proceeds. For example, in my own experience of working as a new teacher in a London secondary school I was constantly faced with finding ways of reducing the amount of time I spent on any given task. Working with an individualised teaching scheme, the potential nightmare scenario of half of the class raising their hands simultaneously, demanded that I develop effective time management skills when dealing with individuals. My teaching strategies were also highly dependent on how well the class were behaving. Strategies such as telling the student to "think about it" were not appropriate at times when I needed to be more directive about what a student should have been doing. I imagine that most classroom teachers face this sort of pressure to some degree.

In attending to a student's raised hand the teacher needs to investigate the nature of the difficulty and decide which strategy might be most effective given the time constraints. She needs to proceed with this investigation before she can become aware of the options available. Only as she proceeds with one of these options does she begin to build a clear sense of the time which needs to be allocated to it. In the background to this is an evolving sense of how she should be allocating time elsewhere and how this conflicts with some of the better options for work with this current student.

We need to be careful in selecting the metaphors we use in describing the teacher's task. "Guiding thinking", "working on knowing", "communicating an idea" and "identifying and resolving the student's difficulty" are all possibilities. Nevertheless, they are all inadequate in describing the flow of time, experienced differently by teacher and student. The teacher in some ways needs to build a picture of how the student experiences this flow and the effect this will have on their subsequent activity. However, the teacher can only grasp this picture and contribute to it as her own understanding of the situation evolves through time. Further, her attention may oscillate between ensuring her own agenda and understanding what the student has actually come up with. Saying "you need to do this..." is much quicker than carrying out a detailed investigation of where the student is and then customising an appropriate strategy.

Meanwhile, the student meets the interaction in a completely different context and sees its elements with rather different flavourings. In raising her hand the student indicates that she has certain intentions based around some sort of exchange with the teacher. Her actions are governed by a rather different framework to that faced by the teacher. She may wish to maximise the length of the interaction whilst the teacher seeks to minimise it. The student may, hitherto, have been involved in private work and is now faced with the rather different task of verbalising her thoughts. The student may wish to understand the mathematics, or may be simply concerned with getting things done. The student might just fancy a chat or some reassurance. The economics of the student's time allocation will not be in alignment with the teacher's. Any shared components present within the exchange, mathematical or otherwise, are seen in very different ways. Also, the interaction derives its meaning through its relation with the other parts of the lesson as experienced by the student. Any ideas raised are shaped through the way in which they are encapsulated in the exchange. Furthermore, the student selects things to attend to according to her particular priorities but it is only after it has been heard that it can be valued. Additionally the student must attend to the signs that suggest the imminence of something of value but there is no guarantee that these suggestions are fulfilled. What is then possibly perceived is a collage of fulfilled and unfulfilled expectations which have only a distant association with any linearity inherent in the teacher's speech.

I shall attempt to tease out such distinctions in an example. I describe two successive teacher student interactions in a lesson using an individualised scheme with 12-13 year old students (first reported in Brown, 1987 b). A student teacher is working alongside the regular class teacher and I contrast their approach with a boy who is working on a worksheet concerned with adding positive and negative numbers, by using arrows on a number line. He has completed half of the questions without making a mistake. He now faces the question +2++6. The sheet is printed incorrectly so that it is not possible for him to draw on the correct solution. He has only drawn the +2 arrow, The +6 arrow would go past the end of the scale and cut through some print. The boy raises his hand.

a) The interaction with the student teacher
A student teacher approaches him but seems hesitant and says nothing initially. After mumbling incoherently he walks away leaving the boy still with his hand raised for the main teacher.

The student teacher returns a few minutes later.

Student:	I see. Do you see the example there? (+2++3=5) plus two plus three altogether you've got five
Boy:	How do I fit it on? *(The student teacher seems nervous)*
Student:	Do you have to mark it on? No!
Boy:	Yes.
Student:	Yes, but not on that sheet of paper. *(It was to be put on that sheet)*
Student:	Ask Mr H.

The boy sits and looks at the sheet for a while and raises his hand again as the regular teacher returns.

b) The interaction with the regular teacher (a few minutes later).

Boy:	Do you know when you're doing this........(Interrupted by teacher)
Teacher:	How did you get on with the other ones?
Boy:	Quite easy Sir.
Teacher:	Were you always adding positive numbers?
Boy:	Yes.
Teacher:	All of the first side were adding and these are still adding.
Boy:	I know how to do these.
Teacher:	Ah! The number line doesn't go far enough - Ah! Yes it's just the line isn't long enough.

ST: That's what I wasn't sure about.
Teacher: It was just a mistake.
Teacher leaves. The student stays to watch the boy complete it.

In the first interaction, the student was at liberty to spend as long as he wanted with the boy since he was under no apparent pressure to attend to other students in the class; the atmosphere in the classroom was very quiet and the regular teacher was present elsewhere in the room for most of the time. The student chose to acknowledge and approach the boy who had raised his hand. In this way the interaction was initiated and both the student and the boy might have certain motives and expectations in respect of it.

The boy had raised his hand and stopped work. However, he had stopped on a problem which, from my view, appeared to be no more difficult than those which he has already done. It seems his difficulty arose as a consequence of the misprint on the worksheet and that he simply required confirmation that the sheet was wrong.

The student teacher, however,was unfamiliar with the scheme and had not witnessed much of the boy's progress so far but felt under some obligation to help the boy. Perhaps he imagined that he could himself read and understand the task and then explain it to the boy. However, as he approached the boy, the boy kept his hand raised for the regular teacher. My reading is that the boy did not acknowledge the start of an interaction, preferring to wait for the regular teacher and, as a result, the student became even more tentative. In this first attempt the student teacher ended up saying nothing. However, this in itself seemed to bring into question his ability to face the task. The interaction was complete, although any projection, backwards or forwards, by the student teacher failed to productively inform his immediate actions. He perhaps projected forwards to imagine a scenario where he, himself, might have problems and where the regular teacher might do better, and so he turned away.

The student teacher would now be faced with choices regarding his attempts to work with the boy. He could stay away or go back to the boy but he could not return to his original choice, of helping the boy or not, since his first encounter had changed the context of this choice. The initial attempt flavoured expectations of any subsequent attempt. The student teacher did, however, choose to return, although, it would appear that the boy was still waiting for the other teacher, and kept his hand raised throughout. This time however, the student made a more determined attempt to find out what was required and to see what the boy had done. However, although he succeeded in learning more about the task he still failed to come to grips with the specific nature of the boy's problem. Consequently, he had to reconsider his choice between "projects of action" (Schütz) again but in the light of this second failure.

His motives seem to have been; to understand the problem; to help the boy; and to "save face". It seems to me that he had a number of possibilities (projects of action). He could: a) leave again; b) continue studying the problem, but with a revised time scheduling; c) ask the boy about it; d) try explaining to the boy with the little he has grasped; e) wait for the regular teacher to return and ask him; or f) refer the boy to the regular teacher. He could also combine aspects of more than one of them. However, for this the order in which he combines them would clearly be significant.

He, in fact, combined them in a way which seemed to suggest that he was projecting forwards and acting in respect of various scenarios, but failed to fulfil them. He finally suggested that the boy asked the teacher. This choice however, was not the one which had been available to him before he approached the boy for the first time.

The boy's interaction with the regular teacher contrasted quite markedly. The teacher responded to the boy's raised hand and approached him quite confidently. The boy started to describe the problem but the teacher interrupted him with questions of his own, thus investigating the nature of the problem. It is as if the teacher had a project of action that he wished to fulfil and did not want the boy to divert him from this. This comprised a projection backwards to see which questions the boy was facing and what he had completed. His time scheduling on this did not seem to allow the boy to set the rules for the interaction. The boy was expected to fit into the structure being set up by the teacher.

The teacher, in approaching the boy, was faced with various possibilities with regard to the interaction but the situation became more clearly framed as its nature revealed itself. The teacher chose a project of action to tackle the problem which assigned very specific tasks to the boy, which enabled the teacher to stick to his planned strategy. In this way, the teacher explored the situation and identified the misprint that seemed to be causing the problem. On finding this he felt that the boy would be able to continue. That is, in his projection backwards he was able to assess the success the boy was having on the particular task and in the light of this in his projection forwards he could imagine the boy succeeding on the subsequent questions now that the misprint had been confirmed.

This interaction with the regular teacher was very different in character as a result of what seemed to be this teacher's greater confidence, perhaps based on his familiarity with the type of task which allowed him to proceed, knowing that he would probably find the source of the problem fairly quickly. Furthermore, he was responsible for the whole class and may have felt under pressure to attend to other students. Such things may explain his style which is certainly economical in terms of time. His approach to teaching is fashioned by an approach to time-scheduling built on his expectations about this type of lesson. He decided the student could do problems of this sort and that there was little to be gained from a detailed investigation of the boy's understanding.

Interactions with the whole class

As I have suggested, the teacher ordinarily has some sort of a lesson plan in mind through which she seeks to influence the actions and thinking of the students. She may achieve this through managing the visual, aural and physical environment. Within this she may employ, as a teaching device, the strategy of publicly engaging in some sort of dialogue with members of the class, with view to discussing some of the ideas being addressed. Students' contributions may help her in building such an exchange, stage-managed, for the benefit of those observing. The teacher can often rely on being the most dominant person in the class and can assume the power to select who speaks and influence the context in which this speech takes place. By posing the question before and by speaking after, the teacher can condition how a student's offering is seen, whilst pursuing her interest of fulfilling her intended framework. With this is in mind, the teacher's responses to the students' offerings may be governed by her concern to keep "on course" in some way. Further, the teacher may emphasise the bits of the student's contribution most likely to affirm her intended structure. Students may not be allowed to disrupt this too seriously.

The student observing such an exchange may decide to listen to some or all of the teacher's speech, along with the contributions of other students. He may also pay attention to other things, such as marks on the blackboard or instructional materials. But he cannot interpret everything as the teacher intends. The teacher cannot control the elements of the lesson which are selected or constructed by the student, nor how they are composed. The students may have expectations in respect of the lesson and look for the things they want to see. They select the things they see as important and give them a weighting and an accent according to their own specific interest. They may contribute or seek to contribute but their motivations may not necessarily be in line with the teacher's.

I reproduce here a transcription of the section of video-tape produced by the Open University, that contains an exchange, involving a teacher and a class of twelve year olds (cf. Brown, 1987 b, 1990 a). They are sitting around in a circle, at the centre of which there was a box of Cuisenaire rods. The lesson started with the teacher, going around the circle, asking each student in turn to take a particular rod. My viewing did not equip me with a full understanding of the task as seen by the teacher. My comments are based on my own recorded transcript of a section occurring about fifteen minutes into the lesson.

Teacher:	Let's have a think about adding them *(Cuisenaire rods)* because you're quite good at telling me what colours they are going to be. Let's.... Somebody over there. What colour have you got there Michelle?
Michelle:	Pink.
Teacher:	You've got pink right. Catherine. What colour have you got?
Catherine 1:	Pink.
Teacher:	You've got pink as well have you - We'll move on to the other Catherine. You've got?
Catherine 2:	Yellow.
Teacher:	What colour will we get if we add a pink and a yellow?
Mumbles	
Teacher:	What number are you Catherine? *(to Michelle)*.
Michelle:	4
Teacher:	Sorry Michelle.
Michelle:	4
Teacher:	Catherine. What are you?
Catherine:	10
Boy:	14
Teacher:	14 *(then inaudible)*

A variety of children speak; a few say 'pink'' The teacher waits *casually looking around.*

Teacher: It's going to be.....?

Mumbles

Teacher: It was a pink and a yellow added together.... Catherine.

The teacher points to Catherine.

Catherine: If you start off with a yellow and then add a white as well you would add the colour above but you haven't got the rest of the set you'd get the colour above and you'd get the same colour.

She gesticulates throughout.

Teacher: Right. *(said slowly with an air of doubt).* Well Catherine's telling us what we would get if we had a full box - We've only got half of the colours haven't we Catherine - that's what you're saying - now can you tell us again if we added a yellow and a pink.

Catherine: We got pink...it wouldn't work really.

Catherine tightens her eyes, mumbles then turns to her neighbour. She holds the rods. The neighbour looks back at her. Catherine mumbles again.

Catherine: No. *(she shakes her head.)*

Catherine looks to the teacher. The neighbour then looks to the teacher and shakes her head.

Teacher: No? Let's see. What do we get? We added a yellow and a pink. What colour was the answer?

The teacher looks away from Catherine.

Boy: Pink

Teacher: It was pink. So we added a yellow and a pink and got a pink.

This interlude centres on an interaction between the teacher and Catherine performed before the entire class. I do not wish to enter into a detailed discussion of the mathematics. In any case, my own viewing of the video did not permit me to fully grasp what was going on mathematically. Rather, I wish to focus on how the teacher nudges the discussion, in stressing and ignoring certain features, managing the class to ensure that others listen. She appears to be using her contributions to the discussion to help weave together the students' contributions. There was, without doubt, an air of excitement around the group of students, effectively engineered by the teacher, although, unfortunately, the starkness of a transcript does not communicate this very well. The students were clearly experiencing more than is evident here. I conjecture that the teacher succeeded in keeping everyone thinking about the problem, but this did not seem entirely dependent on all of the children following the finer detail of the teacher's exchange with Catherine. It seemed to be more about providing a framework through which students could engage with their own thoughts.

When the question is first asked, it is not clear how many people apart from the teacher can add the yellow and the pink. A few say "pink", apparently the correct answer, about half way through the transcript, although the teacher prefers not to hear as she waits for others to engage. The answer was overtly acknowledged by the teacher, but only after time had been allowed for more substantial discussion had taken place. I conjecture that, insofar as the other students engage, they attempt to reconcile their understanding of the task with the discussion taking place between the teacher and Catherine.

When asked, Catherine attempts to explain her perspective. The teacher seems unsure about what she says. "Right" is said cautiously. It is as if the teacher wants to reassure Catherine whilst sending the message that further clarification is needed. The teacher says "Well Catherine's telling us what we would get if we had a full box". Here the teacher is making an interpretation about what Catherine is trying to say or, at least, emphasising the bits she had anticipated. She tells a story that does not "rock the boat." But her speech implies she understands Catherine and that maybe other students can now understand her.

Then she says, "We've only got half of the colours, haven't we Catherine? That's what you're saying."

To whom is this question aimed? Literally it is aimed at Catherine, making a statement to clarify a situation. On the available evidence we cannot say how Catherine reads this. The connotation for others seems to be that Catherine is engaging effectively.

"Now can you tell us again if we added a yellow and a pink."

The teacher seems to be wishing to confirm that Catherine is on the right track.

I would not presume to declare a definitive version of what is going on here. We can only speculate how the thoughts of the teacher and students are connected with the speech described above and the movement of rods. Nevertheless, it seems clear that the teacher is seeking to manage a discussion to be witnessed by the class, with the intention of airing a mathematical problem for public scrutiny. The underlying statement; namely “yellow and pink give us a pink”, is embedded in a discussion seen from a range of perspectives. The teacher is contributing with view to assisting Catherine in connecting her thoughts with the problem in hand, as seen by the teacher. In doing this the teacher seems to be weaving her words around Catherine’s in an attempt to show how Catherine has contributed to the final outcome. I conjecture that, for the teacher, the students’ contributions, are assessed and managed, to some degree, to support the assertion of the teacher’s intended structure - seen by the teacher, as framing the ideas being targeted. However, the target statement, as yet unformulated by many, is not such a strong anchor for many of the students. The teacher concludes by saying:

“It was a pink. So we added a yellow and a pink and got a pink”.

This might be interpreted as an assertion of the teacher’s perspective rather than as a confirmation of what the class as a whole had been thinking. The “correct” solution becomes the resolution rather than a deeper understanding of what Catherine was trying to say. Catherine’s speech needed to be understood in relation to the structure being asserted by the teacher, seeking to cohere the various contributions, before it found a comfortable space in the overt story in the public domain. However, the manifestation in speech and action in the public domain of thinking by participants is only part of the picture. In my reading of events, the teacher’s strategy seemed to be more about creating points of reference, enabling students to orientate their own thinking.

Mathematical ideas, as encountered in classrooms, comprise thinking framed by markers in both time and space. However, any two individuals construct time and space differently, which presents difficulties for people sharing how they see things. Further, mathematical thinking is continuous and evolutionary, whereas conventional mathematical ideas are often treated as though they have certain static qualities. The task for both teacher and student is to weave these together. We are again faced with the problem of oscillating between seeing mathematics extra-discursively and seeing it as a product of human activity. As teachers we cannot ever be sure that students are partitioning their thinking into objects that we share.

Teachers' actions, meanwhile, are conditioned according to whether they see their task primarily as being about nurturing the student's own mathematical creations or, on the other hand, enabling the student in employing conventional mathematical skills. In communicating conventional mathematical ideas, for example, a teacher may do this through making some attempt to influence the stream of attention experienced by a student. For this, the teacher needs to have some understanding of both the "ideas" themselves and the way in which the students might "receive" them. (Ball, 1991, considers how we might conceptualise the creation of "pedagogical content knowledge" in this way.) Insofar as the teacher is assuming the task of offering insight in to conventions she needs to highlight characterisable ideas within a continuing flow of individually perceived phenomena. The teacher's management of the flow of the lesson needs to ensure some affirmation of the lesson structure she intends to support a particular style of activity among the students. Contributions by students, however, may have certain meanings in the students' own schemata, yet may take on a whole new character when included in the public arena managed by the teacher.

If, however, the teacher focuses more on the student's own constructing we need to be clear about the scope for doing this in a field dominated by conventional ways of seeing things. As suggested in earlier chapters, the student can only go so far with personal work before needing to capture her thinking in socially conventional frames. Also, it is very difficult for the teacher to constantly de-centre from her own way of seeing things, which may be closer to these conventions. The teacher cannot access a student's schema except through evidence in the student's written work or through some sort of dialogue. This perception of the student's understanding can only be built through time in relation to the teacher's immediate and broader expectations. The student's progress through a lesson is guided by explicit instructions, learnt ritual, physical restrictions, etc. which may be emphasised by the teacher in support of her intentions. The teacher is faced with the task of managing this progress through attempting to understanding how the student's mathematical thinking is both constrained and enabled. If the focus is on the teacher enabling the student to generate their own mathematics there is still a need to understand how to trigger this through the teacher's management of the classroom as a whole and the individual student's learning within it.

I would suggest however, that an adjustment of teaching policy towards emphasising the student's constructing is far from straightforward. In working with an individual student a teacher may try to account for how the student has spent the lesson. From within the interaction she may have projected back to the beginning of the lesson and projected forward to the end. She might aim to do this for all of the students in her class - so that for a seventy minute lesson with a class of thirty, she has thirty, seventy minutes to observe. Whilst attending to one student's thoughts, the other twenty-nine students rest in the back of her mind. This increased complexity also adds other problems. At any one time up to thirty-one people might be speaking, or up to thirty people might be listening. Add to this the diagrams that have been drawn, the symbols that have written, bodily movements and the signs that remind us of things done in earlier lessons, and we approach a situation that transcends the immediate grasp of any individual present. The individual participant selects what she sees. The various interactions will be witnessed differently by each individual. For example, a teacher and a student working together may, in referring to the student's work both think back to an earlier exchange between them, but, inevitably, they will have taken different things from it with some consequent effect on their behaviour now. Each has their memories and intentions anchored differently. The sequence of interactions that anyone passes through, forms the immediate history of that person. It is difficult for the teacher to predict what any student may have taken from the lesson. Nevertheless, it is within this environment that students will have developed many of their mathematical understandings. Clearly, there are limitations to the teacher's ability to pick up on what individual students intend, although the teacher can more easily keep in mind their own agenda. The odds seem to be in favour of mathematical agendas being inherited rather than renewed.

In general, the teacher's view will be conditioned both by her immediate concerns and by the broader project within which she sees herself engaged. This will be governed by the reading she gives to her own teaching based within the teaching philosophy to which she subscribes. In the next chapter I wish to turn to an analysis of how teachers might see the broader aspects of their practice framed within their developing philosophy of their own teaching.

CHAPTER 7

DEVELOPING TEACHER PRACTICE

Throughout this book I have taken the view that mathematical ideas are not things to be distributed bodily from teacher to student. Rather, mathematics is an inseparable aspect of human practice, a form of structuring placed on various forms of activity. It is located in individuals and only has life in the acts of these individuals. The theoretical perspectives assumed have various implications for the teacher's classroom practice and how this might be developed. If mathematics is a style of activity rather than a list of content on a curriculum, the task of teaching is not one of delivery but one of initiating students into specific styles of activity. Since the mathematics "owned" by the teacher is part of themselves, his task may be seen in terms of helping students to build the mathematics they already have in themselves. In addressing these concerns, teachers need to work on understanding their own professional practice and how this practice governs the way in which students carry out their work in the classroom. Even if teachers have a clear sense of how children learn mathematics, they need to build an understanding of how their actions as teachers function in guiding students towards achieving specified learning objectives.

In this chapter I consider the teacher's perspective of their own practice and in particular teachers looking at their practice in terms of what they can do to improve matters through a process of professional development. Essentially, I question how the individual teacher can be enabled in working on knowing their practice with view to developing it. I offer strategies for addressing this task and provide examples of initial training students and practising teachers reflecting on issues concerning their intended or actual practice as teachers of mathematics in primary schools.

There are however, many ways of understanding a classroom. There is not an unproblematic state of "how things are" with straightforward implications for practice. Changing practice cannot normally rest solely on instantaneous substitution of techniques. There is a need for the practitioner to make any changes part of themselves. I shall pursue the phenomenological perspective of the last two chapters in seeing the individual teacher as acting in the world he imagines it to be and learning about this world through the way in which it resists his actions through a circular hermeneutic process of perceiving, describing and acting, where the individual constantly checks out his expectations against his experience. It is through this process that the individual's particular way of partitioning the world into things continuously evolves in an on-going process of framing professional practice. The location of the teacher in the teaching act and changes to this are necessarily problematic and can only be understood through time. Self-reflection is integral in the teacher's self-positioning in the teaching act and in assessing its affect on the student.

Within the broader field of education, the trend towards "practitioner research" increasingly accommodates an understanding of how researchers are practically related to the situations they investigate, where their actions, as teacher-researchers, are seen as an essential part of situation being described. There is a broad spectrum of recent work addressing these concerns (for example, Schon, 1983; Elliott, 1993, pp. 193-207; Adler, 1993; Silcock, 1994; Brown, 1994 b; Lomax, 1994). Such research paradigms are beginning to find a home in programmes of professional development within both initial training (for example, Calderhead, 1987; Zeichner et al, 1987; Liston et al., 1990; McNamara, 1990; Francis, 1995; Hatton and Smith, 1995; Hanley and Brown, 1996) and masters level work (for example, Cryns and Johnston, 1993; Brown, 1994 e, 1996 b). Associated with these moves is a burgeoning literature on teacher narratives, emphasising the teacher's perspective as represented through the accounts they give of their professional situations (for example, Connelly and Clandinin, 1988; Weber, 1993; Olson, 1995; Beattie, 1995).

Given this shift towards practitioner perspectives there has been an associated reevaluation of the role of theory. In addressing specific professional concerns the teacher may choose to introduce theoretical perspectives but for theory to be used in a meaningful way there is a need to build in a self-reflexive dimension which positions the evolving individual in his evolving professional situation. Here I return to the debate between Habermas and Gadamer concerning the role of language in developing understanding. Habermas' early work has had some influence in theoretical work on practitioner oriented educational research (for example, Carr and Kemmis, 1986). This perspective has since been challenged by Elliott (1987) who assumes a more Gadamerian perspective.

Although Habermas himself has now moved away from his earlier work, Carr and Kemmis (1986) utilised the way in which he differentiated between sorts of motivation present in deciding how one should act and applied this to teachers in classroom situations. It provided an approach to analysing attempts by teachers to articulate their understanding of their own practice with view to creating strategies for developing this practice and moving to new ways of understanding what they are doing. Habermas (1972, pp. 308-311) saw various levels of concern functioning within attempts to articulate practice. He identified three "knowledge constitutive interests" that operate in making sense of one's task and for Carr and Kemmis this framework provided a way of viewing the levels at which a teacher can work on his or her practice. The first of these is the interest of achieving *technical* control. This is the realm of "means-ends" strategies (for example, "if you do mathematical investigations with students they will be more creative in their mathematical thinking"). However, "good advice" of this sort has questionable merit as an effective strategy for change beyond a fairly perfunctory administrative control. There are limits to the applicability of techniques designed for all. Habermas, whilst recognising the power of such a scientific approach, claimed that theoretical stances need to offer more than a technological control over practice, since the individual needs to understand the functioning of such techniques within his own personal style of practice.

The *practical* interest arises in a practitioner getting to know their professional situation. Through spending time "on the job" practitioners begin to understand what they need to do, to be a successful teacher, to be able to implement school or government policy in their own classroom, to do mathematical investigations with children etc. There is a strong personally reflective dimension to this where the practitioner seeks to understand how they are practically involved in their professional situation. For example, whilst it may be unfashionable to talk of mathematics being a body of knowledge, such a notion still resides in much colloquial terminology to do with the teaching of the subject. A teacher working on shifting away from practice grounded in a set of beliefs, supposing a "delivery" model of teaching, cannot switch styles overnight. Their practical knowledge of teaching is embedded within a history of understanding their practice, and the specific strategies they employ within it, in a particular way. To move away from this requires a reconciliation between new readings of what they are currently doing and possible tangible alterations to this way of working.

Both incorporating and transcending this practical interest is the *emancipatory* interest - the quest to break free of the ideological distortions intrinsic in the language itself. (For a fuller discussion see for example, Gallagher, 1992, pp. 245-275; Elliott, 1993, p.197). For example, over the last few years the introduction of the National Curriculum (DES, 1989) in the United Kingdom has radically affected the language teachers use in describing mathematics and its teaching. An emancipatory aspect of knowing may be the attempt to see school mathematics and one's teaching of it outside of this frame, perhaps seeking to understanding how such a language serves the people who created it. Similarly, it may allow the examination of the motives of someone promoting investigational styles!

Elliott (1987), in his discussion of what constitutes appropriate educational theory, suggests that disciplines in their raw form for example, psychology, sociology etc., are not sufficient in themselves as a framework for understanding one's own teaching practice and so supports attempts to develop practical knowing built through a self-reflection. Thus far his task is not far from that conceptualised by Carr and Kemmis. Elliott (1987), however, following Gadamer, prefers to see this as the beginning of an on-going hermeneutic process reconciling experience with ways of describing it. He thus introduces the notion of an *evolutionary interest* (Elliott, 1993, p. 197). The teacher cannot merely operate on the classroom situation with view to changing it. He or she is necessarily an integral part of that change - positioned in the very discourses which describe the change. In addressing specific professional concerns the teacher may choose to introduce theoretical perspectives but for theory to be used in a meaningful way there is a need to build in a self-reflexive dimension which positions the evolving individual in his or her evolving professional situation.

In this way Elliott (1987, 1993) criticises Habermas's model which, he suggests, implies a motivation of moving from a bad state of affairs to good state of affairs and replaces it with a model more focused on an on-going hermeneutic process of substituting successive accounts. We are not so much concerned with emancipation from a particular social tradition to a new one free of ideological distortion but rather involved in the rewriting and reconstitution of the tradition in an on-going way. This avoids the risk present in Habermas' approach where the individual is simply initiated into another "distorted" ideology, or of setting up idealism as an unachievable goal. The task for Elliott is to construct stories which resonate with the ways in which things are seen. In addressing the changes in practice the central task is not to learn new techniques but rather to locate oneself in one's own current practice and build a notion of a way forward. As such we are not engaged in a project with an end but rather in a process that always moves forwards, built around a dialectic of action and description of it. The task here is to construct stories which resonate with the ways in which things are seen. In addressing the changes in practice the central task is not to learn new techniques but rather to locate oneself in one's own current practice and build a notion of a way forward. It is Elliott's approach that I see as most helpful in examining the data to follow.

As has been seen earlier, a new prominence has been given to the role of text and discourse analysis in building understandings of human action. The emphasis in recent studies concerned with language has been on how language is used, by individuals and by societies, and on the reflexivity this language usage displays. And this perspective has taken precedence over the study of the structure and system of language per se. The world is increasingly seen as being understood through the filter of socially derived words which individuals use to describe it. Conversely, in seeking to change their actions, both individuals and societies can, in the first instance, work on changing their use of language. As examples; individuals undergoing psychoanalytic therapy seek to change their actions through reframing the way they see them; recent change in educational practices within the United Kingdom was brought about through introducing a curriculum which reorganised the way in which learning was spoken about. In Habermas' work, social evolution is seen as being brought about through such a process of attempting to reconcile social practices with descriptive practices. This approach can also provide a framework for individual growth and, in particular, teachers working on developing their own professional practices.

My task for the remainder of the chapter focuses on the question: How can the teacher move towards offering descriptions of their classroom which become instrumental in shaping subsequent practice and research into it? I work with the assumption that the practising teacher's understanding of their task cannot be captured in static terms. Rather, the teacher needs to see their plans and immediate actions within an on-going process of development. Statements about their practice are never absolute but rather can be seen as position statements, characterising the here and now. I shall begin with an examination of how professional writing can facilitate this process and outline a theoretical framework which employs professional writings as a mechanism through which such the process of professional development can be initiated, monitored and influenced. I suggest that professional writing can be seen as a form through which teachers structure and account for their practice. I then develop these issues in relation to two examples of teacher education courses where practice is reconciled with ways of describing it. In the first I focus on the mathematics strand of an initial training course for prospective primary school teachers. Here students are developing an appropriate language through which to describe their embryo practice. In particular, I examine the transition they make from understanding the teacher student relation from the point of view of the school student, to using an increasingly conventional professional language grounded in the perspective of the teacher. In the second example I focus on practising teachers engaged in masters level practitioner research examining their common sense understanding of their everyday professional practice. The written descriptions they offer function in framing proposed as well as current practice. In particular I focus on how these descriptions provide a vehicle through which change in practice can be understood, monitored and influenced.

WRITING AS A MECHANISM FOR PROFESSIONAL DEVELOPMENT.

How can writing produced within school-based practitioner research, perhaps as part of an award bearing course, function in framing and guiding both classroom practice and the research process itself? Here I work from the premise that the practitioner researching in his classroom brings about changes both through acting in the classroom itself and in producing writing commenting on this classroom practice. That is, descriptions of classroom practice, made by the practitioner, effect changes in the reality attended to by him. I suggest that actual professional practices and the ways in which these are described can function dialectically in influencing each other. The writing generated in this process can be seen as both responding to past action and guiding future action. In short, in describing my classroom, I affect the way I see it, thus the way I act in it and hence the way I subsequently describe it (since it has been changed by my actions). In engaging in this hermeneutic process, teacher-researchers pass through a sequence of perspectives each capable of generating various types of writing and each susceptible to a variety of later interpretations. Here I will examine how such writing can be processed as data towards stimulating this dialectic. In particular, I seek to demonstrate how writing produced within such work itself becomes scrutinised as an integral aspect of practice and instrumental in the process of self-reflexive practitioner-led change. In doing this I employ a method based on the linguistic model of Saussure. In this, absolute understandings of any individual piece of writing are not sought but rather each successive piece added modifies the flavour of the growing collection. I show how this emphasis on writing can be instrumental in promoting the development of professional practice.

As has been seen, post-structuralist analysis has radically rewritten the notion of the human subject. Rather than being a subject in themselves the subject is seen as being positioned in language, as an identity held in the stories told by him and about him. Similarly, the situation in which this subject acts is also constructed. Such a view asserts an instability in both subject and situation so that there is a need to analyse both, which can be seen as part of each other, as processes. The subject and the structure in which he acts are asserted in the ways they are represented in linguistic categories through time. This is always subject to change as more stories can always be told. These linguistic representations are not mere labellings but are instrumental in the construction of subject and structure. It is the very process of signifying in language that brings into being the notions described and these notions then serve in shaping subsequent actions. Unlike the "picturing" role language displayed in early analytic philosophy, modern continental philosophy sees language itself as being very much part of the world being described. Attempts to capture the world in writing are thus instrumental in creating the reality being described. Writing describing the world conditions the way we see the world and thus the way we act. The world and descriptions of it are in an on-going dialectical relation with each other. Such writing however, can provide an anchorage or point of reference as we develop new ways of seeing things.

Saussure's linguistic model works from a premise that a word in a text does not have meaning in itself but rather derives its meaning from its relation to the words around it. To understand the meaning of a text we need to understand how the individual words interrelate. As we have seen, this idea has become a guiding principle within post-structuralist writing. Derrida's use of this notion is encapsulated in his use of the term *différance* - a play on the French words for deferral and difference (1992). For him, the meaning of a text is always *deferred* since the play of *differences* between the terms is never finally resolved and he firmly rejects any emancipatory quest. Here, I am drawing the analogy between the sequence of words in a text with a sequence of pieces of writing produced within a practitioner research enquiry. That is, the meaning of a research enquiry is a function of how the different pieces of writing are seen as interrelating. Seen this way the process of building this sort of research enquiry is inextricably linked with the process of generating new pieces of writing. There is no ultimate understanding of practice since it always continues to evolve. Nevertheless, writing is a product which can be held on to in a fixed form and offers an approach to accounting for the reality to which we attend. So although the courses to be discussed might be regarded as phenomenological enterprises, textual analysis provides an instrument for working on developing understandings.

The parameters of the space for professional action are negotiable as is what can be done within them. Both this space and how it is seen are governed by the language used in describing it and, I will argue here, this can be operated on through the medium of written text. Writing can be used to tell a story about what is going on. There are, however, many ways of doing this and practitioners can seek to be creative in developing productive ways of seeing their practice through this medium. Nevertheless, although such an approach has a liberating feel to it, there is a sobering aspect to this account of post-structuralism that we need to guard against in examining the relationship between a text and that which it seems to describe. As indicated above, any accounts offered by individuals reflect the society from which they come and have, built within the language itself, layers of assumptions endemic in that society's view of the world (cf. Foucault, 1972; Habermas, 1984,1987). The social values we may wish to bring into question can be embedded deeply within the fabric of the society's way of talking about things. There cannot be a clearly defined boundary between creating and inheriting ways of seeing things. The parameters individuals confront and the way they are understood are conditioned by social norms. These norms might, for example, embrace the tradition of understanding teacher practice through positivistic models (Olson, 1995). Such norms can serve to constrain the individual's sense of what is possible, or realistic, in their own particular situation (cf. Buchmann, 1987). Teachers working on building a picture of their practice face a necessary task of developing a sense of the context in which they see themselves working and identifying their position within it. Such a process can help move the practitioner towards developing a language for describing their practice and the situation in which they see it arising. This can be seen as being very much to do with building categories for describing practice; an on-going process of rescripting that increasingly asserts what the practitioner can do in respect of the situations he faces. By capturing successive accounts in writing the practitioner can become aware of the changes taking place in himself, in the situation and in his way of describing it. The commitment of thoughts in writing, such as in the form of a diary, can provide for the practitioner, a device for reorganising his perceptions of the situations experienced. The construction of self implicit in this self-reflexive process locates a notion of human subject susceptible to chronological change and successive pieces of writing can provide markers of time passing.

In a sense, the meaning of any particular story is dependent on its usage in another story. If, as a researcher, I produce a piece of writing, its meaning is dependent on how it relates to other pieces of writing in the enquiry and with the enquiry as a whole as it currently exists. This relationship, however, is not resolvable in an absolute way. The way in which any two pieces of writing relate with each other is dependent on my understanding of my current task. This will evolve through time as I pass through a variety of perspectives on what I am doing. As we have seen, the meaning of an action can be seen as being related to how it is described. The sort of actions I wish to focus on now, however, are the productions of pieces of writing within practitioner research. I suggest that the meanings of such productions are dependent on how they are understood and referred to in other pieces of writing. Each piece of writing produced functions in a particular way in relation to the others. None has an absolute meaning since another story can always be placed alongside. They support different new stories according to how they are used subsequently (cf. Sanger, 1994, 1995). A space is inserted between the event and the description of it. By creating sets of stories relating to practice, the author produces points of reference, which enable him to orient subsequent practice in relation to characterisations of past practice.

In the examples that follow we will see various attempts at framing practice or understandings of mathematics in written or verbal descriptions. For the initial training students in the first example this is associated with a struggle to talk in the "proper" language to satisfy the demands of their training. For the masters students in the second example the process is more explicitly about finding their own voice.

BUILDING A PROFESSIONAL DISCOURSE OF MATHEMATICS TEACHING WITHIN AN INITIAL TRAINING COURSE.

Research, with my colleague Una Hanley, who should take principal credit for the work described in this section, looks at first year undergraduate students following the primary mathematics strand of an initial training course (see also, Hanley, 1994; Hanley and Brown, 1994, 1996). Our project concerns how students use language in developing an understanding of their future professional task. Our particular perspective focuses on the way in which student teachers build on their past knowledge, as pupils, of mathematics and its teaching, in becoming initiated into using conventional ways of talking about teaching mathematics. Our technique has been to work with anecdotal accounts offered by the students in building our own understanding of how students perceive their task. Whilst recognising that we are working with accounts which offer individual perspectives on teaching mathematics, these accounts conform increasingly to socially determined norms as the process of initiation proceeds. Our attempt has been to capture the reality of individual teachers as evidenced in their speech and writing, whilst at the same time recognising that these accounts serve to anchor this reality in some way. We posit the notion of students perceiving "expert" ways of talking about the practice of teaching mathematics (for example, government issued curriculum documents), a practice to which they aspire. We are examining how students seek to employ this supposed discourse of the expert as a template for their own practice.

The following pages of this section are drawn from Hanley and Brown (1996).

The course

The content of both the college and school based components of the initial training course at the Manchester Metropolitan University is not defined in a conventional way. Student's focus on understanding themselves as potential classroom practitioners and working towards designing a style of practice which suits their intentions. The central purpose of the course is to enable student teachers to build and engage in a dialectic between these intentions, their actions, and their reflections on both of these in writing and in discussion with peers and tutors. For this reason work on "the disciplines", as present in many older-style initial training courses, is replaced by greater attention to individuals building their "practical knowledge".

In the early stages of the mathematics strand students work on their own learning of mathematics as a vehicle through which they become aware of possible issues in the learning of children. Understandings developed in this way are then checked out in school placements. Implications for their own practice are then explored, in the first instance, with small groups of children and in limited tasks with a whole class. Eventually this leads to the student taking responsibility for the programme of work for a whole class for an extended period. The assessment of the students rests largely on their skill in representing and justifying their intentions and achievements, both verbally and in writing. The central item in the assessment of the final school placement is a written file through which students monitor change in their practice with view to controlling that change.

I wish to focus here on the initiation students undergo in becoming familiar with conventional ways of talking about professional practice. After outlining our theoretical perspective I offer some examples of students negotiating various aspects of this learning process. First of all I look at some students attempting to reactivate the mathematical language they learnt in school. Secondly, we meet a student attempting to plan and evaluate a lesson, making use of the "official" language. Finally, I offer some examples of students attempting to frame their own opinions and actions in response to reading literature cited within the course.

Students on initial training courses are faced with a task of describing aspects of their task within writing as part of their course. They are however, caught between a number of perspectives as to the nature of mathematics and how we go about teaching it. Part of their task is to reconcile these perspectives with their own experience. For example, within their training courses they become aware of more "official" ways of talking about things - ways they need to learn if they are to successfully negotiate their course. Here I attempt to capture aspects of the transition students experience between attempting to fit a form of words to their past mathematical learning and their attempts to find experiences which fit official ways of talking. Throughout this, words hold on to notions which are in the process of evolving. Students oscillate between description-led experience and experience-led description.

Student memories of being taught in school.

How does a students' experience as school pupils inform her developing understanding of her future task of teaching mathematics in the classroom? In the early days of the course students are encouraged to reflect on their own learning experiences with mathematics. Although in their first year they have very little experience of teaching themselves, their memories of being pupils in school and their knowledge of what our culture expects of teachers, offer an initial frame for clarifying their perceived task. It is from the perspective of being past pupils that most students encounter college sessions.

I offer an example of students grappling with such a task. I focus on how students use language in holding on to their experience as they move from a language rooted in their memories of their own schooling towards the expert language associated with teaching mathematics in school. The students are seeking to recapture the mathematical language they employed as pupils in schools. We join them as they attempt to recall what they know about the volume of a cylinder.

Colin. *Volume of this is in the inside, area on the outside. It's 3.D., you have to talk about surface area ...volume is the bit … the liquid...you have to do surface inside ...*
Angela. *Do you have to do it in the inside?*
Beth. . ... *area of a square, you've got to double your answer.*
Colin. *But it's weird. If it's* 4 times 4*, the area isn't 16.*
Angela. *No...*
Colin. *It's 8.*
Angela. *It's not.*
Colin. *There's some trick. You have to add 2. Cube is multiply by 8, I promise.*
Angela. *Cylinder, volume equals area.*

Beth. *Volume is circumference times length and the area of two ends.*
Colin. *I wish I'd bought my G.C.S.E. notes.*
Angela. *Why do you add a two?*
Colin. *I've spent my life saying add 2, I promise*

Whilst the words used are very much in the domain of "mathematical language", this recalled learning seems to be a little muddled. The mathematical labels (for example, "area" or "volume") no longer hold the ideas associated with them and the students seem to be left with relics of their learning experience rather than with functional tools. In addition, the students seem less concerned with understanding than with demonstrating that they have the means by which an appropriate answer might be found. Many difficulties, it appears, follow as a consequence of misquoting a particular fragment rather than tackling the mathematics of the problem itself.

Whilst not wishing to make too much of one transcript I feel it highlights the sort of difficulties experienced by students on such courses. It certainly provides an example of students attempting to capture their experience. However, whilst the language offered by the students provides a marker of some past experience, in this case, it seems to be rather like forgetting why you knotted your handkerchief. I turn now to considering how students move towards capturing their understanding of the professional language of the mathematics teacher, in their own writing.

Using the language of the professional in the classroom

In the first instance, students' writing is primarily concerned with reporting on work done with fellow students in college sessions on particular mathematical activities. Students keep a diary of such work and review it regularly. Over the course of the first year, the writings become more school-focused as students extend their experience in classrooms and become more familiar with the statutory documentation and a range of professional literature. Through this process they become increasingly aware that they need to be familiar with the specific style of language used in education. Various terms and phrases begin to appear in their writing as they experiment with specialist language. However, I wish to suggest that very often the student's motive is to be seen using the language, if only for the demands of the course, and that the terms appear in the students writing prior to their ability to employ the terms in the conventional way of the expert teacher.

The following extracts offer examples of this. In the first of these, the student offers an outline of her intentions for a class lesson in which she proposes to introduce the concept of balance.

I took into school a number of heavy and light objects. Working with small groups, I allowed the pupils to choose objects and fill two shopping bags. The pupils split into two groups, were to equal the weights in both bags to balance them. We then followed with a discussion. The objective of the lesson was to use better mathematical, more advanced terminology and to grasp the concept of balance. To meet the attainment targets, the children had to respond to questions, make predictions based on experience, select the materials to use for a practical task, to participate as speakers and listeners in group activity and lastly, to respond to instructions.

From this paragraph we get some sense of the student being guided by the language of college sessions and curriculum documents. She has expressed a desire for the pupils to move towards something “better” and “more advanced” where the task seemed targeted towards the children grasping appropriate “terminology”. Her writing appears to be heavily burdened by her perceived need to employ specialist language. The second part of the paragraph displays language borrowed from curriculum documents. They are presented as a string of requirements asserting the authority of the document rather than an expression of practical efforts to support children’s learning. Nevertheless, they seem to be offered here as the means by which the aims of the lesson will metamorphose into action. Students are aware of teacher specific language and that they need to learn to couch their own experience in such terms. The student here seems to be attempting this task. However, her plans as described above appear to be in a rather different language to that which she employs in reporting on the lesson after the event.

Since the groups are of mixed ability, I have a mixed response to how the activity went. Out of the five groups, three are of a higher level than the other two. The two lower level groups found it quite difficult to grasp the points I was making and they did not understand the concept of balance very well. I felt that I needed to plan the lesson in an even more simple way for these groups and *concentratefully on the same ideas before moving on to new ones.*

The student seems not to refer back to her original plan and abandons the language she used in its construction, as she resolves to *"plan the lesson in an even more simple way"* . A residual "expert" term seems to persist, however, in the notion of *"mixed ability"'* which she links closely to the notion of a *"mixed response"*. These seem to be used as "expert" terms where there is no need for further qualification, with an apparent presumption that the meaning would be sufficiently shared. She seems unable to employ the language of the expert in such a way as to connect her actual experience in the classroom with her use of it in the plan. However, her customary way of talking seems equally unhelpful. It is as if she found the circumstances of the lesson so overwhelming that she was unable to correlate what happened with any of the language available to her. Her use of language fails to function as a framework for her own understanding but in reporting on the lesson she seems to be attempting to use bits of expert terminology to hide her own lack of expertise.

Responding to professional literature

Coming into contact with professional literature carries similar risks. Students on the course are required to read various books, as well as statutory documentation. In practice, students seem to focus on very few sources. The task is not seen as an easy one and they have particular difficulty in adopting a critical stance. The texts often seem to be taken as "truths"; the very appearance of the article as a handout or as a book on the recommended reading list lends support to this assumption. The literature offered to students, tends not to be written in a tentative style and, given their lack experience, criticism may be seen as too risky. The following comments were written by a student in response to an article on games.

"Games in maths teach maths as well as teaching other skills.
Games are fun and entertaining so therefore put forward positive attitudes towards maths.
Communication and listening skills are also used so are even more developed.
Games encourage children to build up their confidence and to be

involved in the activity.
Games reinforce andpractice skills that have already been learnt.
Maths games encourage children to think logically and to practice trial and error.
Children, when playing games, they become strongly motivated, they get involved deeply in the game."

This particular piece of writing, shares with others the sense that the student has identified certain issues raised in the article and itemised them. "Positive attitudes", for example, a term raised not only in the article, but also within statutory literature, remains both unexplored and unconnected to the notions of "confidence", being "strongly motivated" or "involved deeply", which appear in other sentences. The student's involvement with these words itself seems to be one of "trial and error". The task set in relation to the article seems to have been understood as an exercise in using the correct language. The contexts for the games, carefully provided within the article, are disregarded in the search for appropriate terminology.

The government issued curriculum documents governing practice in British schools are an example of the categorisation of the learning experience on a large scale. Lists of categories describe knowledge which must be learnt and the classroom strategies which need to be employed. The bulk of documentation offers further instructions on how this learning might be evaluated, described in very specific terms. The practitioner has to negotiate a "fit" between these descriptions and what happens in the classroom. The first step towards creating this sense of fit may be to opt to use the language on offer, even where this can only be accommodated with difficulty. For students having their initial experiences in school, particularly, the quantity as well as the complexity of activities under view make the fitting and fixing of language highly problematic, not least because the language itself offers difficulties.

For example, the student clearly has difficulties in making sense of the language employed in the professional article relating to games. Her experience of games within college sessions has not served as a means for making sense of her reading. Typically, the language that she was expected to employ, as embodied in the article, was not offered to her until after her "hands on" experience. Whereas some other students would work more successfully towards creating a relationship between the language and the experience and finding some sense of "fit", the student here appears to be unable to do this. Instead she appears to offer some labels which may possibly be significant.

BUILDING AN UNDERSTANDING OF THE TEACHER'S TASK THROUGH PRACTITIONER ENQUIRY.

Practitioner research within a masters course

In discussing the issue of generating pieces of writing as data within practitioner research I will focus on my own teaching on a part-time masters degree at the Manchester Metropolitan University, designed for practising teachers (as discussed in Brown, 1994 e, 1996 b). The course attracts a broad range of teachers from those working in the primary years to those working with adults in colleges or universities. My particular concern here is with how pieces of writing reporting on practice become data within practitioner research enquiry.

The course does not have a syllabus as such given its intention of enabling teachers to develop their own framework of analysis. This is expressed clearly in the course documentation:

> The course ideology is concerned with what Aristotle called phronesis - practical knowledge or wisdom. Such understanding is not reducible to technicalities or technical knowledge because praxis cannot be reduced to means/ends analyses. Because the course experiences are radically constrained by *concepts* of practicality its intellectual origins lie in pragmatism and in continental philosophy in its concern with the phenomenology of lived experience. The hermeneutic enterprise has to begin, we believe, with "finding one's own experience" (Pearce and Pickard, 1994).

This attitude is reflected in the approach taken to assessing student's written work where there is an attempt to minimise imposed structures and assumptions. For example, the assessment of the final dissertation attends to criteria where the emphasis is on students building their own defence of their work through what is intended to be a relatively neutral frame:

1. *Conceptual understanding* - the student's capacity to understand and explain their own professional beliefs and practices.
2. *Research methodology* - the student's enquiry is systematic.
3. *Intellectual context* - reference to relevant debates in current

educational research and related fields in analysing the student's own specific concerns.

4. *Local professional situation* - pertinent analysis of the student's own professional situation, for example, their school.
5. *Broader professional context* - pertinent analysis of the student's broader educational context.
6. *Strategic implications* - evidence of developmental engagement with practical professional concerns.
7. *Professional intent* - evidence of the student identifying his or her own wider professional development within the course and outlining the implications this has for future practice.

For teachers on the course, the initial task when beginning the course, entails capturing the context as they see it and identifying their position within it. This implies a stressing and ignoring process consequential to their engagement in their situation. This process, which takes place over a year, moves the practitioner towards developing a customised language for describing their practice and the situation in which they see it arising. This is very much to do with building categories for effective description of their practice. This might be seen as an on-going process of rescripting that increasingly asserts what the practitioner can do in respect of the situations he faces. By capturing successive accounts in writing the practitioner can become aware of the changes taking place in himself, in the situation and in his way of describing it. The commitment of thoughts in writing in the form of a diary provides for the practitioner a device for reorganising his perceptions of the situations experienced. The linguistic categories passed through in this process are transitory holding devices facilitating the shift from a receptive to an active mode. This process is in some ways akin to Schön's (1983) notion of "naming and framing". In deciding to act, however, it is in a sense necessary to "suspend doubt" (Schütz, 1962, p. 229) and act asif the current way of seeing the situation, the present framing, is accurate.

An important aspect of the course contributing to the development of this customised language is collaborative work with fellow students and tutors. Verbal and written accounts offered by the students are scrutinised by colleagues in small group discussions aimed at tightening up the language used and sharpening subsequent action in school. As part of this process students visit each other in school so that alternative perspectives might be offered about the situations being described. As the course progresses an increasing emphasis is placed on more extensive critical reading to enable the developing notion of self and situation to be contextualised more broadly. The practitioner is positioned in a multitude of other discourses (for example, gender or race of teacher, experience and status in school, political stand etc.) which need to be examined in the on-going construction of self. It is through reading and consultations with tutors and peers that ideological insularity embedded in common sense attitudes can be challenged.

In describing things in the world the teacher is by implication describing their relation to these things. As she continues to build up a collection of such descriptions of things in the world around, she is positioning herself in relation to them. In this way, through the reflexivity of such acts, the teacher is characterising herself by the way in which she perceives the world around. In building such a picture of themselves practitioners on the course are asked to describe, incidents arising in their professional practice as a vehicle through which they get a sense of their being in the world, captured in the categories implicit in the language being used. For example, a practitioner might describe the first two minutes of a lesson she gave, or a single piece of work by a child, or a minute long conversation with a child. This exercise simultaneously generates stories about the practitioner and the situation of which she is part. Initial stages of the course might then be seen in terms of collecting a variety of such pen portraits, each capturing some specific incident which in some way reveals the practitioner acting in her professional context. By analysing and comparing these pen portraits the teacher can build up a picture of herself in their professional setting. Any stories told by the practitioner about links between these pen portraits can be seen as a move in to a more generalised view of this practitioner but still grounded in specific incidents.

This whole process is concerned with the construction of the practitioner's professional identity. By using a process akin to the psychoanalytic technique of free-associating links between the incidents described the teachers can be asked to say what this tells them about themselves. Their field of practice as they see it is captured through this selection of localised pen-portraits each offering a snap shot of the teacher in action. These almost arbitrary snippets can be seen as nodal points around which the overall field of practice can be oriented. Through comparing and contrasting pen portraits the space between these nodal points can be filled as more general characteristics of the teacher emerge through the telling of successive stories. These generalisations however, are grounded in the specific incidents through which they were themselves generated. On the surface, this is rather in the spirit of Levi-Straws's structuralist project of exploring the myths prevailing in certain primitive societies. By collecting and analysing the myths of a particular society he identified common threads which emerged, providing some sense of an essence to non-members. In this way he argued that it was possible to locate and describe some objective structure underlying the mode in which the society operates. The task of the practitioner, however, is to learn about themselves as an acting subject through identifying characteristics that reoccur in the various pen portraits. These pen portraits are only temporary anchorages which resist any move to a more stable account. Through these methods the teachers move towards categorising the situation of which they are part, but within structures that move with them.

For a teacher on the course, however, there is a need to categorise his practice with a view to action. By shifting attention in this way, writing done for the course, in respect of professional practice, becomes a mechanism for clarifying objectives and possible outcomes. The writing provides a way of holding the categorisation, if only for a moment. The teacher then acting as if the present categorisation is valid has a framework through which to assess his actions. Changes in categorisations arise through the passage of time as the perceived field of action shifts in relation to successive sets of concerns. In describing developing professional practice there is a need to build in an effective mechanism to account for the time dimension implicit in this process. Any action can be seen as having both a responsive and intentional component. That is, any action simultaneously has a cause and is a cause. In dealing with this Schütz (1962, pp. 21-22) draws a distinction between "because" and "in-order-to" motives to separate two different sets of concepts.

a) We may say that the motive of a murderer was to obtain the money of the victim. Here "motive" means the state of affairs, the end, which is to be brought about by the action undertaken. We shall call this kind of motive the 'in-order-to motive"...

b) We may say that the murderer has been motivated to commit his deed because he grew up in this or that environment, had these or those childhood experiences etc. This class of motives which we shall call "because motives" refers from the point of view of the actor to his past experiences which have determined him to act as he did.

It is often a characteristic of practitioners starting on the course to emphasise "because" motives in their writing. The emphasis is on the situation as they see it; how the school operates, what the teachers and children are like, their views on the school administration, how they judge themselves as teachers, etc. The school is constructed according to the categories through which it is perceived by the practitioner as an observer and participant. Such accounts are perhaps seductive, especially for in-service practitioners on an evening course wishing to off-load after a hard day teaching in school.These pieces of writing tend to locate the teacher as a recipient of a given situation. In the turmoil of things happening in a stressful day, accounts of how the world appear seem more immediately pressing than a reflective response concerned with identifying the intentional component of what the teacher did. The fatalism endemic in this sort of writing can be seen as dis-empowering where the teacher is passively receiving that thrown at them. Their writings emphasise their response rather than their resolve.

In many respects the course's principal function is to enable the practitioner to redescribe their situation in terms of what they can do about it. This is to do with building a more assertive voice, categorising their practice according to the control they can have over it. Given a particular situation how do I act now? The focus moves towards responsibility and control. The task for the practitioner becomes more to do with learning about how he does things in certain situations. Accounts now capture the practitioner's view of their intentional and potential actions rather than descriptions of arbitrarily chosen situations. The picture the practitioner constructs of themselves becomes one of someone making decisions about how they need to act, in-order-to bring about a certain state of affairs.

Teachers entering the third year of the course will have compiled a huge body of writings - a mixture of small and large pieces, transcripts, lesson plans, anecdotes, responses to reading, responses to sessions, etc. A principal task during this year is to consolidate and extend this work so that it becomes more clearly targeted on a specific theme for focused enquiry. The task of constructing such a theme is to serve as a guiding principle for third year work and, in particular, in the production of a dissertation at the end of the year. A strategy employed in the first few weeks of the third year, is specifically directed towards the clarification of this theme and with how pieces of writing function within it. This strategy is, firstly, to ask the teachers to choose a small piece of work (maximum of one page) written by them in the past that they see as having some resonance with their chosen theme, as they currently see it emerging. Secondly, having selected this they are asked to set up a situation in their teaching during the following week which will result in another piece of writing which they see as being about working on this theme. At the following week's session they bring the two pieces, old and new, together with a one sentence statement of their title as they currently see it. Copies of each of these three items are made for the members of their subgroup. The next session begins with these pieces of writing being circulated to all subgroup members. Each person is asked to write a paragraph for each of the people offering the three pieces. The paragraph is to say how the reader sees the two pieces of writing being concerned with working on the given title. On receipt of these paragraphs the writer is asked to make a statement about their proposed theme, and how they see themselves working on it in a way, which makes explicit reference to the comments made by their subgroup colleagues. This statement then forms the basis for the next cycle. It is through this process that the structures inherent in the writing become *realised* in*formatting* actual practice (Skovsmose, 1994, pp. 42-58).

The act of writing is inevitably associated with an act of reading. In writing this student needs some understanding of how he will be read. In conversing with others, resonance is important. I show my understanding of your story by offering a related story. I substitute your example for another in an attempt to emphasise and extend your point, but also to see how it fits with my own experience. In doing this I bring meaning to your story for myself and perhaps, in revealing my perspective, shift the way in which you understand the significance of your own story (cf. Cryns and Johnston, 1993, pp. 149-152). Such a dialogue conditions the way in which subsequent action is planned and reported on. Another technique employed within the course described above, concerned with confronting this task, has some similarity with the game of Chinese Whispers. In framing my experience in a story it can be quite illuminating to examine how my ideas sound through the voice of another person. Subgroup colleagues are frequently invited to make comments on someone's writing or verbal delivery so that the writer/speaker can hear himself being "played-back" through the voice of another. It may also be that someone from another subgroup or a "spare" tutor might be "borrowed" to witness this summary and to write a one sentence statement which for them encapsulates the summary. The original speaker is then asked to make a statement saying how they see their original statement differing. In this process the author uses earlier pieces of writing and responses to them by peers as points of reference in creating new pieces, under the umbrella of the revised thematic title. By reassessing past writing in the light of peer response he becomes clearer about the way in which he might generate and work with new pieces.

The function of these exercises within the research process is to integrate writing into the framing of the research enquiry. The writing produced in respect of the enquiry is not only about mapping the action on the ground. Writing is an integral part of the action being described. It provides a way of framing experience in a fixed form so as to pin down some aspects of this process with view to orienting this process. In doing this the writing itself becomes part of the substance of the research enquiry. Like the actions in the classroom it becomes part of the "thing" being reported on. The conceptions in the writing become *realised* as they frame actual practice. Further, it *formats* the reality attended to for future action (including future writing) (cf. Skovsmose, 1994). As a consequence classroom practice by the individual becomes increasingly conditioned by the linguistic framings being brought to it by them. For example, in having selected an old piece of writing with view to creating a new piece the teachers are structuring a piece of actual practice for the purposes of creating a new account. There is embedded within this an attempt at creating a resonance between actual practice and ways of describing it. Practice and description of it become mutually formative in an hermeneutic relation.

Another function of such exercises within the course is to enable the researcher *to* become aware of how their research is developing. Of particular concern to someone in the middle of action-oriented research is to decide where to go next. An important dimension of practitioner research is that it entails going through a sequence of different perspectives, where each perspective is informed and flavoured by those which have preceded it. The next step cannot be preplanned since often I will not understand the circumstances until I am confronted by immediate possibilities. In practitioner research, which downplays any notion of a detailed over-arching plan, I need to be rigorous in making the next step. On this point Mason's work (for example,1992, 1994) is helpful in examining the state of being *inside* a problem. He has addressed a variety of types of problems, both within mathematics and within practitioner research and professional development. In particular he has worked on the task of deciding what to do next. A key aspect of his work is learning to recognise in current problem situations characteristics one has experiencedbefore. This might be seen as being a task in assessing the environment in problem situations so that features of current situations might be associated with past ways of reaching a resolution. In this paper, pieces of writing are being offered as a way of marking the environment of the teaching problem and thus providing an orienting framework. Elsewhere (Brown, 1994 e) I have suggested that this is akin to the work of Dockar-Drysdale (1991, pp. 98-111) with emotionally deprived adolescents. For these children, who experienced difficulties in orienting themselves in their everyday lives, the teacher employed a technique of helping them create and remember stories to which they could return, so as to provide points of reference for new stories. Employed within practitioner research this technique provides textual constructions against which the meaning of new stories can be constructed - the meaning of the new stories being relational to those already in place. Such a framework can become instrumental in understanding how practice is changing.

To pursue an "evolutionary interest" within practitioner research there is a need to build an understanding of change. I suggest that the task of practitioner research enquiry is, firstly, to understand this change, secondly, to monitor it and thirdly, to influence it. I am proposing a notion of change as evidenced through markers separated by time. Such a marker in this instance will be a piece of writing within the research process. For the classroom practitioner there are many strands evident in change. Further, the researcher's perspective of this change is susceptible to change, as is his way of describing it. In the classroom the children change because they get older, because of the change of teaching style and because ways of monitoring their progress change. The teacher changes because they get better (or worse!) with practice, because they bring new structures to their ways of describing their lessons, because the children change etc. For the teacher researcher, change is something of which you are part, something you observe and something you report on. There is a need to experience yourself as part of it before you can report on it. Making sense is done retroactively. Pieces of writing can function as markers in time, capturing how things are seen at a particular moment. By comparing pieces of writing produced at different junctures the writer can understand how certain things have evolved.

Practitioner's writing

Before concluding this section I offer some writing by teachers following the course. My examples will be restricted to those who teach the normal range of subjects in primary schools but have chosen to include a focus on their teaching of mathematics within their work for the course. These teachers will not be mathematics specialists and may have done no formal academic study on mathematics teaching since the mathematics component of their initial training as primary teachers. Whilst recognising that in this book I am talking primarily to the mathematics education community, I need to stress that the separating out of the mathematics from the rest of the curriculum is, for these teachers, slightly artificial. Their work in mathematics is understood in a broader context of all the school subjects they teach and their classroom strategies are developed on a broad front and not created in respect of specifically mathematical concerns. For each of the two teachers cited I offer extracts from early chapters in their final dissertation where they engage in a detailed analysis of their classroom practice through an examination of their work in mathematics. In later chapters this early writing is used in addressing broader teaching issues and their own developing personae. Although my intention is to communicate something of the flavour of the course through some brief extracts, this is doomed to failure given the impossibility of representing three years of sustained writing by the people I mention.

a) As a first example I offer writing by Linda Chamberlain, a teacher tracking her own development through the writing she produces. I offer some extracts appearing together in the final dissertation. Chamberlain (1995) chose to focus on the way in which her early practice was governed by her understanding of a child centred pedagogy. Her approach in her later work for the course entailed collating pieces of writing produced during the early stages of the course with a view to understanding the way in which her practice was motivated by such beliefs. Having engaged with left wing writers offering critiques of such a style of work (for example, Walkerdine, 1988; Sharp and Green, 1975) Chamberlain then seeks to provide a revised reading of how children are grappling with mathematical tasks and in the light of this develop her teaching strategies. As such her work for the course revolves around her attempts to reconcile a number of discourses, namely, child-centred pedagogy, Marxist critiques of these, her school policy, the curriculum and past and present interpretations she has of these with regard to her own classroom practice. I offer the sequence of extracts appearing in her dissertation together with some of her commentary on them:

Chamberlain, writing in 1995, is looking back on earlier accounts of her teaching, produced during the first two years of the course:

I was "getting it wrong" and thought the solution lay in the reinforcement of child-centred practice. I was convinced that these beliefs were the "truth" and that the only possible action was to better match the practice to the ideology. To confront this problem I embarked on the strategy of keeping a journal, I recorded actual incidents in the classroom together with any subsequent analysis. I purposely selected for examination areas that caused me concern. For example, after a 'free-choice" session in 1992 I wrote a piece that contained both my perceived ideal and my major misgivings:

Extract 1

A group of children are involved at the art table. The class has lots of activities going on: some children are using the tools, others are using the construction kit to make a maze for the hamster, three are working with Logo on the computer, there is a table of girls working through the maths scheme. I have had to tell the children to share, to be quieter, but generally there is an air of activity. I look back and think this is the ideal, that is, until I look closely at the children involved in the art work. I then become anxious. They are splashing paint on the paper again and folding it over to make butteflies. I have a contradiction. I want freedom to be creative. I hold a belief that the children are creative and should be allowed to develop through self-motivation, yet they are still painting butterflies.

Extract 2 (1992) written shortly after was an attempt to theorise my thinking. I labelled it as "an attempt to theorise my position regarding the classroom"

Extract 2
My ideal situation is one where children are responsible for their own learning and the motivation to work comes from within. The children would determine their work programmes. Deep down inside Ifeel that the children would gain great personal satisfaction, learn more successfilly and thereby achieve far more at the end, if this was the basis of the practice I employed in the classroom Indeed there would be no end, as such, to their learning as the basis for personal development would he built by the child, not by the teacher or some other outside influence. In simple terms the child would be responsible for his/her own learning. This theory is built on the premise that children have inquiring minds, that they are lively, inquisitive and self-motivated beings, not lethargic, disinterested automatons.

How do I now see this version? At the start of my research my writing places children as individuals who, if allowed, are able to take total responsibility for their own actions... Thus the child within this "reality"has an innate desire to "learn" and knows what it is necessary to learn. He or she will learn about everything and use the knowledge rationally Extract 2 suggests that the children can place themselves within a form of curriculum beneficial to themselves, as well as being educationally viable to them... I imply (extract 2) that learning and personal satisfaction are mutual partners and both noble. I accept these beliefs as axiomatic. For me this "natural state "for the child is an idealised "reality". I had read the writings of Dewey, Rogers, and radicals such as Illich which cast children in this light. Ifound their ideas persuasive and wanted to offer children as much chance as possible to interact with their environment. Like these Progressive writers I too found that children seemed alienated and negative to much that was going on in the classroom, ..School did not seem like the ideal learning environment.
Extract 3 (1992) is an attempt to analyse the description of my practice as recorded in extract 1 (produced earlier that year). However, my then child centred faith clearly clouded my judgments:

Extract 3

The children were investigating colour. They were discovering symmetry They were active. I need to show them other pictures of nature where symmetry is present. I felt that I needed to take them on from this stage to another. ....I am concerned about open-ended questions. I ask them why they are painting butterflies. They look at me with puzzlement, They said they do not know. One child said that she likes doing them I said, "Can you see the symmetry? Have you seen other things that are the same? Look as if they have been folded over like this? "They suggest petals and faces are symmetric. I ask if they have seen a honey cone. I left them because the maths group was asking me for help. I hope they paint something else next week. Every class in the school has symmetric butterflies of the wall.

...I wanted to facilitate their learning so they could experience the classroom in an active personal way. I wanted the children to be as independent and responsible as possible. They had to be active in their own learning... The children painted butterflies, I looked for an educational angle and selected symmetry. I identified this as the children's stage of development and was determined to capitalise on it.

Chamberlain then follows this analysis of her past practice, through examination of earlier writing, by reevaluating this practice in relation to Walkerdine's critique of child-centred pedagogy. Nevertheless, she works in a school where certain procedures are on the syllabus and she is obliged to teach them in line with school policy. Given these restrictions Chamberlain asks how she can still pursue her own educational priorities. For example, she describes some work with children where she is introducing the "Decomposition" method of subtraction. To shift the emphasis away from her own explanations she encourages the children to discuss their task with each other. After extensive accounts of such discussion, with various transcripts in support, Chamberlain continues:

They are questioning each other about the sense they bring to the situation.. They call upon their experience of previous sessions to recognise that I wish them to discuss the work. The message I attempt to give them in this session is to talk about and try to make sense of the work. I wanted them to use their past experience in *these terms. I avoided helping them and simply left them to it. When asked to assist I reply with, "What do you think?" This worked quite well...*

In her conclusion, Chamberlain gathers some of her thoughts:

I began to see the children and myself as individuals attempting to make sense of their situations. Given that interaction was the fulcrum, enriching the classroom experience would lie in expanding the range of each concept and its link to language. For example I now realise the action of completing the sum (in an earlier chapter):

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-476

was actually enriched by using various upproaches to produce an answer. By enlarging the experience, the children's concept had been broadened. The language (labelling of concepts) can be either spontaneous or teacher provided. But great benefit would be derived from the children recognising the link in that it could be used as a point of reference. However, it is the interaction that has the intrinsic value.

I had recognised the participants in interactions as having different positions... I hoped to empower them to take better control of their own thinking. The use of research methodology has enabled me to recognise the relationship between theory and practice. Through this my experience could be captured in text and analysed critically. This enabled me to journey to a "reality" which feels more authentic. At this stage in my journey I still see the school as my vehicle where active research had let me look out of the windows. This methodology has endowed me with a new skill of reappraising past "realities" to make the affect change.However, while on my journey I will always be trapped within discourses. Any research cannot remove discourses as it merely serves to set up others. Nor can remove relative positioning occupied within schools. Recognition of all this has been the important factor. This is the stage I have now reached in my "journey of awareness". A final thought is to follow Foucault's (1984, p. 374) advice:

"A demanding, prudent, "experimental" attitude is necessary; at every moment, step by step, one must confront what one is thinking and saying with what one is doing, with what one is."

b) Lorraine Dooley, at the time a Deputy Head in a primary school with class teaching responsibilities, focused initially on her questioning strategies within her mathematics teaching as a trigger to a broader examination of what it means for children in her class to become "independent" and "autonomous human beings". I offer some selections from her chapter which addressed her mathematics teaching, where she considered her perceived difficulties in giving over more responsibility to the pupils whilst still equipping them with the skills prescribed on the curriculum. In these extracts there is a strong emphasis on revisiting dialogue with children, as evidence of past ways of working. However, unlike the Chamberlain extracts cited above, here we have textual analysis operating in a more immediate way, with analysis of transcripts produced last week being used as a device in framing intentions for next week. Dooley (1994) offers a selection of transcripts, tracking her progress over a number of weeks. Initially she characterises her practice as being grounded in a belief in open ended questioning styles, where she reads her own actions as being in line with this policy. She expresses bewilderment when challenged about this. Later on she moves towards building a more sophisticated notion of what she means by independence and styles of teacher input facilitating this.

Dooley (op cit., pp. 38-40) introduces her concerns over her teaching of mathematics:

The tension which emerged is that although I feel I encourage children to exercise some control in certain areas of the curriculum, I assume tighter control, mainly in mathematics. So there are times when the way the curriculum learning is managed blatantly denies the children a sense of ownership. Do these instances contradict, and therefore counteract, what I am trying to encourage and are there, therefore, equally or even more important aims that I feel take priority. Do I believe that there is a curriculum which exists independently of the children and that to developing independence and autonomy included the process of children coming to know these things which are included in curricular areas like maths? If I value responsibility and ownership in other areas... How can I enhance the development of these desirable qualities in other areas of the curriculum over which I have tighter control?

I made the following statement, after producing evidence and talking about situations in the classroom which I perceived as children developing autonomy: "There are still things I need to teach them (the children) especially in Maths". This was quickly *jumped on by the tutor when he asked "Why? Why do you feel you need to 'teach' them?" The question stupefied me somewhat, but I answered: "Because by teaching the children 'new' mathematical concepts, which sometimes had to be didactically, they would have a good grounding or base for the more open-ended mathematical activities and problems presented at a later stage". The discussion progressed and I argued that I did encourage thinking in the children, by my use of questioning, which I viewed as more 'open-ended' than 'closed' type or reversing questions asked. However, I was encouraged to explore the notion that children can find things out for themselves, even in maths and therefore create opportunities for extending these boundaries that I had erected for myself I therefore started to look closely at my teaching practices in maths and explored the different ways I could influence the nature of the decision making process, with particular reference to my interactions with children.*

Dooley (1994, pp. 40-50) offers some transcripts capturing some of her dialogue with children:

Teacher: *What number do you multiply first?*
Lisa: *The 2.*
Teacher: *Why the 2 Lisa?*
Lisa: *Because the 2 is in the units column, so we multiply the units first*
Teacher: *Good (nodding)*
Kim: *Can you see whether I'm doing this right...*
Teacher: *Can you tell me what you've been doing with the ones you have already done.*
Kim: *Well, for these . I set them out on some paper like this (starts to write H T U) .. then I put the numbers in under here like this ... then I added them.. like this*
Teacher: *That looks fine to me Kim How could you check them?*
Kim: *Use the calculator*
Teacher; *There you've answered your problem yourself, haven't you (smiling)*
Kim: *Yes.*
(A little later)
Teacher: *Well remembering what you've just done for 76x4, can you do the same for 76x20 here?*
Ben: *Yes I know ... we can put the 1520 here and then*

put an arrow showing 76x20 here. Is that right?
Teacher: Good. Well done!

These brief extracts show children asking direct questions and being given more direct answers, utterances and positive gestures for *example, smiling of nodding of the head) and showing my approval at what they have done for example, Good! That looks fine to me. Well done!). In these instances I cannot really gain an understanding into what the children are thinking because they are answering direct questions.*

This transcript was discussed with colleagues on the course. She continues in her dissertation (op cit., pp. 42-44):

As I begin to modify my teaching style, I adopt a different questioning technique and it becomes a more powerful tool for encouraging children to think, calling for more thought and explanations, calling for analytical reasoning and informed judgment.
This second extract, taken a few weeks later, illustrates this
(Here I extract some examples of teacher speech from a long transcript):

-How are you doing these Kim?
-Why? Tell me why you thought that?
-...but how did you arrive at 11x11?
-Yes, I understand that but...
-So whats a square number?
Well, David said that 11 is a square number. Do you agree with him?
-Can you tell David what you think a square number is then?
-Do you understand that David?
-Fine. Just see if you can discover what a square number is and what isn't...
-So what have you found out about square numbers?
-Can you tell me a bit more?
-Why?
(A substantial discussion between pupils follows this.)

After examining the details of this transcript Dooley (op. cit., pp. 46-47) continues:

This transcript has brought me to a clearer understanding that the way children think and learn, in a mathematical activity, can he influenced by the teacher's questioning technique. The children de-centre, think about and reflect upon their thoughts and explanations, and consequently become more analytical, less impulsive and achieve more effective control over their learning. By modifying my own teaching style to adopt different questioning techniques, I am giving the children time to elaborate and reason out loud, learning how to express their ideas, formulate their thoughts and say what they know, providing opportunity to regulate, reason and explain themselves, so improving their level of performance... I am now beginning to see my role changing from that of a person who imparts knowledge to that of a person who is responsible for carefully structuring the learning experience of the children, only intervening where necessary. I am now inviting interaction, negotiation and shared constructions of experiences which will enable the children to learn. Although I have not taught these children anything 'new' about this mathematical concept, I have encouraged them to use skills and knowledge they already possess to understand more clearly. I see my role as now challenging the thinking going on instead of explaining how I think it should be done. I can also see the changes with the children to which I am giving more status.

So this is where I have got to, which I feel is more successful. Earlier I was speaking in negative terms - "I can only teach maths this way". By monitoring changes - hence raising the question I was asked a few months ago - Do values shape our practices, or do practices shape our values? I am beginning to think the latter could be true!

Dooley (op cit., pp. 49-53) follows this with a detailed description of some work with a child in her class. I offer a very brief extract from an extensive transcript provided by Dooley in capturing the flavour of her new approach to questioning:

Teacher: *You look happy about something, David!*

David: *Well, I got the shape like you said and I measured all the sides . But I knew they's be the same length. Then I thought, well the angles must be the same. I mean, even though I was making a bigger one ... this is just the same only a shrunk down version, the angles will be the same.*

Teacher: *How do you know they are all the same size, David?*
David: *well ... I know an angle of a square is always 90 °- it doesn't matter about the size and a triangle with all the sides the same is always 60° so I thought this is a regular pentagon, the same will apply to this.*
Teacher: *So what did you do?*
David: *So then I measured up all the angles and I marked it down as 70 at that stage, but David O'Shea asked me how I did it and then I said I just measured the angles like this (picking up his protractor) ...a and then, I saw then that they were all slightly above 70°, so I added them all together, Actually they were 72°.*
Teacher: *How did you get 72°, David (confused)*
David: *Well I just measured them again to show David And I saw that the protractor ... this bit on the bottom here (the protractor base line) and it wasn't straight, so then I measured it again and it came up as 72°.*
Teacher: *And what make you think this was right?*
David: *Well, I thought 360° is a full turn and I thought well this round the edges here is nearly a full circle with the edges cut off and I thought well it loses some as you cut off the rounded part and it gained some as the turning is sharper. Then I thought, well it's logical . it's 360° ...so then I measured all the sides again on my drawing and they were all the same.*
Teacher (confused by David's explanation)
Show me which angle you measured.
David: *This one here ... look, it's just of 72° (measuring outside angles) and I worked it out - 7 by 72° is 360°.*
Teacher *Oh, I see now (although still a little confused)*
David: *But also ... if you measure this here (inside angle) that comes to 108° so it must be right, because if you think about it, this here (putting his ruler across the shape) is a straight line and 72° and 108° is 180° which is a straight line.*
Teacher: *Right I understand now. What did you do then?*
David: *First I decided the length I wanted my sides and then, working with the angles, I worked out the pentagon.*

By allowing David to reflect on his experiences and the kind of thinking her was engaged in, he became more aware of the activity in which he had been engaged. Tackling a new problem, he *brought his own past experiences to bear on it and made productive use of them He invented a workable method to solve his problem, a method which I couldn't have envisaged and one which had confused me initially when he was explaining the process he went through to arrive at his answer... When I began this chapter, my belief was that mathematics was something which was outside independence and autonomy, but I now realise that this was an indication of my state of mind, my attitudes, my values and belief about mathematics. But is it also a general message I have, of what children are capable of? ... My own notion was that the conceptual structure of mathematics as a subject was one where certain mathematical content had to be "taught" in a linear way. In other words, I felt that "basic" concepts had to be taught and learned before more difficult concepts could be tackled. This therefore limited the range of my practices, which then served to define and reaffirm the academic nature of the subject. If I believed that children needed a structure, I therefore questioned their ability to create structures of their own... Being somewhat forced to experiment with maths has made me peel back old beliefs in order to examine previous assumptions. But what are the beliefs and values I hold, in particular to the teaching of mathematics, and from where have I got them. I showed earlier that my beliefs and values came from a variety of sources, which included my own education in school, college training and my experiences in different schools. How then have these influenced my beliefs about teaching mathematics?...*

In her conclusion she contextualises this earlier work:

I now know that it has nothing to do with techniques I use in the classroom, it is not just me coming up with some good questions as far as maths is concerned, it is about me opening up and creating a set of possibilities for children to explore the world in all kinds of significant ways and to be objective about themselves, their role, their conduct and who they are.

Dooley (op cit., pp. 123-128) leaves the evaluation of the outcomes in her classroom to some of the children working with her:

Carly: Like with our work organiser, we get to choose, like what to do like, what we want to do. Like you're not told "you're *doing maths now" or "You're doing English now". Like we know how much we've got to do '... like we've got a lot of maths to do, so we think we'd better get that done or do the rest of the English say.*

Julia: *well... like when we've been in other classes, some people would understand bits of it and other people understood other bits ... you don't really know really why you work something out - the teacher has just told you how to do it.*

Carly: *Yes like its in maths, like instead of working it out like the text book tells you, you can use your own ways, which means you have a bit more independence and you understand it better... a lot ... well I do.*

Kim: *Like now in maths, you feel why you are figuring it out like that and you're not just doing it because you were taught that way. Like it helps you to think about .. like you've not just got someone at the front going blaa, blaa, blaa ...*

Carly: *Yes like when we did circumferences, I remember Mrs Dooley saying 'What if I was an alien, how would you describe a circle? and we all had to figure it out like...*

Kim: *Yes, I'll always remember that the circumference is like ... you know... when you put a piece of string on a point ... then all the points on that circle are always equal.. that is the radius ... and then we measured all those cylinders, Pritt Sticks etc. and found out that the circumference was always three and a bit more than the diameter - it was really good.*

Carly: *What I like about the way Mrs Dooley teaches, is that she ...erm.. she doesn't really tell you how to do it immediately, we have to we have to try and do things ourselves first ... work out our own methods of working out sums sand things like that ... and then she'll give us some clues if we get stuck and then ...*

Kim: *Like at first I panicked in maths, when Mrs Dooley asked me to say how I had worked something out ... like I'd done the work but I'd not realty understood it, so I was confused ... Now I feel I can work out things for myself ...I'm not doing it one way, 'cos I'm meant to be doing it one way.*

Julia: *I used to feel ... stupid before... like you'd think ... like if you said something, like before the teacher would say like 'Well you do this' and explain it again and then you won't ask again 'cos you feel really stupid because you haven't grasped it.*

The teacher's task has a close relation with how he describes it. Such describing, however, evolves as it is progressively reconciled with actual practice. This chapter has focused on how the teacher can work on developing his practice through analysing the things he says about it. We have been chiefly concerned with teachers in a reflective mode, in particular, with those consciously seeking to describe what they see themselves doing with an accent on what they can do to develop things. Through working on this perspective, either explicitly or implicitly they engage in a process of self-formation, positioning and identifying themselves through the way in which they speak of the things in the world around. I have suggested that within this process teachers can assume certain liberties in deciding how to describe their practice. A practising teacher may for example be able to assume a positive attitude in writing the script of their practice. That is they can be pro-active in controlling the way in which they choose to construct themselves.

The university courses described here displayed a deliberate policy of enabling students to construct their professional space and develop strategies for navigating around it. I have sought to emphasise two key aspects of the role of writing within this process. These are:

i) writing as an integral aspect of the classroom action being described,

ii) writing as an important marker of time in monitoring change.

In producing writing as part of the practitioner research process I am creating part of the reality to which I attend. Further, I construct an understanding of time through selecting and composing sequences of pieces of writing. Consequently, the process of research becomes a task of, firstly, positing a way of doing things in writing and secondly, assessing this writing in relation to how things are actually done. Neither of these can be understood independently of time. In order to capture time, moments in time are characterised through pieces of writing which serve as position statements for those moments. These pieces of writing, however, become anchorages for the constructed reality simultaneously capturing the past and positing the new, according to their particular usage in newly generated stories, constructed by the researcher as they move between being a writer and being a reader in response to, and in creating, their evolving research interest.

This is most overt in the masters course where students acknowledge their own development during the course and select pieces of work from different stages of the course in an attempt to capture their own personal history on the course, as evidenced through pieces of writing, each piece giving some insight into how they saw things at the time of writing. The collation of such pieces results in the material for a story about the individual's development. An account frozen for the purposes of examining the very practice generating the pieces of writing. A yardstick created to be held against living experience to help us to observe change and to bring change about. The two examples in the last section offered accounts of teachers examining their own classroom practice through the medium of writing produced about it. In both cases, although in different ways, writing was being used to suspend action in time so that the teacher could look back at her practice and decide about different ways of seeing actual or potential change. The teaching of mathematics was being reconceptualised in line with broader professional concerns about their own teaching rationale. In the first the instrument of analysis was writing produced a couple of years before, which served to reveal changes in professional rationale during the intervening period. By better understanding the parameters of her earlier "child-centred" pedagogy she felt more able to locate and develop the dimensions of her practice which most effectively addressed her professional concerns. In the second example, the instrument was an analysis of teacher speech and its function in teacher student exchanges. By adjusting her speech in the light of this analysis the teacher felt the children in her class were more able to develop their responses and build agendas of their own.

Meanwhile, less experienced teachers may be under more obligation to construct their professional development in more conventional terms. That is, they are obliged to frame their own experience of teaching in the early days with the professional language used by more experienced practitioners. Initial training students, for example, can only go so far in employing their own personal language before the come up against a real or imagined demand to employ a more "professional" style. They are caught between writing creatively and writing conventionally in their evolving professional task. The language they use is not a straightforward map of their experience. Rather, on the one hand they need to bring meaningful experiences to the vocabulary with which they are presented whilst at the same time find ways of capturing, reflecting and acting on their experience. Over the four years of their training, students must learn to survive in a range of contexts of varying complexity in order to satisfy the professional requirements of the course. They must learn to analyse their experience using the language of the expert as offered to them through sessions in college and in recommended literature. As a starting point for this, they resurrect those aspects of previous experience which are available to them. In the first piece, when looking at a mathematics task, aspects of previous experience arose as snippets of learning which over a period of time had become jumbled and were no longer usefully connected to understanding. A feature of the recall offered in this example, lay in the students' preoccupation with a particular kind of mathematical language rather than an attempt to rediscover the mathematics of the task. The students worked hard in locating appropriate snippets of language both as a place holder for possible understanding but also to demonstrate that they are aware there are correct ways of talking about experiences. These were also features of the pieces which followed which had an apparently different focus.

Before completing this chapter I wish to raise one final concern; namely the validity and dissemination of practitioner research conducted by teachers. While researching into my own classroom I am both writer and reader of my research. In capturing, in words, a certain view of my work I can use this to orient future action. But what of the reader uninvolved in the research project, how might the written product of such work help him? The modes of dissemination normally associated with traditional research seem not to apply. The product of practitioner research does not result in statements of practical implications common to all. Rather, it gives an account of a practitioner examining specific issues within their practice and how these were addressed as problems within the research process. As in an ethnomethodologist stance, the "science" comprises both search and outcome. Similarly I suggest the practitioner, with his perspective and his way of working, is an essential part of the situation being described. In post-structuralist accounts, the self, and the situation he is in, are non-dualistic but rather, are mutually formative, as part of each other. Further, the self/situation has an essential time dimension understood by the individual through engagement in their situation. To understand the situation involves an appreciation of how the self/situation, and the decisions faced, evolve. An account of this cannot be given except by an individual addressing specific professional concerns. For the practitioner reading the research report the loss of supposed "objectivity" is replaced by an account of what might be seen and how best to see it - a traveller's guide rather than a map or an encyclopaediaentry. It remains for the reader to assert his right to tell stories about how it connects with their own practice, and in this sense the writer's "science" can be seen as reproducible (cf. Sacks, 1992, p. xxxi). As such the task of research is not to provide a mapping of "how things are" but rather is about production that triggers renewal.

PART 4

CONCLUSION

Recently, there was a conference dedicated to the work of Derrida. Derrida was in attendance. A colleague and conference attendee, Antony Easthope, described how Derrida himself patiently sat through numerous papers speaking of his work without passing any comment. However, at the end of the conference he made his own presentation. Having declared his delight to be in a conference celebrating his work he was, nevertheless, uncomfortable listening to so many people describing his work. He spoke of how, in attending the conference, he had experienced a sensation of being already dead. Having witnessed numerous attempts to sum up his work and integrate it elsewhere had made him feel as though his work had already been frozen for eternity, as if people were no longer seeking his present thinking. Elsewhere he has spoken of the impossibility of subject and object coinciding in time, the state of objectivity always being after the fact.

As I move towards my own conclusion I am reminded of a piece of writing by Derrida which he offered at the beginning of his book "Dissemination". It discussed the purpose of prefacing a book, an act which he sees as a way of killing off the book itself. He begins:

> This (therefore) will not have been a book.....Here is what I wrote, then read, and what I am writing that you are going to read. After which you will again be able to take possession of this preface which in sum you have not yet begun to read, even though, once having read it, you will already have anticipated everything that follows and thus you might just as well dispense with reading the rest (Derrida, 1981, p. 7).

My task in writing a conclusion is similarly torn between, on the one hand, describing the book that has been (summarising and framing for eternity maybe) and, on the other hand, activating what is to come, at least in my self and hopefully in others as well. It feels like mixture of fixing and releasing.

I have already done quite a lot of fixing. In the transcript offered in the first chapter Chester and his chums, as six year olds, were working on a counting exercise. By now Chester will be eighteen, possibly growing bananas or fishing for a living, like so many of his fellow countrymen. Yet I can only picture his grinning face as he moves bottle tops. My image is held in place by some words created at the time as part of a research project. It was one of many attempts at freezing experience in time for the purposes of talking about it. This sort of freezing has appeared throughout the book; from the students talking of how "for a five sided shape the degrees is 540" or teachers looking back at past writing to see how they used to describe their way of teaching. Such fixing preserves experience whilst, at the same time, killing it off. Yet we are forced to kill off our experience in this way if we are to live it. Any assertion of how the experience was, is associated with a loss. Dockar-Drysdale's work (1990, pp. 98-111) has, nevertheless, focused on the necessity of capturing and symbolising experience in this way and how we need to provide help for those unable to do this. It is a basic requirement in making sense of life. The very act of reflection is a reorganisation of lived experience into a caricature of it that helps us get our bearings. But in fixing we normally use language which asserts our community and our preferred way of talking of experience. Whilst mathematicians are accustomed to fixing things and releasing them, demands on the discipline become ever more complex and contingent. As time passes ever faster and the linguistic structure of mathematics becomes more transparent, the state of "being fixed" becomes ever more temporary.

CHAPTER 8

CONCLUSION

MATHEMATICS AND LANGUAGE

How then can we hold on to our mathematical thinking so that we can talk about it and understand it more clearly? Addressing this has been far from easy since the whole notion of finding a way of holding things still is complex. Can we, for instance, allow ourselves to suggest that something can be seen as being stable in itself, even before we get on to considering the way such a thing evolves in individual or collective minds? So far we have considered how language might function in organising such mental activity. Whilst we have seen that language provides some sort of medium for creating, preserving and communicating mathematical thinking, there are limits to what it can achieve, unless we broaden its scope beyond its traditional frame. Mathematical thinking, it has been suggested, is susceptible to changes in time, space and the varying perspectives of people experiencing this, which makes it difficult to create a stable relationship with language. Both the "seen" and "who sees it" shift continuously, if only because the person "grows older". Further. language would need to transcend its role as being primarily descriptive for a functional association to be possible with mathematical experience so viewed. Many traditional presentations of mathematics regard mathematical language as being coterminous with its symbolism, where human performance is irrelevant to the meaning of the symbols being used. This book however, has looked at the interface between mathematics and humans and, in particular, focused on views of mathematical learning where meaning is created within mathematical activity. We have also questioned how far language and mathematics themselves can each he understood independently of human performance of them. We are not just concerned with its locutionary properties, we are also interested in language's effect when it is performed in a social situation.

This chapter, which comprises three distinct sections, will summarise and develop some of main issues raised in the analysis. In this first section, after sketching how the various views of language introduce'd might connect with recent constructivist mathematics education research, we will locate this in a broader hermeneutic frame. This frame will be used in discussing how linguistic and mathematical performance are related and, in particular, how language functions in both describing and activating mathematical experience. Armed with an expanded view of language we focus on its association with the human experience of reality, making particular reference to the work on language by Derrida (cf. Brown 1996 e). Finally, we consider how the apparent dichotomy between positivistic and interpretivist views of developing mathematical understanding is located in the broader frames of hermeneutic and post-structuralist analysis. In taking this perspective I examine how language functions in organising mental activity and suggest that since language is so fundamental to the social formation and individual construction of mathematical ideas, it conditions all mathematical experience (cf. Brown, 1994 d, 1996 c). In this spirit I argue that linguistic reduction is an inevitable aspect of any mathematical construction, both locating and conditioning broader cogitations. As such, loss is a necessary result of the process of stressing and ignoring that underpins any conceptualisation.

In the second section, the emphasis shifts to the individual confronting mathematics in a broader social frame (cf. Brown, 1996 d). After, considering how students derive mathematics from the classroom situation we look at how this is associated with processes of cultural initiation. It is suggested that students find themselves confronted by a multitude of co-existing subcultures each with their own values, demands and appeal.

Finally, the implications of an accelerating social evolution are examined in relation to demands faced by teachers of mathematics in meeting the needs of students preparing for an uncertain future.

Language and constructivism

Many modern theories of language see the generation of language as instrumental in the self-formation of society and of the individuals within it. In this perspective, language can no longer be a seen as providing an unproblematic labelling of the world. Analytic philosophy's positivistic notion of language picturing reality (for example, Russell, early Wittgenstein) no longer holds up as an adequate metaphor for the way in which language functions, although such a belief may well govern the everyday actions of many people. A range of opinions about the way in which language is associated with reality have been met. With a serious risk of over-simplification these have included:

-language conditions all experience of reality (for example, Gadamer, Ricoeur),
-language distorts experience of reality (for example, Habermas),
-there is no reality outside textual analysis (for example, Derrida).

Within mathematics education research, an increasing emphasis is placed on the formation of mathematical ideas in people's minds rather than on a notion of a fully constituted set of ideas to be conveyed intact. As such the teacher's task is to initiate mathematical activity rather than merely communicate certain mathematical ideas. There is, however, some debate over how this is achieved. As we have seen, many constructivist writers of various leanings identify themselves with Piaget but Piaget himself is rather less clear on aligning himself with the stronger radical constructivist assertion, that the learner does not discover a preexisting world. I would suggest the radical constructivist avoidance of an ontological commitment here seems to draw them a little away from Piaget. For a discussion of this see McNamara (1995 b). In the scheme above it is Habermas (for example, 1991) who explicitly draws on the work of Piaget. In particular, they share a view of language, which they maintain, comprises fully constituted (i.e. stable or, at least, "fixed for now") components that can be subjected to critical analysis in relation to the reality they represent. This Habermas/Piaget view of language, as stated, is perhaps closer to the social constructivism of Ernest, although neither Piaget nor Ernest develop Habermas' idea of language distorting experience. For Habermas, language imposes itself on a not always willing reality - the world being described gets created in this very assertion.

I have argued that radical constructivism emphasises the individual constituting reality without sufficient acknowledgement of the way in which society constitutes the individual. That is, they downplay their role in task framing and setting the linguistic agenda. Were they to accept this cultural conditioning as endemic in their approach they would be more clearly identifiable with post-structuralism, in the sense of constructions not being oriented by "facts", although still lacking the ontological position provided by phenomenology or the one suggested by Derrida's work to be discussed shortly. Meanwhile, Vygotskian inspired, socially oriented models seek to acknowledge the social formation of individuals. Vygotsky's notion of "inner speech" co-existing with external objects is faintly analogous with hermeneutics' oscillation between seeing language as part of oneself to seeing it as something separate upon which you can work. However, the increasing dispersal of writers discussing such issues, resists any firm analogies from being drawn. I shall conclude the book by trying to tease out some of the areas where the modem brands of hermeneutics assist us in transcending the limits of constructivist inspired analysis towards providing a fuller account of the relationship between language and mathematical activity. In doing this I will discuss aspects of Mason's overtly internalist approach, as an example of a mathematics educator abandoning positivistic assumptions in reconceptualising the location of mathematics.

The hermeneutic horizon

Hermeneutics has a popular image of being primarily concerned with text interpretation. In this narrow interpretation, hermeneutics is seen as reconciling understanding with its capturing in explanation, for example, in examining the fixation of speech in writing. In this book I have sought to work with a broader Understanding which has been developed within its application in the social sciences. Here the net has been cast more widely to address the issue of how action may be captured in evidence; as Ricoeur puts it, the mark action leaves on time. In focusing on evolutionary processes, such as learning in classrooms, we need to find ways of capturing dynamic qualities. I have argued the creation of fixed points of reference, (for example, mathematical statements by a student, arrangements of physical apparatus, or pieces of professional writing by a teacher) within developing understanding, brings into play a system of orientation which functions, almost simultaneously, both in capturing current understanding and in guiding subsequent action.

As has been pointed out, there is some diversity, between writers in hermeneutics, as to how such points of reference organise the dynamics with which they are associated. In particular, one's view of language, and how it relates to reality, influences how one sees language as functioning in orienting experience. I have suggested, however, that it is not helpful to draw strict distinctions between these varying positions. Hermeneutical views are characterised by a circular movement encompassing a succession of alternative perspectives, for example, between seeing language as embedded in what I am doing and seeing it as a separate labelling device. Differences in views of language held by such writers are essentially to do with the way they choose their home base on this spectrum and how far they stray from this base. Through seeing mathematics as functioning like language, such a home base can similarly characterise the view held of mathematics. Seen in this way, hermeneutics can offer some assistance in describing how mathematical thinking gravitates around constructed objects. Whilst mathematical thinking is evolutionary, its preservation, both for an individual or for a society, leans on fixed statements, physical apparatus and diagrams. Dynamic constructing needs to be reconciled with relatively static constructions.

As teachers, we need to enable students to be constructive in mapping out the space she faces whilst, at the same time, enabling them to be, as it were, constructed through engagement with pre-organised schemata. Emphasising the constructive, radical constructivists would argue, nurtures the student in taking control of their on-going thinking as opposed to working compliantly in spaces left for them. Such creativity may be suppressed if too much stress is placed on the student recreating externally defined end products. In this way the structure imposed by a teacher and the constructions made by the student in respect of it can be seen as comprising a joint sequence of actions. Each takes some responsibility for structuring the space they share. The student's action and the space in which it happens cannot be seen separately. Whether you see the actions of the student as constitutive or constituted depends on your perspective and your particular interest in describing the exchange.

This has implications for the perspective assumed when speaking about the mathematical achievements of students and the sort of overview this presupposes. For example, the progression of a student's mathematical learning in school is often described in terms of tackling a succession of topics each containing various concepts, procedures, key results etc., taught and assessed as if they were stable facts. In such formulations symbol and meaning are coterminous, with the former assuming a neutral labelling function. By shifting to a more hermeneutic understanding of mathematics, where mathematical ideas are encountered through an on-going circular process of reconciling expectations with experience, we disrupt any apparent stability in mathematical structures held in the mind of an individual. Assessment of this would need to emphasise on-going constructing as well as facility in making constructions. In this formulation, physical mathematical symbols, or other embodiments, take on a different role in relation to evolving mental phenomena, assisting the human mind in organising and anchoring its mathematical endeavours. This use of physical phenomena, nevertheless, conditions the ideas it supports. Indeed, mathematics' own archaeology makes extensive reference to the physical world (Brown, 1994 g; Pimm, 1995). Insofar as we see mathematics as having associations with the physical world we are often guided by how we see mathematics as being embodied in certain features of the world. Ideas are often experienced in relation to framings we suppose in physical manifestations. If, however, language is no longer seen as a mapping of the world, the mathematical objects cannot be a spatial mapping of the objects of the world, since they play an important part in forming those objects. Mathematical notation does not simply describe mathematical phenomena, it activates it. Language does not just describe action, it is part of it, displaying locutionary, illocutionary and perlocutionary dimensions; namely a descriptive function, an operational function (in the mathematics itself) and a social effect.

Is there something to be transported from the mind of the teacher to the mind of the student? All but the more traditional writers seem to think not. Nevertheless, as educators we are faced with initiating students into particular skills and styles of thinking. In order to hold on to our thinking and, in particular, when we want to share this thinking with others, there is a need to translate it into something tangible. We require some medium of exchange, whether this be a verbal explanation or an exercise that will result in the student encountering an intended line of thinking. In the latter scenario, words and diagrams, offered by the teacher may be the trigger of an activity rather than the anticipated content of that activity. For the mathematics teacher the functions of language include both, a) describing mathematical ideas, b) triggering activity (cf. Pimm, 1995); it has a locutionary function and a perlocutionary effect. Both literal meaning and resulting actions can be seen as being associated with the initial linguistic performance. In hermeneutics, inscription in writing punctuates a flow, simultaneously marking the past whilst nudging the future.

In a teaching/learning environment, the teacher performs and the students respond. Whilst the teacher may have certain intentions, the students, however, will respond in line with their own agenda. The teacher's words might well help anchor, in the mind of the teacher, a definite set of mathematical ideas or a course of actions by the students but there are no guarantees in the style of student response. This has been discussed in relation to the way in which the teacher-student interaction is discussed variously within conservative, moderate, critical and radical hermeneutics. In a sense, theories of teaching and learning can he discussed separately or together; a staunchly conservative teacher has no control over the reading of his teaching provided by a radical commentator. For example, I can often learn from someone else's overarching perspective on "what" mathematics is. Whilst I might be skeptical about the speaker's plans for me, I may well find myself learning in the space I share with her. This is not to say I receive her message, or even believe her message if I do feel I have received it. I do not see my task as learner as being about receiving the intention of my teacher. Nevertheless, I occasionally work with expositions from such believers, as a part of my becoming clearer about what I believe myself, in organising my mathematical task as I see it. In this sense, I do not very often see presentations of mathematics as representing an overarching perspective, although that may well be the intention of the presenter. Certainly, in its effect on me, its significance is more oblique than its delivery might suppose. I am not interested in what her presentation "means" but rather with where it is coming from, what does it presuppose? (cf. Derrida, 1981, translator's introduction, p. xv). A piece of mathematics comes about through personal work no matter how much the presenter of it thinks otherwise, nor no matter how much curricula prescribe particular performance skills. A point made by some constructivist writers is that students learn through making their own constructions even if their teacher gives straight lectures (for example, Pirie and Kieran, 1992). The teacher's performance may well influence the student's response, whatever teaching philosophy the teacher assumes, but the teacher's recognition of this is not necessarily matched by such a recognition by the students. The student's have an understanding of what they are interested in, how they learn, how they feel about responding to the teacher, and act according to that agenda. Just because a teacher drops their reliance on lectures does not mean her students change their perceptions of how they learn.

The ontology of mathematical experience

The relationship between mathematics and language needs further unfolding if we are to understand how language might serve as a medium through which mathematical ideas are shared. If we accept the analogy of mathematics as being like a language and, in particular, mathematical activity as being a form of linguistic performance, mathematics finds itself subject to the scrutiny of modern day critiques of language which emphasise its situation in history, in culture and in personal accounts. The human subject engaged in mathematics is positioned in a number of co-existing social agendas which flavour the style of engagement. Insofar as we see mathematical meaning being generated in the mind we cannot escape the formative influences on the mind. Also, we cannot partition a section of it and label it "mathematics". For example, those introducing insider perspectives into mathematics teaching and learning require appropriate apparatus for capturing individual experience. Some such writers, however, see a clear distinction between mathematical experience and the linguistic description of it.

Here I shall focus on the position taken by John Mason in associating mathematics with language, as an example of a writer in mathematics education moving away from assumptions of language picturing reality. The following quotes give a flavour of his view:

Words generate more words in explanation, hut often draw us away from the experiences from which they stem. Mason (1994, extended version, p.176)

Express to yourself in action (by doing it) and in words (by talking to yourself or a colleague) a nile for continuing thefollowing array..... Honsberger quoted by Mason (1989 a, p.3)

On the one hand we have the experience, on the other, the description of it in words. In my conversations with him, Mason defends the content of his mind as not being reducible to description in words. Whilst I may report on my experience as a mathematician, in so doing, I insert a gap between experience and report, resulting in the precise nature of my experience being rather elusive, being partly lost, at least as regards its capturing In language.

How then can we locate mathematical meanings in relation to mathematical and linguistic performance by humans? How do hermeneutical accounts assist us? Traditionally, the task of the teacher and learner may be seen as sharing a preexisting mathematics not susceptible to individual interpretation. Such an account is governed by positivistic notions where the teacher seeks to direct the student's attention to a specific way of seeing an objectively understood mathematics. Truth is embedded within the mathematics and the student seeks to locate this. Insider views of mathematical problems, meanwhile, as exemplified in the work of Mason, focus on directing the student on a journey around problems and reflecting on the affective and cognitive experiences of doing mathematics. Whilst the emphasis is more on the student reconciling her experience with ways of describing it, Mason argues, the attempt to describe in words might draw the student away from the mathematical experience itself. The teacher's intention is rather less didactic, but this may not necessarily imply a less conventional view of the underlying mathematics. Mason follows Gattegno in seeing truth gravitating around personal awarenesses, that is, truth is located in the mind of the individual.

So where mathematics is located? In more traditional views the mathematical meaning is independent of individual human performance. Meanwhile, according to Mason, the emphasis is on the individual human's personal awarenesses of mathematics. Nevertheless, both appear to see the description of mathematical activity in words as being outside the realm of mathematics itself.

Contemporary accounts of language provide a rather more transcendental view of language that infiltrates, whilst coalescing, the reality it serves. I wish to argue that the framing of mathematical experience in words by individuals should be seen as an integral part of the mathematics itself, inseparable from less visible cognitive activity. I shall briefly indicate the positions of Ricoeur, Gadamer and Habermas before developing Derrida's more radical post-structuralist position.

Ricoeur and Gadamer assert that experience itself is conditioned by any attempt at a linguistic framing. For them language mediates truth. That is, whilst they have ontological beliefs, they see any underlying truth as being obscured by our attempts to access it. The chief consequence of this for our current analysis is that mathematical experience and description of it in words are drawn closer together. Phenomenology has offered an approach, supportive of their hermeneutic framework, which assists us in discussing how the individual confronts and works with mathematical ideas. Here, the material existence of the world is fully accepted but it only presents itself according to some particular phenomenology subsequent to being carved up in a time dependent categorisation by an individual. The material world lights up as it is touched by the human's gaze. Objectivity itself is historically created, defined in terms of the way in which the culturally informed individual consciousness perceives the material. This partitioning of the material world into phenomena is closely related to the descriptions made in respect of it. Mathematical objects then, present within such thinking, are not unproblematic entities for all to see, but rather, are understood differently by each individual. The distinction between such phenomena and the perception of them is softened with phenomena and perception evolving together through time. In this perspective mathematical ideas, as located through notation, are not endowed with a universal meaning, but rather, derive their meaning through the way in which an individual attends to them. Thus mathematical "object" and human "subject" are seen in a more complementary relation as part of each other. The emphasis in this phenomenological formulation is on the individual's experience of grappling with social notation within his or her physical and social situation. This provides a framework, seen from the individual's point of view, in which the distinction between the individual and the social is softened. In building his or her understanding, the individual is obliged to work through the social filter of language. My strategies for making sense of and acting in the world are always underpinned by cultural stylising derived through language, whether I be mountain climbing, dancing or doing mathematics. All such activities can be seen as specific "discursive spaces" (Stronach and Maclure, 1996, p. 262).

Habermas sees the language of mathematics itself as being strictly outside of experiential language. Nevertheless, the activity that takes place around it is still susceptible to hermeneutic analysis. His focus is more explicitly on the agendas built into styles of language use. In the light of Habermas' concerns we might step outside and assume a critical distance in examining how the styles of language employed in mathematical activity serve the respective interests of the people involved and what these interests represent. For example, the style of work advocated by a mathematics teacher is associated with a particular view of what mathematics is and the values that view presupposes.

In his post-structuralist perspective, Derrida (1978, pp. 278-293) is more fatalistic in claiming you cannot step outside of language in this way, neither to look at it, nor to experience something else. You can only observe the linguistic performance of others from the home base of your own linguistic frame. One is always positioned within culturally derived ways of seeing and so experience itself is textual insofar as it is understood through inherited schemata embedded in language usage (Brown, 1994 b d). In short, language is always about a world already conditioned by language. Any human performance can be read as a "text" in the philosophical sense of the word. Indeed, Derrida famously asserts (1976, p. 158), "there is nothing outside of the text", nor are there truths to provide points of anchorage. He sees differential structures as being inherent in explicit language, consciousness and unconsciousness. Like Lacan he identifies this as a general feature of the mental world, with both conscious and unconscious being "structured like a language". The mental world, so seen, is a system of differences, part of which is claimed by explicit linguistic structuring (Derrida, 1982). It should be stressed, however, Derrida does not dismiss the experience itself, rather, experiences are in a constant state of flux as attempts are made to associate them with a never ending linguistic flow. He would see mathematical involvement as necessarily textual, brought about through human partitionings of the world - a framing that is, in a sense, already there, brought about through cultural linguistic heritage. "My own words take me by surprise and teach me what I think" (Merleau-Ponty, quoted by Derrida, 1978, p. 11). Derrida builds on this quote in discussing how inscription in words (and maybe also in symbols, in diagrams) orients psychologically produced phenomena. If I may again risk using his own, rather slippery, words (1978 , p. 12):

> If writing is inaugural it is not so much because it creates, but because of a certain absolute freedom of speech, because of the freedom to bring about the already there as a sign of the freedom to augur. A freedom of response which acknowledges as its horizon the world as history and the speech which can only say: Being has already begun.....(Writing) creates meaning by enregistering it, by entrusting it to an engraving, a groove, a relief, to a surface whose essential characteristic is to be infinitely transmissible. Not that this characteristic is always desired, nor has it been; and writing as the origin of pure historicity, pure traditionality, is only the *telos* for a history of writing whose philosophy is always to come.

I take Derrida to mean, crudely, that inscription in writing functions closely in relation to the psychological phenomena it locates and, indeed, becomes part of it. In reading Derrida one never gets to what he means but rather one experiences the on-going sensation of being moved on before you are ready. His words never frame the final version of his "present" thinking. In this respect, Derrida's position is not that far away from the more moderate line of Gadamer and Ricoeur who permit an on-going renewal within the co-evolution of phenomena and perception. However, Derrida's refusal to allow any anchorage in truth makes his work quite distinctive and more radical in its ability to reject orientation around universal structures.

Derrida's position takes language well beyond its traditional scope towards embracing the whole of human experience. Objections sometimes arise when we attempt to nudge language into this extended domain. For example, on the surface at least, such views appear unsatisfactory to those who wish to defend the power of their own mathematical experiences as being outside the realm of language. Nevertheless, Mason does speak of manifestations in the "outer" which have some sort of association with "inner" experience. Indeed he seems distinctly Buddhist when he suggests that we need to acknowledge "a world of experience that is not material, not phenomenal, but inner, with access through what we are able to read in the outer" (Mason, 1994, extended version, p. 7). This, I feel, invites a degree of compatibility between his understanding and the line taken by Derrida.

We need, however, to ask about the nature of this association and question how these outer manifestations attach themselves. Are they like the tips of icebergs (i.e. part of the thing being signified) or like road signs (i.e. separate to the thing being signified)? I suggest words, diagrams and other manifestations of mathematical activity, can be seen as functioning in either way, according to current interest and the emphasis one assumes. Indeed they may be seen as two points on the hermeneutic cycle connecting ways of seeing experiencing and ways of describing (Brown, 1991). The physical environment, for example, is textual, in Derrida's sense, insofar as the human eye organises it differentially. Thus "seeing" is always in relation to an a priori conditioning. Any attempt at inscription reflects this broader but maybe more elusive differentiality. As with Saussure (1966), Derrida sees the signifier/signified duality as inseparable. But as with Lacan (1968), Derrida sees relatively stable signifiers being associated with a fluid underbelly, comprising a signified field which sweeps out to occupy the whole of consciousness and, indeed, the unconscious. Both presence and absence are located by the signifier. The loss incurred in the attempt to articulate remains attached to the signifier seeking to replace it. Meanings are derived only through retrospective examination of the flow of signs. The component signifiers do not have implicit meanings, only relational associations with other signifiers in the chain. There are no independently existing meanings in the chain since any attempt to frame in words, any attempt to "mean", creates a gap between "being" and attempts to explain it. Lacan speaks of an indefinite sliding of meaning to convey the "impossibilities" of attaching one word with one meaning. We have no truths to provide orientation apart from those generated through this system of differences. Derrida (1981, translator's introduction, p. ix) suggests that self-present meanings are illusions brought about through repressing the differential structures from which they spring. However, as a note of caution Derrida seems to have back-tracked a little from the extreme way of thinking many associate with him:

it was never our wish to extend the re-assuring notion of text to a whole extra-textual realm and to transform the world into a library by doing away with all boundaries, all frameworks, all sharp edges (Derrida, 1991, p. 257).

In describing mathematical experience we may suspend the "presence" of the experience, in a sense, but the experience itself was textual (i.e. understood differentially) and thus already a suspension, so no more nor less the "real" experience. There is no experience outside the text, only retroactive constructions of it asserted by the individual. To make a strict distinction between experience and description of it in words, as does Mason, requires a relatively restrictive view of language. Whilst this might offer a valuable rhetorical device in initiating or analysing mathematical performance, such a distinction suppresses the historicity endemic in anything commonly recognised as mathematical performance, or even mathematics itself, and thus obscures the values associated with this (cf. Derrida, 1989). In particular, the linguistic forces driving (and being driven by) mathematical constructing get squeezed out of the picture. Mathematical constructing, I would suggest, is always linguistic to a degree, oscillating in a hermeneutic circle, between more or less sturdy linguistic frames.

Whilst the constructivist writers address these issues fleetingly I feel there is a tendency to be a little evasive. Radical constructivism explicitly by-passes objectivity, saying that it does not affect the "viability" of their activity oriented concerns. Such writers work on the premise that mathematics can only ever be perceived from particular positions and perspectives by observers with individual interests. An overview is never available. Further, mathematics can only ever be perceived through mathematical activity which further mediates any access to any supposed externally defined objective mathematics. Notions of mathematics where it is constructed outside of the individual consciousness are seen as being less helpful in addressing issues of how individual learners gain access to this. This however, results in a rather incomplete account of the ontology of mathematical learning. Radical constructivist writings seemingly stops short of offering a more robust path through this maze. Meanwhile, social constructivism (for example, Ernest) offers accounts of objectivity and subjectivity yet the words are used in unconventional ways. Ontological concerns are again by-passed. Further, through associating objectivity so closely with a notion of intersubjectivity seen at the level of community, social constructivism brings in a view of mathematics as something existing outside the mind of the individual knower. Both hermeneutics and its radical form, post-structuralism, I believe, offer accounts which, through being more flexible in their understandings of language, engage with the material qualities of the world. Phenomenology accepts the material world but intercepts perception before it assumes shared notions of categorising this material world into objects. Post-structuralism meanwhile uses language itself (or more accurately textuality, that is, differentiality) as its home base and so subjects meet in their shared use of the manifestation of this in speech or writing. For both post-structuralism, and more moderate hermeneutics, the association advocated between language and reality resists stability between signifier and signified. Rather, both reality and language are caught in an historical process of mutual formation which is never complete, nor even pauses long enough for one to map the other. In such a perspective, the historicity present in both the genesis and the current performance of mathematics is recognised. Seen in this way there is no mathematics outside language.

In this perspective, the learning of mathematics moves away from being concerned with recreating existing ideas but instead emphasises the tightly knit relation between language and understanding. On this point Mason and Derrida seem close. In assuming the teacher's task himself, Mason is concerned with enabling his students to generate their own mathematical experiences. That is, he does not explain the mathematics in his head but rather, initiates an activity which he hopes will enable his students to experience some mathematics and in this, perhaps, encounter certain ideas. He sees learning as a journey of self discovery:

> it is important to research, re-collect, re-connect, re-learn, re-integrate, and re-cast insights in the discourse of the times. I see working on education not in terms of an edifice of knowledge, adding new theorems to old, but rather as a journey of discovery and development in which what others have learned has to be re-learned, reinterated and re-expressed in each generation (Mason, 1994, p. 177).

Ideas are not inherited prepackaged and intact, but rather, each new generation will engage in tasks that give rise to new understandings of what might be seen as old ideas. There is a need to work on ideas, they cannot just be "received". This way of thinking bears a striking similarity with some of Derrida's recent work:

> Inheritance is never a given, it is always a task... there is no backward looking fervour in this reminder, no traditionalist flavour, Reaction, reactionary or reactive are but interpretations of the structure of inheritance. That we are heirs does not mean that we have or that we receive this or that, some inheritance that enriches us one day with this or that, but that the being of what we are is first of all inheritance, whether we like it or know it or not. Derrida (1994, p. 54).

Whilst the student's task may well oscillate between bringing language to mathematical experience and bringing meaning to language through reflection on one's own experience, both experience and linguistic production forever continue, resisting attempts to settle on a particular version. Mathematical meaning never stabilises since it is caught between the individual's on-going experience and society's on going generation of societal norms as manifest in its use of language, in particular, those pertaining to society's view of mathematics. Mathematics, language and the human performing them are always evolving in relation to each other. There is no final version to be learnt, since we lack universal truths to hold this in place. I conclude with a quote another post-structuralist thinker, writing shortly after the student uprising in Paris in 1968:

> Just as psychoanalysis, with the work of Lacan, is in the process of extending the Freudian topic into a topology of the subject (the unconscious is never there in its place), so likewise we need to substitute for the magisterial space

of the past- which was fundamentally a religious space (the word delivered by the master from the pulpit above with the audience below, the flock, the sheep, the herd) - a less upright, less Euclidean space where no one, neither teaches nos students, would ever be in *his final place* (Barthes, 1977. p. 205).

In short, within the very limits of the teaching space as given, the need is to work patiently tracing out a pure form, that of a floating; a floating which would not destroy anything but would he content simply to disorientate the Law. The necessities of promotion, professional obligations... imperatives of knowledge, prestige of method, ideological criticism - everything is there, but floating (ibid. p. 215).

MATHEMATICAL CULTURES

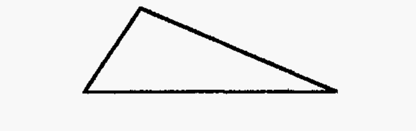
This book has also sought to present a framework through which we can examine the individual's experience of a mathematics lesson. In Part Two lessons were described as social situations involving a number of people each acting according to their own particular interests, within the space they each perceive themselves to be in. Each of the people present see the lesson in a different way according to the roles they assume, whether this be as pupil, teacher, researcher or whatever. Whilst everyone co-exists in a tangible world of things and people, each comes into this world with a personal history, a set of motives and a will to act. Consequently, each individual attends to this world by placing a personal emphasis and accent on each of the elements she perceives to be forming it. This requires both a reading of things and a reading of people. Furthermore, it is necessary to make an interpretation of the surface appearance to understand the world more deeply and decide how one might work within it. By acting in the world she imagines to exist, the student learns about this world through the way in which it resists her actions. It is through this process that the individual's particular way of partitioning the world into things continuously evolves.

I have suggested that this model can be used in describing how students derive mathematical ideas from the social and physical environment. The hermeneutic process can be seen as shaping and flavouring mathematical notions in the mind of the individual. As we adopt successive new perspectives, our way of seeing the world and our expectations of it are renewed. In engaging in mathematical tasks we are generally faced with deciding where and when to use particular ideas, especially in problem-solving situations, and we often remain unsure until after we have checked things out. Our phenomenologies, in particular, our mathematical phenomenologies and the ideas contained within them, evolve as we experience successive situations. These ideas never become "fully formed", rather they are subject to successive modifications through time as they are encountered in new situations - or perhaps become fragmented as memories fade, as with the initial teacher training students described in the last chapter. As another example, the holding qualities of a label, such as "line of symmetry" evolve and dissolve through time as stories are told and forgotten.

Schütz's theoretical frame points a way to accommodating the social aspects of mathematics education, consistent with radical constructivism. This has been achieved by denying an overview to any individual learner or teacher. The social world is accommodated by focusing on the perspective the individual has of this and the possibilities open to them within the world they see. This side-steps social constructivism's apparent reliance on an agreed upon "objective" world and focuses instead on the meeting of individual perspectives, where only individuals can have a perspective on "taken-as-shared'' ideas, Nevertheless the acceptance of the individual's interest as being grounded in their "biographically determined position" allows for the social conditioning of this individual. Any notion of an over-arching objective reality, however, is never encountered if we focus on the task and perspective of the individual learner. Similarly, the teacher is only ever confronted by their own particular teaching task. Whilst they may see this as having some relation with their understanding of an over-arching structure, this can never be with an "actual" over-arching structure, with "actual" components. Nevertheless, it may well be that in responding to the mathematical work of a student "the teacher's mathematical image functions like an objectively given standard for the orientation of his correcting and supplementing actions" (Bauersfeld, 1992, p. 479). To some extent the teacher speaks the society from which they come and the student's learning to speak in this way is an essential part of their cultural initiation but such things are only ever perceived by an individual in relation to the social norms they suppose. Practice-related language generated within this framework of social norms functions in guiding the reality of both parties.

Having built this model which captures the individual's experience of the social and physical world, the gateway is opened to Habermas' recent work and, in particular, his attempt to find an approach to reconciling individual perspectives with over-arching structures (for example, 1987, 1996). Nevertheless, Habermas (1987, pp. xxv, 126-132) sees serious limitations in Schütz's model as outlined earlier, resulting from its one-sidedness. In particular, he sees the emphasis on individual consciousness as problematic, feeling that by centring around an individual identity there is an overemphasis on social integration and the reproduction of cultural knowledge, to the relative neglect of the formation and transformation of group memberships and personal identities. For example, by understanding learning as initiation or enculturation, shifts in culture itself and choices between cultures, are underplayed. I assert my own identity through asserting my identifications with various groups, by participating within them. Through doing this, both the group and I, myself, evolve (Habermas, 1991). Habermas' own broader project focuses on what it means to create commonality and communicative links between different forms of life (see White, 1988, p. 154). He sees the building of these links as an integral element of growth, where both social and individual evolution are bound up with attempts to reconcile social practices with descriptive practices (Habermas, 1987,p. 60). Such practices however, can be highly localised and for this reason he seeks to develop a multi-dimensional concept of the world, as experienced by its various inhabitants, towards integrating alternative practices.

To engage in mathematical learning, not only is it necessary to share understandings with other individuals, one also needs to be able to participate in and move between a variety of mathematical subcultures; "everyday" mathematics, various types of school mathematics, examination mathematics, novice vs. expert mathematics,etc. each with their own particular language, scope of interest, values and associated skills (cf. Carraher et al., 1985; Lave, 1988; Lave and Wenger, 1991; Cobb, 1994). Insofar as mathematics is socially constructed there is a need to examine the way in the particular subculture flavours the mathematics it uses and how these various demands made on mathematics evolve as social needs change. As an example, the variation in styles of questions employed by different examination boards within the United Kingdom, results in students not being examined in mathematics per se, but rather, in the particular style of questioning the board chooses to offer. Similarly, the "geometry" I did in school twenty five years ago with protractors and set squares is qualitatively different to the "geometry" present in work with Cabri software (cf. Laborde, 1993; Pimm, 1995). My own school's penchant for wobbly compasses had considerable effect within my formative experiences of what constituted a circle! Classroom technology both encourages and facilitates a shift of focus. The mathematics itself is different, not just its presentation. Old points of anchorage get eased out. For example, a triangle in Cabri environment is qualitatively different to one in a pencil set square and compass environment, or one made with plastic strips, or one created on a computer using BASIC graphics or LOGO. In each situation the triangle appears in an environment where certain things can be done to it; it is a function both of available operations and of things that can go wrong. Each environment hosts a particular style of work or mode of culturality. The triangle appears in the specific cultural discourse, differentiated from the things around it, in terms of both its innate qualities and the way it is produced in the particular field. David Pimm (1995, pp. *56-58),* in homage to Magritte's painting of a pipe popularly known as "Ceci n'est pas une pipe", speculates:



"This is not a triangle".

Magritte's joke however does not transfer successfully. For it to work we would need to assume the primacy of a transcendentally defined mathematics, existing outside most cultural practices. In many cultural discourses a pencil drawn triangle is indeed a triangle, not just "as good as the thing itself for our purposes" (ibid.), not merely a nice drawing that helps us see the thing itself.

Mathematics in learning situations is generally subject to such cultural and stylistic flavouring. The activity of the girls doing symmetry in Chapter 5, is governed by certain conventions prevailing in their classroom. These conventions might be characterised as those associated with "investigational" style mathematics of the sort practised in many London schools in the eighties. An article written at the time suggested that these conventions focused on learning process such as; processing information, symbolising, illustrating mental pictures and/or physical actions by diagrams, searching for patterns, seeing connections etc (Billington and Evans, 1987). I suggest that the girls' expectations which have evolved through familiarity with these conventions condition the responses they offer and the way they proceed.

This sort of concern invites a shift from having an overview of mathematics *qua* mathematics, towards understanding how it is to engage in particular versions of it, within given social settings (cf. Mellin Olsen, 1987, pp. 18-76). Further, there is a need to understand how students participate, with their teachers, in the constitution of classroom mathematical practices (cf. Cobb, 1994, p. 15). To address Habermas' concern with conservativism (see Huspek, 1991), that is, to prevent this constitution from being mere reproduction, participants in mathematics lessons would need to build a clearer understanding of how their actions relate to norms inherent in the particular subculture and how the criteria might change as they move into a new subculture. That is, students need to become more aware of the parameters of their own learning, towards being able to take a critical stance of how these parameters govern their situation. For example, Skovsmose (1994) advocates an increased emphasis on thematic project work within mathematical learning to enhance student awareness of how problems may be contextualised. Meanwhile, Mason's work focuses more closely on the parameters of mathematical problems themselves. The insider perspective needs to be understood more closely in relation to the contextual forces operating on it. An essential aspect of this that needs to be addressed by empirical mathematics education research is an analysis of how the symbolic framework employed within a given subculture mediates access to the understandings of that culture.

In my view, Habermas' objection to Schütz's work is legitimate but overstated. Schütz's Cartesian framework does not necessarily exclude the individual's awareness of the social parameters of their own individual space. Like Cobb (1994), I feel it is inappropriate to insist that either individual or social perspective takes precedence. In looking around me I have some awarenessof the conventions inherent in my culture and how they influence the way I see things, quite independent of any explicit educational programme alerting my attention to this. As an individual I have some concern with how I fit in. This seems inevitable in a world where so many subcultures confront each other and maybe some offer me membership. Consequently, I do not see a need to abandon the model based around the individual's perspective. However, in line with Habermas, I suggest that the task of education must, in part, be concerned with enabling the student to take a self-reflective critical stance in relation to the perspective she assumes. More than ever, students are preparing themselves for a world undergoing fundamental structural changes, comprising rapid growth and increasing diversity. This creates pressure to regenerate styles of teaching and learning mathematics whilst moving away from treating mathematics as though it could be conceptualised in a stable state in relation to the reality it serves (cf. Brookes, 1993, 1994).

There are, however, a few remaining difficulties, which make the policy implications of such a view less than clear. Initiation will always remain an indispensable dimension of mathematical activity. Indeed it would be hard to conceive of mathematics teaching without initiation; the whole enterprise would dissolve into thin air. We need to set something up before we can engage in a critical attitude. Nevertheless, in an environment of rapid environmental change we are increasingly concerned with enabling students to respond positively to constant renewal. The old ways cannot be our only concern. Indeed as adults we can only assume partial control in making the decisions. We have seen how teachers and pupils invariably share responsibility for structuring the space they share. But as my colleague Olwen McNamara has pointed out, adults often find themselves very much at the beck and call of their offspring in such shared space. The child's voluntary action bringing into play an involuntary resistance on the part of the adult trying to cope; the adult's behaviour forming itself around the child's whims. (Apologies to my baby son Elliot!). But things need to be in motion or, at least, anticipated, before resistance is possible by either adult or student.

I return to David Pimm's intriguing account, where he examines some of the issues associated with a move towards a more critical mathematics education. He offers examples of a number of questions which could only arise in a discourse designed specifically for the teaching of mathematics. The stories are daft in the sense they would not have any function outside of a pedagogic frame. (for example, If one Confederate soldier kills 90 Yankees, how may can 10 Confederate soldiers kill? (op cit., p. 64)) But what are the more honest alternatives? To engage in critical mathematics education it is a prerequisite that we embed the mathematics in a social practice, even if this is simply the social practice of doing school mathematics, but all too often the attempt to embed mathematics results in a curious world that exists only for mathematics classroom. On the other hand firm emphasis on a more pure mathematics, situated only in the practice of doing mathematics, can result in an alienating discipline restricting access to all but a few and concealing its applications in broader social practices. I am reminded of a story I was told by an Israeli professor talking about a new colleague of his who had recently arrived from Russia. Asking his new colleague about how he had presented a certain topic the colleague replied by saying that he had given the students the required formulae. "But how did you motivate the students, how did you contextualise it, how did you connect it to their experience?" asked the Israeli. "I simply gave them the formulae" replied the Russian. I suggest the gap between the Russian and the Israeli comprises a culturally specific pedagogical discourse which hosts certain assumptions about styles of learning, the content of mathematics, the role of the teacher etc. Such a pedagogical layer, I suggest, is present in both versions, which, in their different ways, maintain the inevitable distance between students and the mathematics being "offered"; a gap perhaps bridged by interpretation. In one there is the cultural obligation to be seen to be "motivating" students". In the other, a cultural preference for minimising clutter.

Pimm meanwhile (ibid., pp. 153-158) questions the virtue of giving primacy to critique in the way Skovsmose seems to. There is a risk that if students focus too much on critique they do not learn mathematics. Skovsmose's approach which emphasises teaching mathematics within social themes, certainly would reorientate the way in which we value mathematical concepts, placing more stress on those with practical potential, but then which practice? We risk introducing all sorts of artificial worlds as a proxy for social realities. However, if we reject such a move we are still left with deciding what it is we do promote. Do we make some serious investment in embedding mathematics in order to develop skills of critique or concentrate on going with some version of mathematics itself? John Redwood, a right wing government minister, recently questioned the value of comparative religion in British schools on the grounds it meant that each religion examined received no more than lip service. To embrace a religion you need to make a total commitment, dwell in it as an act of faith. He argued that students are not equipped to hold on to too many versions simultaneously and they should instead focus on one. But, of course, such a policy results in one religion asserting its power over all others. Similarly, if we invest too much faith in one version of mathematics we risk forcing students in to conservative reproduction of the status quo and providing short cuts for getting there. But, of course, we will never have such an extreme polarity of choice.

At some point we need to concern ourselves with the issue of how far the teacher can take responsibility for facilitating initiation, in relation to the students building significance for themselves but, before this, we still need to decide what it is we are initiating students in to; various cultural practices, some sort of disembodied mathematics or, more probably, somewhere between. Following discussion on the hermeneutic circle it seems to me we are necessarily in a state of moving between situated mathematics, of which stories can be told, and a mathematics understood as if existing outside of everyday human discourse or, at least, functioning as a discourse with a very specific structure. Whilst the stories that Pimm cites lack credibility in the real world, all such stories would to a degree. They more or less embody some view of abstract mathematics and we can never have anything more. We need such stories to mediate experience, to help associate sense and reference. "Maths-speak" is, from a post-structuralist perspective, just as much a story as all the other stories, although some stories may appear more overtly mathematical than others and so more transparently reflect the status quo? I have spoken of Pool's writing in introducing the idea that you have to live a way of life before it becomes meaningful, its not a case of switching between alternative visions. Our overarching task is to nurture a dialectic between reality and stories told about it, a dialectic which transforms both. As Pimm suggests, we need to combine descriptive with generative language and, I suggest, not just generative within the existing frame. Stories cannot usefully function as a mere map but also need to trigger renewal.

In summary, there are many versions of mathematics engendered through practice in a multitude of subcultures. For a teacher there is always a decision as to whether mathematics should be situated in "practical" examples so as to make it more meaningful and accessible. However, many attempts at situating mathematics result in an artificial world created purely for pedagogic purposes. Meanwhile, the teacher may or may not wish to make initiation of their students into the status quo their principal concern. Whatever they decide it may be that students construct their own agenda anyway. Combining the teachers perceived intention with an outside view on how this intention is associated with significance to the student, we may begin to speculate on how much a teacher's input ensnares the student into reproducing the status quo and whether this is desirable or not. The task of the teacher seems to always involve initiation to a degree but if too many short cuts are taken in effecting this initiation, learning becomes compliance, resulting in students being ill equipped for facing new situations. We also risk facing a broader scale version of the Brousseau's "didactical trap", namely, the more we seek to specify the nature of a more pure mathematics the more risk there is of students not experiencing it. Teachers, however, need to find ways of enabling students to engage mathematically beyond the frames teachers themselves offer.

THE SOCIAL CONSTITUTION OF MATHEMATICS AND PROFESSIONAL CHANGE

In the current world wide climate of rapid economic restructuring, choices for teachers are not clear. Conceptions of mathematics change variously in a multitude of practices whilst governmental preferences in curriculum design fluctuate in response to rather different agendas. Criteria for professional advancement are not always commensurate with meeting perceived local needs. Teachers, however, must assume some sort of professional identity if they are to build up resistances to increasing environmental pressures and conceptualise the changes they face. The teaching environment has become too complex for all demands to be complied with and so teachers are forced into making choices, both constrained and enabled by the structural framework they meet. Rapid change seems to have become a permanent condition rather than being a mere short term phenomenon seeing us through to a new stable state.

In the mid-eighties I worked with a group of teachers doing doctoral studies under the supervision of Bill Brookes. The group's principal aim was to tackle issues in education resulting from rapid environmental change. This work resulted in the emergence of the journal "Chreods". In the first issue we outlined certain tenets underlying our concerns:

-practice is governed by belief and that in changing circumstances there is a need to work at those implicit presuppositions which prevent accommodation to new conditions.
-When the rate of change of environment is faster than the rate of human generation there is an implicit destabilising.
-The co-existence of an increasing variety of possibility in the world with the

constant universal commonness of growth from birth to maturity implies unavoidable conflict.
-This is seen as a constant state of resolving; that is of control in changing from state to state. It implies dynamic stability; homeorhesis (stability of flow) instead of homeostasis (stability of state) (Brookes, 1986).

These principles locate a desire to address the difficulties faced by teaching, learning and mathematics to keep pace with demands placed on them. Like any curriculum subject, mathematics is, to a large measure, a function of the demands placed on it. As these demands evolve the subject itself comes under increasing pressure to change. What appeared static begins to strain as the pace of environmental evolution overtakes that of generational change (i.e. pertaining to changes consequential to normal biological growth) with substantial changes taking place within a lifetime, within a period of schooling even. Yet the task of creating new curriculums is a complex affair and overseen by governments often lacking the insider perspectives of teachers. For example, Brookes (ibid., see also 1978) argues that the recent penchant for criterion referenced testing in curriculum documents within the United Kingdom is

> an intellectual product of engagement with a strongly behaviourist form of thinking involving a belief in identifiable aims and objectives for any particular educational enterprise.

Such a move, he sees as an attempt to force adaptation to a new order, sterilised in the language of the old. Here mathematics is set up as something relatively static where teachers are forced into assuming a "technical" task of implementing a particular version, according to externally defined "standards". Brookes continues:

> The recognition that the world is changing rapidly casts doubt on any programme which depends on rigidly defined propositions embodied in a static educational theory not capable of responding to environmental change.

The way in which mathematics is socially constituted is subject to considerable turbulence. There are multiple strands of change in the ways in which it is understood as a discipline to be taught. These include:

-Changes in mathematics as utilised by society
The composition of mathematics itself, understood as a discipline

supporting human endeavours in the physical world, is constantly on the move, as are the procedures through which we apply it. There are shifts in economic and social structure. Communities grow and decline. Human skills become increasingly specialised and short lived. Mathematics moulds itself around perceived needs and thus can never be in a stable relation with the world it serves.

-Changes in mathematics as seen by specialist mathematicians
Research in "new" areas and popularisation of others shift the relative emphases received by the various topics within mathematics. Whether it be pursuing Chaos theory, Cabri geometry or spreadsheets, mathematicians enjoy promoting their own specific patch and ensuring that it receives appropriate space

-Changes in mathematics as described in school curriculums
Developments in the curriculum such as handling data and an increased emphasis on mathematical processes have altered the composition of mathematics lessons as have computer technology and calculators. Curriculums are a function of many forces and interests created by variously perceived needs. Brookes (1994) offers the example of "computer studies", as taught in British schools until recently, where "environmental" change of computer knowledge and skills was far greater than the response time needed for the necessary change in the exam system. It was supposedly a "modern" subject but permanently out of date.

-Changes in adult-child relations
The skills of the mother are less often those used by the daughter. The teacher increasingly trains the child for a world of which the teacher has little knowledge.

-Evolution of learning theories and teaching practices.
Learning theories and their implementation are themselves increasingly susceptible to the intellectual turmoil that surrounds us and impinge on the way teachers conceptualise and carry out their practice. (A recent graduate from a British school may be a little confused: *"Well yes, when I was five I started off learning my tables, hut then they decided I needed to understand them more, so they put me on my own to learn at my own pace, that is, until they decided I needed to investigate more and chat about my findings, but frankly just before I left the teachers had just given up on all that and simply told me what to do and tested me to make sure I*

knew it.")

Teachers faced with presenting mathematics to children have to navigate a careful path between all of these conflicting demands but very often, as seen earlier with initial training students, their skills in conceptualising mathematics do not necessarily extend far beyond that which they teach. There are, for example, widespread difficulties among primary level teachers attempting to convert the mathematics they learnt as students in school into pedagogical content for the students they now teach. Recently, there has been a substantial amount of research in many countries motivated by such themes (for example, Shulman, 1987; Peterson et al., 1989; Ball, 1990, 1991; Calderhead et al., 1991; McNamara 1991; Strauss, 1993; Meredith, 1995; Aubrey, 1996). Two related reports discussing teachers beliefs in British schools, have shown that teachers experienced great difficulty in describing their own practice in terms of the language employed within curriculum documents (Millett, 1996; Simon et al., 1996). Indeed the much promoted notion of "using and applying mathematics", which featured prominently in these documents, rarely appeared in teachers' descriptions of their own practice. There seemed to be a significant lag between attempts at written curriculum led change and evidence of this impacting on the teachers' practice and their way of describing this. British teachers have recently experienced a succession of such curriculums, with each new set of guidelines modifying the last to a significant degree. The teachers did not have the opportunity to live in a particular version for more than a few months before yet another change was demanded. The disorientation and fatigue brought about in teachers by such rapid change resulted in a promise from the government not to change anything else for another five years to appease a disillusioned teaching force. At their presentation the writers of these reports suggested that this stability in the curriculum might in itself bring about an improvement. To use terminology introduced in the last chapter, it was as if the collective attitude of teachers towards their motives for teaching had shifted firmly into "because" mode, as they stumbled from one new set of guidelines to the next.

Nevertheless, issues of curriculum reform in mathematics manifest themselves in different ways around the world and are met with different approaches with little consensus as to the optimum content or the level of teacher influence. To cite but a few varied examples that come to hand: In Holland, where the new curriculum is not compulsory, the "realist" approach, which is gaining broad based respectability, makes explicit attempts to situate mathematics within contemporary thematic "real-life'' contexts (for example, Gravemeijer et al, 1990; Streefland, 1991). In contrast, the recent Chinese curriculum reinforces a long standing concern with the "basic knowledge and basic skills of mathematics" (CNEC, 1991) and downplays any notions of "using and applying mathematics" (Dawei, 1992). New Zealand presently seems to be going down a similar path to one one described above with its new curriculum drawing many new-right assumptions from the British model (for example, Begg, 1995; Bell et al, 1995). New curriculums in Scandinavia are generally broad statements of intent that emphasise changes but do not include specific details, leaving rather more to the teacher's interpretation (Begg, ibid.). In Poland, recent political changes have resulted in teachers becoming more involved in the process of curriculum change towards more student centred methods (for example, CODN/SNM, 1993). Reform in the United States also seems to be motivated by an increased emphasis on "inquiry" methods of teaching mathematics (NCTM, 1989). Meanwhile, adjustments in curriculum are common and various within developing countries drawing on international assistance supplied through short term arrangements. Reynolds (1996) provides an outline of recent international surveys of mathematical achievement and approaches to teaching. However, his paper arises in response to widely held beliefs in the need to make international comparisons according to universal criteria created outside of a frame that recognises the cultural specificity of environmental growth. Such comparisons are wholly concerned with generational change, in the sense of children being initiated into the current adult world, in a statistically defined environment.

Whatever the approach to curriculum taken, however, I wish to argue that the rapidity of current environmental change is such that educational policy grounded in the beliefs of the last generation and how they did things, risks losing touch with the challenges faced by the new generation, yet rapid policy change can in itself bring about problems. Such recognitions seem crucial whether one sees the task as implementing existing conceptions of mathematics in changing conditions or of also seeing the conceptions of mathematics themselves as subject to revision. The very relationship between successive generations is challenged as past roles assumed between old and young are subject to new demands and potential conflicts (cf. Lave and Wenger, 1991, pp, 113-117).

To meet these growing pressures students will need to be more equipped to generate and work with their own accounts of the realities they face rather than rely too heavily on the accounts provided by their elders. The structures which govern our actions in mathematics education, however, are embedded in the very language we use; our mathematical language, the way governments describe mathematics on a curriculum, the way we describe our individual intentions as a teacher, etc. Increasingly, in our "post-modern" age, our words are more sturdy than the things they describe but none of these ways of talking exist for very long either and often become invalid before they become familiar. Their chances of surviving and being passed on to the next generation forever recede.

Nevertheless, mathematics must have some sort of identity before it can be resisted. Teachers have to live a style of teaching before they are able to build a clear sense of how they are achieving their particular objectives and how they might achieve new objectives. This requires that they understand how they are practically related to their situation and see their motives for teaching within an "in-order-to" mode. It is for these reasons, I have suggested, that teacher intentions have to be couched within a process of professional development. There is a clear need to reconceptualise professional practice and to do this frequently but you can never do more than move from where you are. Teachers, students and curriculum planners cannot change instantly but need some sense of how they might develop. I suggest the principal task for mathematics students should not be to "reconstruct" the thoughts of ancestors or contemporary experts but, nevertheless, they do need some understanding of this inheritance and its manifestations before they are equipped to introduce and develop structures of their own. Curriculum designers, meanwhile, perhaps need to be motivated more by needs and possibilities, based on movement from existing practices, rather than by criterion referenced "standards" which, arguably, put everyone outside. The task for both teachers and curriculum designers, within this frame, is to understand their practice as embedded within social change over which they have some influence. Language, as employed here goes further than passively describing "how things are" but rather, language use <u>becomes</u> generative action, emphasising how things are changing and contributing to that change, through guiding action, deciding the next best step, and sometimes being the next step. The task for the individual professional involves questioning what she can do to bring about a better state of affairs within this process.

But to conclude, I shall revisit Brookes' words cited at the very beginning of the book where he says that there is a need to

> accept the twin constraints of an environmental framework that is changing non-repetitively and accelerating and a generational framework which is cyclically repeatable and only gently changing. Brookes (1994, p. 45)

I have to question whether we are able to accept these constraints in constructing educational practices since no one can enjoy an overview of the task and, if they did, whether it would be in their interest, or within their power, to change policy in a way which faces the conflicts arising. Since policy makers, in any case, are limited in their ability to facilitate any behaviourist inclinations they may harbour, we may have to remain with imperfect attempts to reach consensus where policy intent has always changed before the previous policy has been implemented. Brookes argues that we have already learned to live with partial success as the norm, whether we admit it or not.

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