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Bargaining in Economic and Ethical Environments

An Experimental Study and Normative Solution Concepts

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Contents

Part II

Normative Bargaining Theories on Economic and Ethical Environments

Chapter 1

Introduction

The discussion on welfarism and bargaining theory has several lines. There are different points of criticism on the Nash bargaining model (1950) and on the class of normative solution concepts like the Nash solution, the Kalai–Smorodinsky solution (1975) and others. Sen (1970) argues, that the status quo of a bargaining situation may be an inappropriate reference point to select a social alternative by bargaining, because of the possible injustice of preconditions of the bargaining situation that are represented in this point. Rawls' comment (1971) points in the same direction: "What is lacking is a suitable definition of a status quo that is acceptable from the moral point of view." Ethical considerations and aspects of fairness and justice play a role in negotiations. How can they be modelled? Redefining the status quo may bring ethical norms into the model. However, it does not enlarge the informational content of the theory.

Roemer points out that the definition of the status quo of a bargaining situation is not able to capture all information that is necessary to model a bargaining problem in a given social environment. "If rights are important, they must be fully incorporated into the threat point. I do not think that all the rights–relevant information can be summarized in the threat point" (1986).

In several papers, Roemer also argues against the definition and the properties of bargaining solutions and solution concepts only dependent on utility information. Normative bargaining solutions defined in the Nash model fulfill what Roemer calls "universal welfarism" (1990). This means that whenever the images of two different bargaining problems in utility space are identical, a welldefined bargaining solution chooses the same utility vector as the agreement point. This is the type of welfarism that is usually discussed in the literature.

Elster (1991) describes that in societies different procedures and rules are applied when scarce resources have to be allocated. The applied methods and principles depend on the type of the resources and on the scale of the situation. There are also differences between the allocations of resources and burdens. Elster calls this phenomenon "local justice". The existence of local justice, non–universal justice, is a contradiction to universal welfarism, where the information on the type of resource or burden does not matter. Do "local" distributive principles also play a role in bargaining problems? In Part I we describe some results of studies by other authors and of our experimental study showing this phenomenon.

Roemer summarizes: "It is not surprising that universal welfarism is ethically unattractive, for our intuitions on what social alternatives are just depend intimately on the *kinds* of social alternatives that are being compared, or the kind of utility that is involved" (1990).

Roemer introduces the concept of "simple welfarism". Here, the set of alternatives, the domain of possible utility functions and the set of individuals are fixed. If under these restrictions a solution does only depend on the image of the problem in utility space, it is simple welfarist. Nash implicitely assumed simple welfarism to prove the characterization of his solution. Roemer shows that a reformulation of Nash's axioms on economic environments without simple welfarism does lead to a variety of possible solutions.

Roemer argues against simple welfarism by showing two main problems of this assumption. "In some cases, the welfarist postulate, even in a positive theory, is a bad one because the agents being studied take ethical norms into account in their economic behavior.... But ethical views are not the only reason that simple welfarism is a poor postulate: the discussion indicting Nash's independence of irrelevant alternatives axiom was not ethical. Generally speaking, focal points are important in the positive description of economic behavior, and those foci are defined in terms of actual resources or lotteries being allocated" (1990). Roemer criticizes not only normative bargaining theories using simple welfarism, but also positive theories. In his reply, Sen (1990) evaluates Roemers contribution. "His rejection of welfarism in the context of bargaining theory extends – in an important way – the critique of welfarism from prescriptive to predictive theory. The failure of welfarism in the context of predictive theory of bargaining is due partly to the impact of ethics and norms on behavior and partly to the influence of initial (non–utility) conditions." However, this criticism on positive theories does not hold for a lot of studies we mention in Section 2.1 that form the background and motivation for our own experiments.

In Part I we describe an experimental study on bargaining behavior. We investigate the influence of some experimental conditions, called environments, on the behavior of subjects. We also try to find out which norms the subjects apply and how they justify them. We also consider variations of bargaining problems in the sense of simple welfarism. We choose a fixed payoff possibility set and embed it into different ethical environments. Some observations from these experiments lead to new ideas for a non–welfaristic normative bargaining theory. This theory and some other results concerning solution concepts derived from distributive principles that we observe in the experiments are to be found in Part II. We conclude by combining both parts to a predictive theory for bargaining problems of the type we investigate in our experiments.

Of course, the argument of Gaertner (1992) is partly applicable to our study, too: "However, many game–theoretical experiments still involve rather simple situations so that these factors from the economic or ethical environment to which Roemer was referring, quite often cannot be taken into adequate consideration. Consequently, the findings from those experiments are only of limited value for problems of distributive justice." We have to restrict the experimental study to a certain type of situations, and we have to define a small subset of variables we want to control. Nevertheless, we try to develop some aspects of a positive theory of bargaining in ethical and economic environments that are generalizable to a certain extent and we describe some ideas for a possible generalization. The normative theories are formulated in the most general way allowed by the special features of the solution concepts.

The aim of this study is to show some possibilities of a specific type of research. Starting with some criticism on welfaristic bargaining theories we find a motivation for an explorative study on bargaining behavior in situations with different economic and ethical environments. From the observations in these games we get the idea to formulate properties of non–welfaristic normative bargaining solutions on economic environments. We also find a way to deal with differences between ethical environments in this normative theory. The third step is to apply the new solution concept to the data of the experiments and to compare the agreements predicted by the theory to the experimental data.

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Part I

An Experimental Study on Bargaining Behavior

Chapter 2

Motivation and Design of the Study *)

2.1 Motivation

In his fundamental paper of 1950, Nash constructs a model of a bargaining situation with two persons and formulates a set of axioms which uniquely characterize a bargaining solution, the so–called Nash solution. In Nash's model preferences of the persons over a set X of feasible alternatives are expressed by cardinal v. Neumann–Morgenstern utility functions. Among the alternatives in X there exists a certain alternative x_0, the alternative of disagreement, often called status quo. In the general case of $n \geq 2$ persons, the pair (X, x_0) is mapped by the utility functions of the persons onto a pair (S, d) in an n–dimensional utility space. (S, d) is called a bargaining situation with n persons, if S is a convex and compact subset of $\mathbb{R}^n$, if d is an element in S, and if there exists an alternative x with an image s in S such that every person strictly prefers x to x_0, i.e. $s > d$. For every bargaining situation a bargaining solution f selects a point $f(S, d)$ in S.

The classical solution concepts make use of the assumption that the

*) Parts of this chapter are literally cited from Klemisch–Ahlert, M.: ***Distributive Results in Bargaining Experiments.*** In: Gaertner,W. and Klemisch–Ahlert, M. (1992)

whole information which is relevant for the bargaining problem can be represented in the utility space. In addition, the solutions are characterized by axioms claiming strong rationality requirements which are expressed by relations between utility levels.

Nash himself pointed out that using utility functions in bargaining models is a strong idealization, as there are many important properties of bargaining problems which are not representable in utility spaces.

Yaari and Bar–Hillel (1984) have run an experiment with questionnaires on the distribution of commodities between two individuals. Among the solutions from which the respondents were asked to choose were the Nash bargaining solution, the Kalai–Smorodinsky solution and others. The results show that for different distributive problems having the same representation in utility space the solutions choosen by the respondents depend on special economic, social or ethical aspects of the situations. Schokkaert and Overlaet (1989) have reported similar results for some other types of distributive problems. They also consider the sharing of losses.

Persons' judgement on the justice of distributions apparently depend on dimensions such as

— needs of the persons involved in the problem
— their possibility to enjoy certain goods
— their endowments and skills
— their effort or productivity or contribution to a cooperative product
— their rights or legitimate claims.

Applied principles may vary dependent on the type of the environment of the distributive problem and dependent on the objects to be distributed. Experimental studies have to concentrate on a reduced number of aspects to be controlled. Gaertner (1992) argues in support of this procedure: "Therefore, it may seem justified to partition a social state into sub–categories such as political rights and liberties, basic health and longevity, the quality of the environment, the provision with material goods and services, and other aspects and then decide with respect to each component." The studies we mention in the following deal with the allocation of monetary payoffs under certain controlled environmental conditions. We concede that this is only a small aspect of what is meant by a social state.

A comparison given by Güth (1989) of the distributive results in dictatorship games, reward allocation games, and ultimatum bargaining games shows that changes in the experimental environment have severe effects on the behavior of the subjects. From the results of several ultimatum bargaining experiments it follows that the subjects nearly never choose the game theoretic solution. At least, when the payments in the experiments are rather low, strategically irrelevant aspects have a strong influence on the bargaining behavior.

In dictator experiments and ultimatum bargaining experiments the strongness of the selfishness of the players seems to be influenced by the moral justification of their position. In the experiments Güth describes, this justification has to be deduced from the experimental environment, e.g. the talent of a person or her ability to win a strategic game or an auction.

In addition, the impact of an economic variation on the behavior of the players can be observed. Higher total amounts of payoffs lead to higher proportional demands of the players. In this case, players also seem to investigate the situation more precisely and to behave more carefully.

In reward allocation experiments, a contribution standard is observed. Inferior players tend to split the total payment proportional to the contributions of the persons. Contributions are the obvious basis for them to legitimate their claims. Therefore, this type of equity principle seems to be stronger than the equal split tendency. The superior allocators more often choose equal rewards.

In Selten's Laboratory of Experimental Economics in Bonn experimental two–person characteristic function games have been conducted. The economic conditions that were varied in these bargaining experiments were the status quo of the bargaining situations $(v(1), v(2))$, the value of the two person coalitions $v(12)$ which is the value to be divided by the two persons, and the scale factor for the payoffs. Uhlich (1988, 1990) introduces a descriptive area theory for this type of experiments, the Negotiation Agreement Area (NAA) for nonnegative status quo points. The theory is extended by Rockenbach and Uhlich (1989) to situations with negative threat points. The authors show that in comparison with normative theories the NAA is the area theory with the best predictive success. The NAA is defined with the aid of three aspiration levels: the maximal aspiration level for the strong player (Player 1) A_1^{max} and for the weak player (Player 2) A_2^{max}, and the attainable aspiration level for the weak player A_2^{att}. The last level is defined by the equal surplus norm. The lower bounds for the payoff of the strong player is defined by $\frac{A_1^{max}}{A_1^{max} + A_2^{max}}\ v(1,2)$, and the lower bound for the payoff of the weak player is $\frac{A_2^{att}}{A_1^{max} + A_2^{att}}\ v(1,2)$. The NAA is then bounded by these values after some corrections with respect to the prominence level. The proportionality factors of the bounds reflect the different positions of the players in the game. The factor of the strong player is defined by the proportion of his maximal aspiration level to the sum of both maximal aspiration levels, whereas the factor of the weak player is deduced from his attainable aspiration level that involves the thought of equal split of the surplus.

2.2 Design of the Study

We mentioned some examples of interviews and experiments in order to demonstrate how solutions to distributive problems may depend on economic, social or ethical dimensions of the experimental environment. We are interested in some of these dimensions and we design some classes of bargaining experiments with identical payoff constellations and different environments.

It is our aim to evaluate the distributive principles the subjects apply in the experiments, when they formulate their bargaining marks and expectations, as well as the justifications for the principles. We are also interested in their considerations concerning the principles and justifications their opponents apply. In addition we observe the bargaining process and the resulting agreements.

We try to answer to following questions:

— Which principles do the subjects apply dependent on their bargaining position?

— How do the applied principles and the agreements depend on the economic or ethical environment of the experiments?

— How does the agreement depend on these principles?

We investigate bargaining situations with two opponent parties, Group A and Group B, each group consisting of two subjects. Group A is formed by Players 1 and 2, Group B by Players 3 and 4. The parties bargain about the distribution of a certain amount of money. In contrast to the experiments of Uhlich and Rockenbach, the sum of the payoffs of the four subjects is not constant. It depends on the agreement of the parties. The situations are asymmetric, i.e. the two parties have different payoff functions. Between the two parties, verbal communication is not allowed. Proposals and answers are written on forms.

We would like to restrict the set of possible variations for our study to the following types:

We choose some fixed payoff sets and embed them into four different kinds of experimental environments. The first environment is defined only by the payoffs. In the second environment the payoffs are enlarged by multiplication with the factor 2.5. In the third type of experiments the positions in the game are filled with subjects dependent on their contribution to a collective task, which has to be performed previously. In the fourth environment payoffs of one party are connected to additional payments to indigents. These payments are remitted as gifts to certain social services, selected by the subjects.

The variants of the sets of feasible payoffs for the two parties are given in Figure 2.1. The Pareto optimal boundary (in terms of payoffs) of a situation and the status quo (0–payoffs for all persons) define the feasible payoff set.

Different variants are generated from Situation 1 by truncating the top or the right part of the triangle at certain levels. The subjects don't know this two–dimensional graphic representation of the feasible payoff constellation. The material they receive consists of payoff tables and offer forms (Figure 2.2 shows an example of a payoff table and Figure 2.3 an offer form).

Situations
(Payoffs per person (DM) of group A and B)

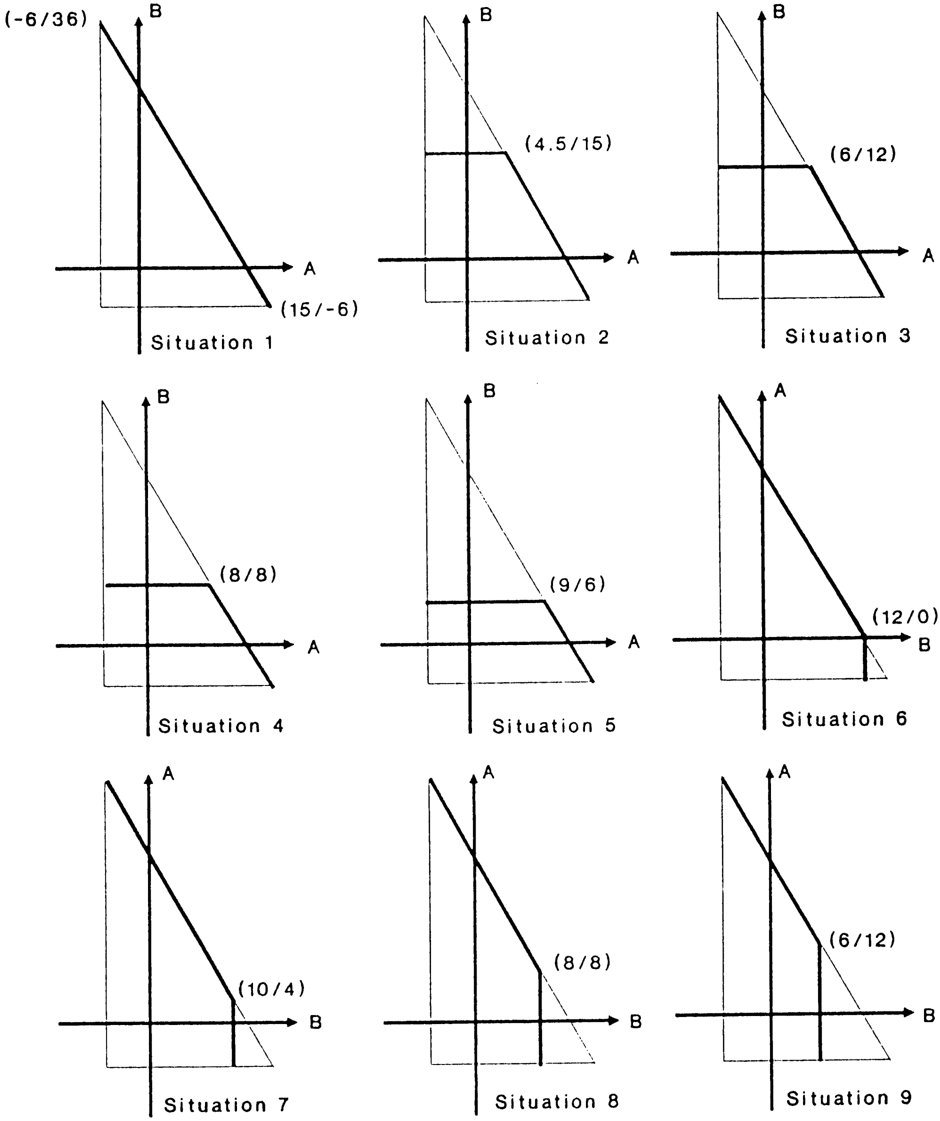

Figure 2.1

Konflikt 2,8×2,5 Auszahlungstabelle

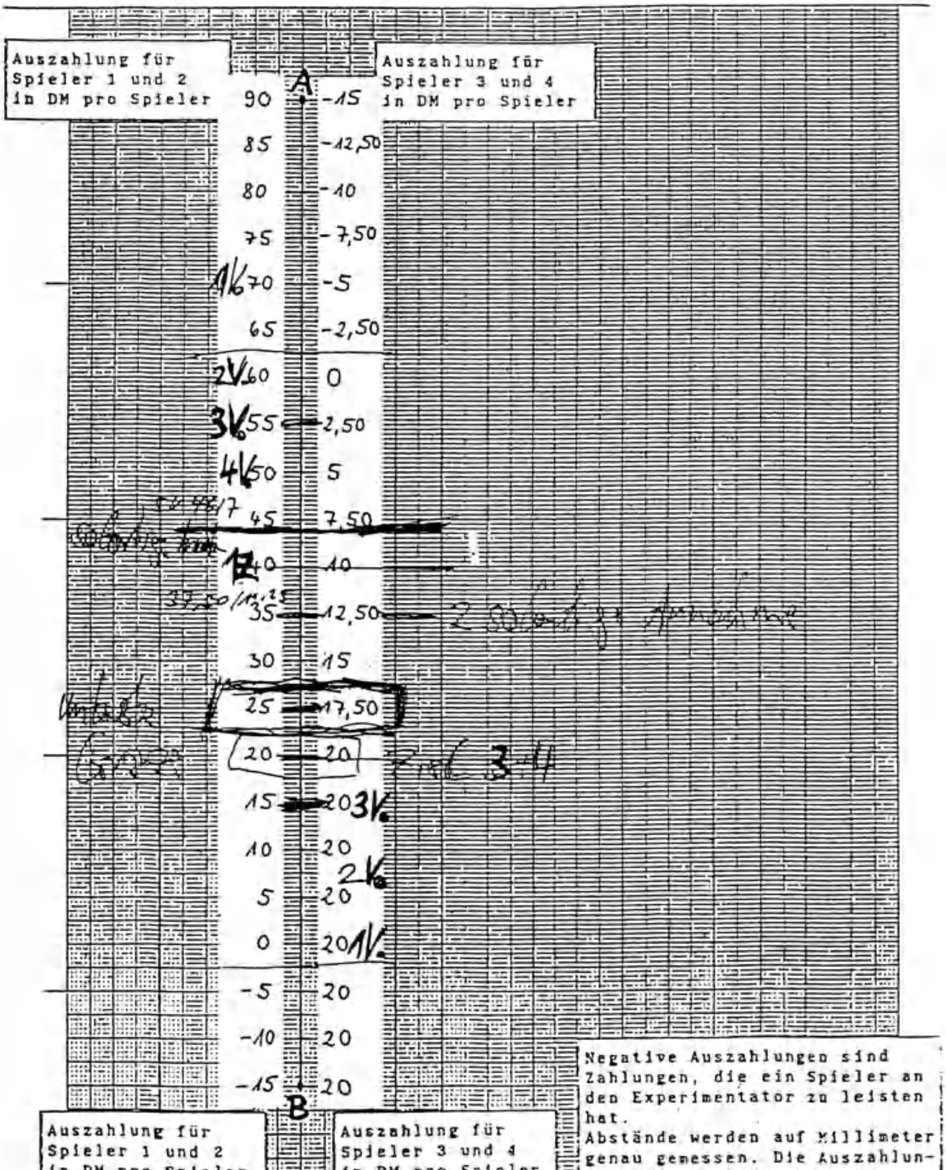

Figure 2.2

Experiment vom 14.11.90　　Konflikt 2.8 × 2.5

Spieler 1: ______　　Spieler 2: ______

Spieler 3: ______　　Spieler 4: ______

Uhrzeit: 18.13

Vorschlag von Spieler 1 u. 2 Runde 13	Spieler 1 / 2 Je 40 DM	Spieler 3 / 4 Je 10 DM	Spieler 3 Annahme ☐ Ablehnung ☒

Uhrzeit: 18.17

Vorschlag von Spieler 3 u. 4 Runde 14	Spieler 1 / 2 Je 22 DM	Spieler 3 / 4 Je 19 DM	Spieler 1 Annahme ☐ Ablehnung ☒

Uhrzeit: 18.19

Vorschlag von Spieler 1 u. 2 Runde 15	Spieler 1 / 2 Je 37,5 DM	Spieler 3 / 4 Je 11,25 DM	Spieler 3 Annahme ☐ Ablehnung ☒

Uhrzeit: 18.20

Vorschlag von Spieler 3 u. 4 Runde 16	Spieler 1 / 2 Je 25 DM	Spieler 3 / 4 Je 17.50 DM	Spieler 1 Annahme ☐ Ablehnung ☒

Uhrzeit: 18.22

Vorschlag von Spieler 1 u. 2 Runde 17	Spieler 1 / 2 Je 34,- DM	Spieler 3 / 4 Je 13,- DM	Spieler 3 Annahme ☐ Ablehnung ☒

Uhrzeit: 18.27

Vorschlag von Spieler 3 u. 4 Runde 18	Spieler 1 / 2 Je 30 DM	Spieler 3 / 4 Je 15 DM	Spieler 1 Annahme ☒ Ablehnung ☐

Figure 2.3

At the beginning of an experiment the experimentor assigns the player numbers to the subjects, without telling the numbers to the subjects. In the first, second and fourth environment, the numbers are determined randomly. In the third environment the persons have to pass a multiple choice test in microeconomics, directly before the bargaining games are played. The ranking of the results of the subjects in the test defines the positions in the bargaining conflicts. The subjects are told that proportional to their commonly achieved numbers of points in the test a factor will be chosen by which a standard payoff situation will be multiplied. In addition they are informed that the strongness of their positions in the bargaining game played afterwards will depend on their individual contribution to the total amount of points.

The two groups are led into optically and acustically separated rooms. There they receive the payoff table. They have ten minutes of time to discuss the table with their team partner. Afterwards they are told to which group they belong, which player number they have, and which group has to make the first offer. Player 1 acts for Group A, Player 3 acts for Group B. The two parties communicate on offer forms, on which they have to write down their offers by turns. The party which has to make the next offer has up to ten minutes of time for this decision. The acting player of the opposing party decides in a third room whether he accepts the offer or not. Afterwards he gives reasons for his decision to his team partner. Every player has the possibility to declare "Disagreement" at any time. In this case the game is finished and the players receive their disagreement payoffs (0 DM). If some acting player (Player 1 or 3) accepts an offer, then an agreement is reached, the game is finished, and the players receive the payoffs specified by the accepted offer.

In each group's room a tape recorder is installed which records the discussions between the two players in the same party and the arguments of the acting player when he explains his decisions to his partner.

The subjects of our experiments are undergraduate students of economics and business administration at the University of Osnabrück. Nearly all of them had no knowledge of Game Theory and none of them had participated in a game theoretic experiment before. The subjects were instructed in a 30 minutes session immediately before the experiment started. Each subject played in two or three different situations (dependent on the types of the ethical environments) with different partners and different opponents.

2.3 Data

We are able to analyze the following data of every game: the economic and ethical environment of the situation; the time sequence of offers by the two parties and the result of the game; the principles and their justifications the players use in their discussions and reasoning of their expectations and behavior.

From the whole set of data we choose the following variables of each game for our analysis:
— the result of the game (in payoffs)
— the number of rounds
— the length of the game (in minutes)
— the time sequence of offers.

From the discussions on the tapes we gather the following aspiration levels of every group. These levels are observable in nearly all of the games we played (cf. Tietz and Bartos (1983))
— the planned bargaining goal
— the agreement seen as attainable
— the lowest acceptable agreement
— the expected planned bargaining goal of the opponents
— the expected lowest agreement of the opponents.

We sometimes have further information on planned threats to break–off negotiations and expectations about break–off conditions of the opponents. Often we know expectations about the first offer of the opponents.

We analyze the data of 47 games with 80 subjects. In the following table we show in which environments each basic situation is repeated and how many repetitions have been played. In this table the basic situations are named by their numbers. If the term $*2.5$ is added to a number, this characterizes a situation which is generated by multiplying the payoffs of a basic situation by 2.5. The letter Q means that positions are assigned according to the results of a quiz. The letters A or B mean that Group A resp. B negotiates also for indigents.

Situation (S)	1	$1*2.5$	$1*2.5$ Q	$1*2.5\,A$	$1*2.5\,B$
No. of Repetitions (#)	4	2	1	1	1

S	2	$2*2.5$	3	$3*2.5$	4	$4*2.5$	5	$5*2.5$	$5*2.5$ Q
#	2	2	2	2	2	2	4	2	1

S	6	$6*2.5$	$6*2.5$ Q	7	$7*2.5$	8	$8*2.5$
#	2	2	1	3	2	2	2

S	9	$9*2.5$	$9*2.5$ Q
#	2	2	1

We tried to repeat every basic situation and multiplied basic situation at least twice. Since Situations 1 and 5 have some special features which we will describe in the following chapters, we played them more often. Because of the explorative character of the study we felt justified to decide this. One difficulty with the quiz–experiments was that we needed eight subjects to participate at a certain date. The second difficulty was that it took more than half an hour to let the subjects answer the questions and to evaluate the test. So the subjects had to invest more than two hours of time alltogether for the instruction, the first game with a basic situation, the test and the second game with the quiz–situation. We were only able to recrute enough subjects for two dates. Therefore we got the data of four quiz–games. Also the experiments with payments to indigents needed more time and additional preparation. We decided to choose one situation, namely situation $1*2.5$, to be played in two diferent variants of this type of environment. The purpose of these games was not the statistical evaluation of the data, but a comparison of the discussed principles and arguments to repetitions of Situations 1 and $1*2.5$.

The aspiration levels, norms and justifications mentioned in the discussion on the tapes are subscribed by two persons independently. The protocolls are compared afterwards. There were not many differences that had to be clarified. The aspiration levels could be recognized very unequivocally. In addition changes in the negotiation behavior have been noted in the protocolls.

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Chapter 3

Monotonicity Results

3.1 Introduction

In the experiments we observe that the subjects discuss and apply five distributive principles.

First, there is the equal payoff principle which means that all persons receive the same amount of money. Then there are two principles that are derived from proportionality considerations. The subjects either choose the maximal payoffs for the two groups and calculate their ratio, or they take the maximal payoffs in the individual rational part of the payoff constellations in order to form a ratio. The principles then claim that the ratio between the payoffs of the persons in Groups A and B should be equal to the ratio of their respective maximal values. In our situations with a status quo of 0–payoffs, the second principle defines the Kalai–Rosenthal (1978) solution and the third principle defines the Kalai–Smorodinsky (1975) or Gauthier (1985) solution in payoffs. The fourth principle is the equality of the ratio between the payoffs of the two groups and the transformation rate between the payoffs of the groups. The transformation rate in our situation is $1:2$. This means that, for example, a concession of 1 DM per person of the first group implies a payoff gain of 2 DM for each person in the other group. This principle which is defined for payoffs corresponds to a property of the Nash solution (1950) in utility spaces. The fifth principle is the maximization of the sum of the payoffs of both groups.

Since we vary the shape of the feasible payoff pairs in a systematic way by changing truncation levels, it is possible to construct functions that describe how the payoffs change with respect to the five distributive principles.

In the following sections we investigate a certain kind of interdependence between the payoff situation of a game and the agreement or the planned bargaining goals of the groups. We compare the observed relations to those defined by the distributive principles.

In ultimatum bargaining experiments a tendency to equality of payments in the results is observed. Higher total amounts of payoffs lead to higher proportional demands of the superior players. There seems to be a competition between the principle of equal payoffs and the selfishness of the superior player to exploit his strategic power. The tendency to selfishness becomes stronger when the feasible payoff increases. In dictator experiments there has been observed a politeness ritual. This means that the "dictators" sometimes resign to exploit their positions. In reward allocation experiments the inferior players tend to split the total payment proportional to the contributions of the persons. Contributions are the obvious basis for them to legitimate their claims. Therefore, they use the proportionality principle and not the equal–split principle. The superior allocators more often choose equal rewards. They do not need a justification for this behavior. From these results (Güth, 1989) we learn that the behavior of players in these games is guided by different distributive norms. Often there is more than one principle that could be applied. Which norm a player chooses depends on the type of game, on the position in the game, on the experimental environment (size of the payoffs, how the players get their positions) and on the possible justifications for the application of the norm.

In the following sections we try to find out which principles the subjects in our experiments use to formulate their goals for a negotiation in a game, and which compromise between different principles is made in the agreements. In addition we investigate the effect of enlarged payoff scales on the agreements and the goals.

3.2 Rank Regression Analysis

The statistical procedure we describe in this section has been proposed by Iman and Conover (1979). We use the notation of Conover (1980). The rank regression analysis is a nonparametric method for a monotonic regression of a bivariate data set. It is used in cases where a linear relation between the two variables cannot be expected, but it seems reasonable to assume a monotonically increasing or decreasing relation. In addition the method has the advantage of being applicable even to data sets with a small number of independent observations like in our study.

Rank Transform Regression

Let $(X_i, Y_i)_{i=1,\dots,k}$ be the data set which is a random sample from some bivariate distribution. There is no assumption made on the type of the distribution.

To find the estimate of the regression of Y on X the following procedure is used.

a) Calculate the ranks $R(X_i)$ of the variables $X_1, \dots, X_k$ and the ranks of $R(Y_i)$ of the variables $Y_1, \dots, Y_k$. In the case of ties average ranks are used. This means in the case of no ties, the smallest value receives the rank 1 and the highest value the rank k.

b) The least squares regression line on the ranks has to be found.

$$y = A + B\,x\,, \tag{3.1}$$

where y stands for the ranks of the Y_i, $i = 1, \dots, k$, and x for the ranks of the X_i, $i = 1, \dots, k$.

The formulas to derive A and B are

(3.2) $$B = \frac{\sum_{i=1}^{k} R(X_i)R(Y_i) - k(k+1)^2/4}{\sum_{i=1}^{k} (R(X_i))^2 - k(k+1)^2/4},$$

(3.3) $$A = (1-B)(k+1)/2\,.$$

c) For each rank $R(Y_i)$, $i = 1, \ldots, k$, we can calculate the estimated rank of X_i, $\hat{R}(X_i)$ with respect to the linear regression (3.1):

(3.4) $$\hat{R}(X_i) = (R(Y_i) - A)/B \quad i = 1, \ldots, k\,.$$

d) For each $\hat{R}(X_i)$, $i = 1, \ldots, k$, we have to find the estimate $\hat{X}_i$.
If $\hat{R}(X_i)$ is equal to the rank of some observation X_j, $\hat{X}_i$ is defined to be X_j.
If $\hat{R}(X_i)$ lies between two adjacent ranks of observations X_j and X_ℓ with $X_j < X_\ell$, then $\hat{X}_i$ is found by linear interpolation between X_j and X_ℓ:

(3.5) $$\hat{X}_i = X_j + \frac{R(X_i) - R(X_j)}{R(X_\ell) - R(X_j)}(X_\ell - X_j)\,.$$

If $\hat{R}(X_i)$ is smaller than the smallest observed rank or greater than the largest observed rank, $\hat{X}_i$ cannot be calculated this way. Linear extrapolation is not possible. Then there is no estimate for $\hat{X}_i$ in this case.

e) In order to find the end points of the regression, choose the smallest observation and the largest observation from $X_1, \ldots, X_k$. Let us call them $x_{(1)}$ and $x_{(k)}$.
We have to calculate $\hat{E}(Y \mid X = x_{(1)})$ and $\hat{E}(Y \mid X = x_{(k)})$ in the following way:
We know $R(x_{(1)})$ and $R(x_{(k)})$ and therefore we can estimate the ranks $R(y_{(1)})$ and $R(y_{(k)})$ by applying the regression (3.1).

(3.6) $$R(y_{(1)}) = A + B\,R(x_{(1)}) \quad \text{and}$$
$$R(y_{(k)}) = A + B\,R(x_{(k)})\,.$$

Now we have to find the estimates $\hat{E}(Y \mid X = x_{(1)})$ and $\hat{E}(Y \mid X = x_{(k)})$ by linear interpolation between ranks of the observations of the Y_i $i = 1, \ldots, k$.
This procedure is similar to (d).
If $R(y_{(1)})$ is equal to the rank of an observation Y_i, then $\hat{E}(Y \mid X = x_{(1)}) = Y_i$.
If $R(y_{(1)})$ lies between two adjacent values Y_i and Y_ℓ with $Y_i < Y_\ell$ and $R(Y_i) < R(y_{(1)}) < R(Y_\ell)$, then

$$\hat{E}(Y \mid X = x_{(1)}) = Y_i + \frac{R(y_{(1)}) - R(Y_i)}{R(Y_\ell) - R(Y_i)}(Y_\ell - Y_i) \; .$$

If $R(y_{(1)})$ is smaller than the smallest observed rank of Y_i, $i = 1, \ldots, k$, $\hat{E}(Y \mid X = x_{(1)})$ is defined to be the smallest observed Y_i.
If $R(y_{(1)})$ is greater than the largest observed rank of Y_i, $i = 1, \ldots, k$, $\hat{E}(Y \mid X = x_{(1)})$ is equal to the largest observed Y_i.
The analogous definitions hold for $\hat{E}(Y \mid X = x_{(k)})$.

f) After all points of the rank transform regression have been calculated they can be plotted with $\hat{X}_i$ as the abscissa and Y_i as the ordinate and with the end points $(x_{(1)}\, , \, \hat{E}(Y \mid X = x_{(1)}))$ and $(x_{(k)}\, , \, \hat{E}(Y \mid X = x_{(k)}))$.

g) If we connect the points with straight lines, this graph represents an estimate of the regression of Y on X.
If $B > 0$, all straight lines have to be increasing, in the case $B < 0$ they have to be decreasing. In the first case, we estimate Y for a given X by a monotonic increasing (not necessarily linear) function, in the second case by a monotonic decreasing function. The definition set of this function is the full range of the observation of the X_i, $i = 1, \ldots, k$, because of the calculation of these end points.

We have chosen this method to analyse the relations between the shapes of the situations of our experiments, represented by the truncation levels of the payoff sets, and variables like payoff pair of the agreements and aspiration levels like planned bargaining goals. We do not expect linear relationships between these variables, but we expect some monotonicity relations. Since we do not have information on the distribution of the variables, we do not want to assume a certain type of distribution. This method does not need these assumptions. It leads to an estimate of the investigated variables (e.g. payoff ratios of the groups) for the range of all possible truncation levels. This means we receive an estimation not only for the situations we have played, but also for all situations with truncation levels inbetween. Since in general we will not receive a linear relationship between truncation levels and our observed variable, the type of non–linearity, i.e. changes and differences in the slopes of the straight line connecting the regression points, will lead to additional observations and interpretations.

3.3 Monotonicity in Payoff Ratios of the Agreements

In this section we investigate the relationship between the truncation level of a situation and the ratios of the payoffs in the agreements of the games. There are four data sets with a realization of two variables. The data sets are defined by the type of the truncation (whether the basic triangle is truncated from above or from the right) and by the size of the payoffs (basic payoff constellations are multiplied by 2.5). We calculate rank transform regressions for the resulting four data sets Situations 1 2 3 4 5, Situations 1 2 3 4 5 $*$ 2.5, Situations 1 6 7 8 9 and Situations 1 6 7 8 9 $*$ 2.5 seperately.

The index i of the variables in the rank transform regression numerates the different experiments within one data set. The variables X_i are the truncation levels, where the basic Situation 1 has the truncation level 36 in combination with Situations 2 3 4 5 ($*$ 2.5) and the level 15 in combination with Situations 6 7 8 9 ($*$ 2.5). From the collected data we take the agreements payoffs as a pair $(Z_A|Z_B)$ = (payoff for each person in Group A|payoff for each person in Group B) and calculate the ratio $\frac{Z_B}{Z_A}$. Since for a given situation different agreements lead to different ratios, the agreements can be reconstructed from the ratios, if the truncation level is known. (For agreements that are strongly Pareto optimal in the payoff set, the information on the truncation level is not necessary.) Therefore, investigating the ratios $\frac{Z_B}{Z_A}$ we do not loose information on the agreement payoffs.

The results of the rank transform regression analysis can be found in the appendix. The estimates of the regression of Y on X are plotted in the figures at the end of this section.

In these figures the proportion of the payoff of each person of Group B to the payoff of each person of Group A with respect to certain distributive principles can also be found.

The line "Equal" belongs to the principle "equal payoffs to all persons" and has the constant level 1.

The line "Prop_{T}" defines the payoff ratio for the principle that chooses a point in the payoff set with a payoff ratio identical to the payoff ratio of the maximal payoffs of each person in the whole payoff set.

The line "Prop_{IR}" does the same for the principle that is defined by the proportion of the maximal payoffs of each person in the individually rational part of the payoff set.

The line called "Nash" represents the ratio of a payoff pair that would be defined by the Nash bargaining solution applied to the payoff set.

These curves have been calculated for all possible truncation levels between 0 and 36 for truncation of the basic situation from above and between 0 and 15 for truncation from the right.

Let us consider the graph "Payoff" of the rank transform regression analysis for the Situations 1 2 3 4 5. The line segments are strictly increasing. This means the higher the truncation level on the axis of Group B is, the greater is the estimated payoff ratio of a person in B to a person in A. The estimated ratios are close to 1, i.e. close to "Equal".

We define a position of a group to be stronger than the position of the other group, if the estimated payoff of a person in this group is greater than that of a person in the other group. We observe that in this data set "strength" switches at a truncation level of 13.28. For levels lower than this, Group A has a stronger position than B, for higher levels vice versa.

There are some estimated points for levels between 6 and 7, where the estimated payoff ratios are higher than the "Nash" curve which in this area is defined by the kink of the payoff situation. This means that agreements are predicted that are weakly Pareto optimal but not strongly Pareto optimal and that this type of agreements has been observed. We describe this phenomenon in Section 4.2

Now we consider the Situations 1 2 3 4 5 $*$ 2.5. Again the rank transform regression shows a monotonically increasing relation between the truncation level and the payoff ratio B/A. Comparing the slopes of the lines connecting the estimated points of the Situations 1 2 3 4 5 and 1 2 3 4 5 $*$ 2.5 we find that they are steeper in the situations with the multiplied payoffs. Here, changes of the truncation levels lead to more drastical changes of the payoff ratios.

The switch point for the strength of the groups is in this case estimated as a truncation level of 8.76. This truncation level is close to Situation 4, where the "Equal" principle leads to the kink of the payoff set ("Nash"), i.e. these two principles fall together. This means that the payoff pair of the kink can be used as an estimate for the strength of the positions. The group with the higher payoff in the kink has the stronger position. Of course, the kink of a payoff situation is a focal point. In the payoff tables the players get, this is a point such that one group has a constant payoff above or below.

The stronger groups can exploit their position better in Situations 1 2 3 4 5$*$2.5 than in 1 2 3 4 5. The estimated payoff ratio in Situation 1$*$2.5 for Group B is 2.92, which is even more than the "Prop_T" principle would predict. If we compare this to the estimated payoff ratio in Situation 1 for Group B, which is 1.39, we find that the ratio is more than twice as large. For truncation level 6, where Group A is the stronger group, we have the payoff ratio B/A of 0.71 for Situation 1 $*$ 2.5 and 0.74 for Situation 1. In this case too, the stronger group is relatively better off in the agreements when the payoffs are multiplied.

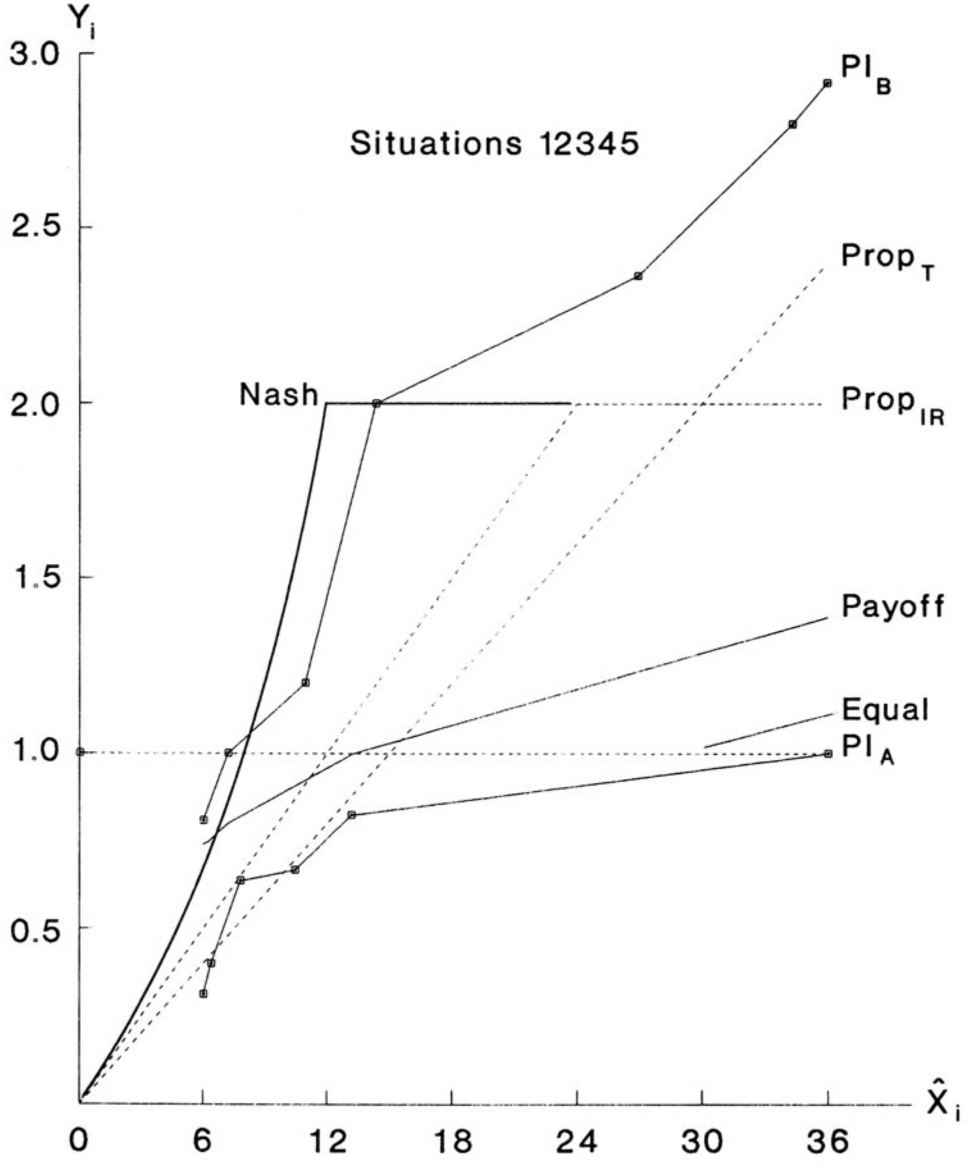
For truncation levels between 26 and 36, i.e. truncations outside of the individually rational part of the payoff sets, payoff proportions are predicted that are greater than the values defined by any principle. For instance in Situation 1 $*$ 2.5, the maximal payoff of each person in Group B is 90 DM and the maximal payoff of A is 37.50 DM. From the tapes we learn that the possibility to win an amount of nearly 100 DM makes an enormous impression on all players. This leads to a more than proportional increase of the payoff ratio in favor of Group B. We will discuss this observation more detailed when we describe the estimation of the planned bargaining goals of the groups.

In Situations 6 7 8 9 the assignment of the names A and B to the groups is changed so that the group with the constant part in the payoff table is again Group B. We also change the names of the groups in Situation 1. Now this situation has the truncation level 15 for Group B (former Group A), which means that no truncation takes place.

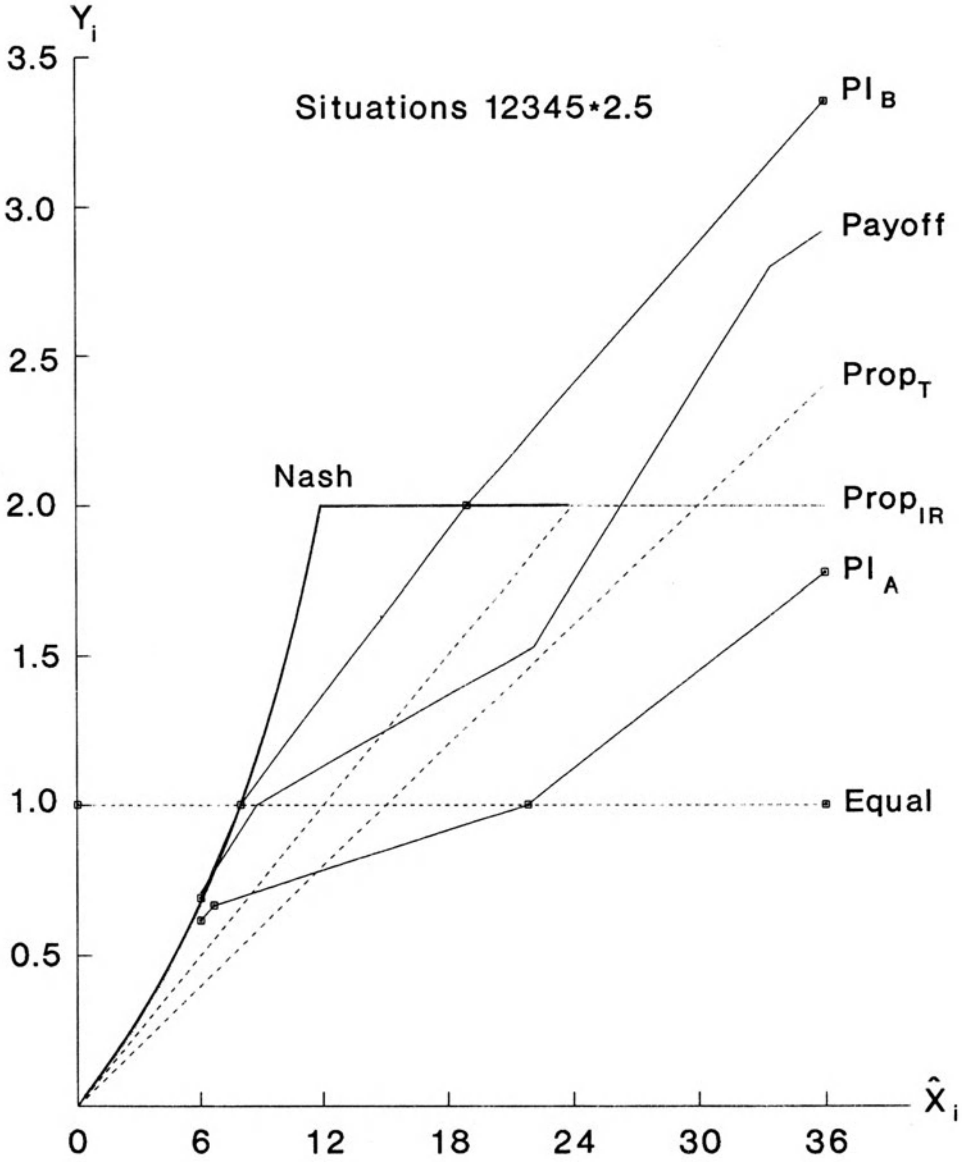
In these situations Group A always is the stronger group. In the rank transform regression of the payoff ratios B/A, the estimated curves for the data sets 1 6 7 8 9 and 1 6 7 8 9 $*$ 2.5 lie below the "Equal" line. Again both curves are increasing. This is the same monotonicity property of payoff ratios dependent on truncation levels as in the data sets 1 2 3 4 5 and 1 2 3 4 5 $*$ 2.5. Comparing the payoff ratios of both data sets we find that the multiplied payoff scale leads to an estimated payoff ratio curve that lies between or close to the "Prop" lines, whereas in the standard Situations 1 6 7 8 9 the estimated values are greater and lie between the "Equal" line and the "Prop" lines. Therefore we have an observation analogous to the data sets 1 2 3 4 5 and 1 2 3 4 5 $*$ 2.5 . The multiplication of the payoffs of a given situation by 2.5 leads to estimated agreements where the weaker party is relatively worse off.

From the four data sets we observe a monotonicity relation between the payoff ratios and the truncation levels. And we find that the multiplied payoff scales lead to a relatively greater estimated succes of the stronger party. This tendency is also observed in dictatorship experiments and ultimatum bargaining experiments (cf. Güth 1989).

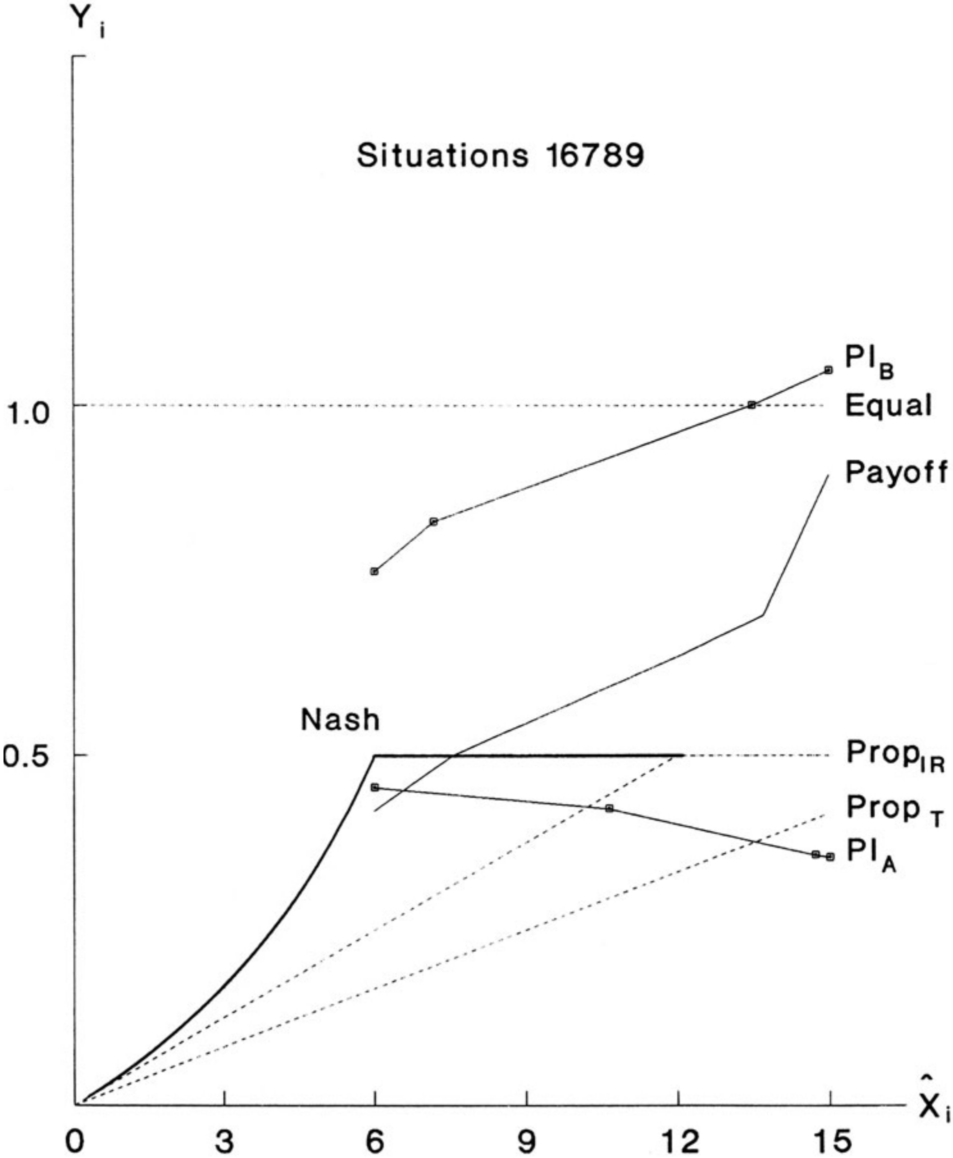
Planned Bargaining Goals and Payoffs in Ratios B/A



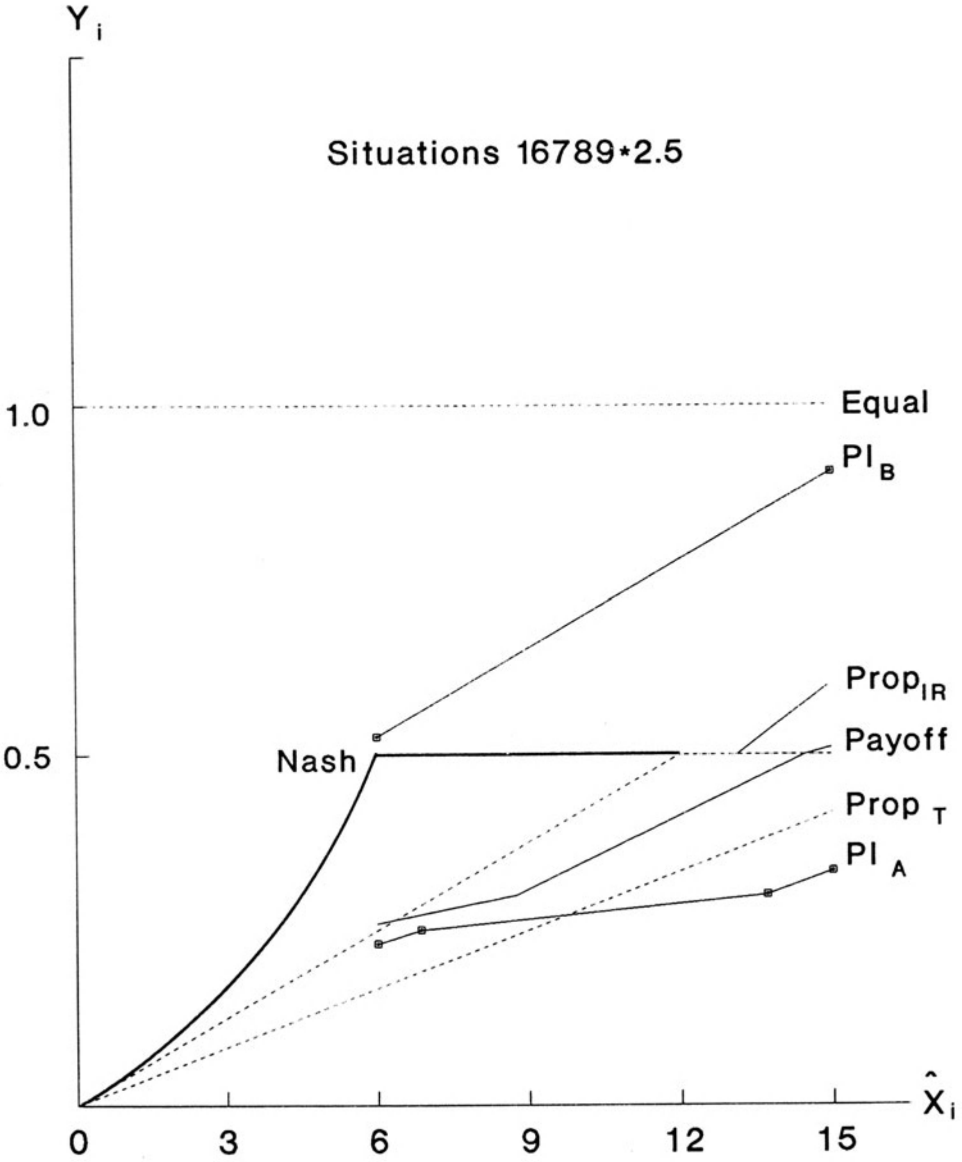
Planned Bargaining Goals and Payoffs in Ratios B/A



Planned Bargaining Goals and Payoffs in Ratios B/A



Planned Bargaining Goals and Payoffs in Ratios B/A



3.4 Monotonicity in Planned Bargaining Goals

Planned bargaining goals depend on the norms and their justifications the subjects have in mind when they are faced with a given bargaining situation. We concentrate the analysis on planned bargaining goals because this kind of aspiration levels is the one that is most independent of the anticipation of the negotiation process. Therefore we can try to explain the goals of a group by norms they apply and the agreement as a solution of the conflict between the different norms of the groups.

In the experiments each group had about ten minutes of time to discuss the payoff table without knowing whether they were A or B. During this period nearly all groups put themselves into the two positions and thought about what they would want to get, if they were Group A or Group B. When they were told which group they are, again they thought about their plans for the game. Sometimes they came up with a revised goal. As the data for the variable "Planned Bargaining Goal of Group A (B)" we choose the point in the payoff table that belongs to the payoff Group A (B) really wants to get in an agreement. Either the data is taken from the discussion before the game starts or from the discussions during the first rounds. If a group does not name a unique point, but describes an interval in which the planned bargaining goal lies, we choose the midpoint of this interval as the data for their planned bargaining goal. If a group only discusses their own payoffs and comes up with a planned goal leading to a set of payoff constellations that are all weakly Pareto optimal, we choose the unique point from this set which is strongly Pareto optimal in payoffs. This problem sometimes occurs in truncated situations, when the subjects in Group B plan to get their maximum payoff, which equals the truncation level. In this case, we choose the kink of the payoff set as their planned bargaining goal. Our reason for this choice is the following: If their plans would be not to give the payoff belonging to the kink to their opponents, they would pronounce this. They would name an interval or a point in the interior of the weakly Pareto optimal set of the truncation line instead of discussing only their own payoff level.

It is interesting to remark that the goals could be determined for both groups in nearly all games. In general the statements on goals are made very early in the discussion, not later than during the first three rounds. There is only one game of Situation 6 where Group B names a goal of 10 for A and 7 for B after the 8th round of 12 rounds. Plans usually are not the subject of the discussions in later rounds. Therefore they can be analysed from the tapes unequivocally and can be assumed to be constant for a game.

For each game in the four classes of situations 1 2 3 4 5, 1 2 3 4 5 $*$ 2.5, 1 6 7 8 9 and 1 6 7 8 9$*$2.5 we observe the planned bargaining goals of each group as a point of the payoff possibility set of a given game. Let e.g. $(G_A^A|G_B^A)$ be the planned bargaining goal of Group A in a given game. Then we define $Y^A = \frac{G_B^A}{G_A^A}$ to be the variable for the rank transform regression. Y^A is the ratio of the payoffs of Group B and Group A that are observed to be the planned bargaining goals of Group A. Analogously we define Y^B for the planned bargaining goals of Group B. We call these variables "Planned Bargaining Goals of Group A (resp. B) in Ratios B/A". These variables are unique representations of the planned bargaining goals of the groups for the same reasons we have given when we discussed the use of payoff ratios in Section 3.3.

The eight rank transform regressions of the planned bargaining goals of each group in ratios B/A on the variable truncation level of the situation can be found in the Appendix. Graphical representations of the estimated ratios are given in the figures at the end of the previous section together with the estimates for the payoff ratios in the agreements.

<u>Situations 1 2 3 4 5, Group A</u>

For the Situations 1 2 3 4 5 the rank transform regression shows an increasing relation between the truncation levels and the planned bargaining goals in ratios B/A for both groups. For truncation levels between 13 and 36, where Group B is the stronger party, the estimated planned bargaining goal of Group A lies below, but close to the "Equal" line. This means that in these situations Group A plans to get about the same payoff as Group B, but for lower truncation levels a little more than Group B. We remark that for truncation levels between 15 and 36, the equal payoff principle is the best distributive principle for Group A from the set of discussed and applied principles. For truncation levels between 6 and 13, the estimated planned bargaining goals of Group A in ratios B/A lie close to the "Prop_T" principle. Four of five estimated data lie below this line, one data lies between the "Prop_IR" line and the "Prop_T" line. For truncation levels below 15, the "Prop_T" principle is the best principle for Group A. This turns out also to be the area where Group A is stronger than Group B.

Situations 1 2 3 4 5 , Group B

The estimated planned bargaining goal of Group B is close to the kink of the situation for truncation levels below 15. There the kink is the payoff pair belonging to the "Nash" principle. From the set of discussed principles, this is the best one for B for low truncation levels. For truncation levels below 8, Group B has estimated goals that are not strongly Pareto optimal in payoffs. This means that, if the kink is lower than (8|8) for Group B, envy plays a role. Group B wants to get her maximal payoff but in addition plans not to give their opponents their best payoff under this restriction, because this would be more than their own payoff. Therefore the estimated plans of Group B are Pareto dominated by the kink of the situation. For truncation levels greater or equal to 15, the estimated planned bargaining goal of Group B in ratios lies above all ratios that are defined by the distributive principles. From the tapes we learn that in these cases the persons in Group B say that they are the stronger group and that they use the proportionality principle over the total payoff set to calculate their goals. In Situation 1 for instance they have to evaluate the ratio 36/15. The groups who discuss this ratio, round the result to values close to 3. Then they search for a prominent point in integer amounts of DM without amounts in Pf in the payoff table that would reflect this ratio (cf. Albers and Albers, 1983). Sometimes they choose a point that is even more favorable to them than the ratio they have in mind. The maximal goal that is formulated in Situation 1 by a Group B leads to a ratio of 4.0. The combination of rounding in favor of Group B and searching for an appropriate prominent point again in favor of Group B explains the difference between the planned bargaining goal of Group B and their best principle "Prop_{T}".

Situations $1\,2\,3\,4\,5 * 2.5$, Group A and Group B

The rank transform regression of the planned bargaining goals of the groups in ratios B/A on the truncation level of the situations is also monotonically increasing for the Situations $1\,2\,3\,4\,5 * 2.5$. In this case the estimated planned bargaining goals of Group A in ratios B/A lead to a curve that intersects the "Equal" line at a truncation level of 21.8. This means that for high truncation levels the estimate implies that Group A plans to get less than equal payoffs. In comparison to the data set 1 2 3 4 5 the plans of Group A are relatively less demanding when the payoffs are multiplied by 2.5. For truncation levels lower than 21.8 we estimate a nearly linear relation between the ratio and the truncation level. For a level of 6, the estimated goal lies close to the kink of the payoff set of Situation $5 * 2.5$.

For the truncation level of 6 the estimated goal for Group B lies also close to the kink of Situation $5 * 2.5$. Here the difference between the goals of the opponent groups is very small. The curve of the estimated planned bargaining goals of Group B for the Situations $1\,2\,3\,4\,5 * 2.5$ is very similar to the curve for the Situations 1 2 3 4 5. The estimated goals are a little bit less demanding in the area of truncation levels lower than 20 and a little bit more demanding for higher truncation levels.

We observe that the gap between the curve of Group A and Group B is smaller in the class of situations with multiplied payoffs. This is mainly due to the observation that the estimated goals of Group A assign a higher payoff to Group B for all truncation levels and that the estimated goals of Group B assign a higher payoff to Group A in situations where A has the "stronger" position. The multiplied payoff tables seem to induce the groups to formulate goals that are less demanding for themselves. The goals are less incompatible for situations where the strength of the position is not very different.

Situations 1 6 7 8 9 , Group A

In contrast to our expectation, the rank transform regression of the planned bargaining goals of Group A on the truncation levels of the situation is slightly decreasing. The values of the payoff ratios B/A of the goal points lie between 0.35 and 0.45. For a truncation level of 15 which means no truncation in Situation 1 with changed names of the groups, the estimated goal of Group A is close to the "Prop_{T}" line. This is the same observation as in the Situations 1 2 3 4 5 with Group B. In addition there is the same rounding–up effect, which leads to an estimated point below the "Prop_{T}" line. The lower the truncation level is, the stronger becomes the position of Group A. The planned goal of A intersects the "Prop_{IR}" line which is the second best principle for A and then approaches the "Nash" curve. For a truncation level of 6 (Situation 9) the estimated goal is close to the kink of the payoff set $(12|6)$, which would be the Nash bargaining solution in payoffs. We think that it is possible to generalize the estimated goals of Group A to points that give Group A a little bit more than twice as much as Group B gets. For very strong positions of A this group seems to have less demanding goals, but goals are nearly constant, nearly independent of the truncation level. This can be interpreted as the generosity not to exploit a very strong position.

Situations 1 6 7 8 9 , Group B

In these situations Group B has the weaker position. The rank transform regression estimates an increasing relation between payoff ratios of the goals and truncation levels. For the level of 15, where the position of B is the best in comparison to the other levels the goal lies close to the "Equal" line. For weaker positions it lies between the "Equal" line and 0.76. This means that for very weak positions, Group B deviates from the equal payoff principle in favor of Group A, their goals are less demanding in these cases.

Situations 1 6 7 8 9 $*$ 2.5 , Group A

Here we find an estimated curve for the planned bargaining goals of Group A that is slightly increasing, but similar to the Situations 1 6 7 8 9 nearly constant. Comparing the games with standard payoffs and the games with multiplied payoffs we find that the enlarged payoff scale leads Group A to goals with higher payoffs for themselves. They are here more demanding than in the standard situations. For a truncation level of 15, again there is the rounding up effect of Situation $1 * 2.5$. For very strong positions of Group A, this group gives up goals that lie close to the "Prop_T" line but uses goals that lie close to the second best principle, the "Prop_IR" principle.

Situations 1 6 7 8 9 $*$ 2.5 , Group B

The rank transform regression estimates a linear, increasing relation between the payoff ratios of the goals of Group B and the truncation level. The only estimated points are a point close to the kink of Situation $9*2.5$ as a goal for B, and a goal close to the equal payoff point of Situation $1*2.5$. Equal payoffs is the best principle for Group B, and it seems to be used to formulate the goal for situations with a relatively weak position. For a very weak position the second best principle is used.

Analogously to the comparison of the data sets for Situations 1 2 3 4 5 and 1 2 3 4 5 $*$ 2.5 we observe that the goals of the opponents are closer to each other for the multiplied payoff scales than for the standard scales.

Goals and Agreements

In the graphical representations of the rank transform regressions for payoff ratios of goals and agreements, the estimate of the agreement payoffs lies between the estimate of the goals of Group A and B. There is only one exception in the data of Situations 1 6 7 8 9. For a truncation level close to 6, the curve "Payoff" intersects the curve "Pl_A". But the difference of the estimates for Situation 9 (truncation level 6) is neglectible small.

Compared with the multiplied situations, in the standard situations 1 2 3 4 5 and 1 6 7 8 9 the "Payoff" curve lies closer to the goal curve or parts of the goal curve belonging to the weaker group. This implies that in the situations with the multiplied payoff scale, the "Payoff" curve lies closer to the goal curve of the respective stronger group, though in these cases the weaker group pronounces already less demanding goals than in the standard situations. For the interdependence of goals and agreements, this means that in the multiplied situations the weaker group tends to make larger concessions, first when they formulate their goals and second concerning the agreement. With respect to the stronger groups we learn that they have more demanding goals in the multiplied situations. In addition in these cases they are relatively more successful in achieving an agreement close to their goal.

The "Pl" curves nearly everywhere have a greater slope for the stronger group than for the weaker group. We remark that for Situations 1 2 3 4 5 and 1 2 3 4 5 $*$ 2.5 and low truncation levels Group A has the stronger position. The only exception of this observation we find in the data set 1 2 3 4 5 for truncation levels around 10. This implies that an improvement of the payoff table for the stronger group leads to an increase of their goal ratio that is relatively higher than what their weaker opponents give up in their goal ratios.

Let us now consider the amounts each group wants to get for herself in her goal. We observe from the estimates that Group B wants to have more, the higher the truncation level is. In all situations except for the data set 1 6 7 8 9 we find that Group A wants to get less if the truncation level is raised. For situation 1 6 7 8 9 we can say that their goal is constant. This means that the ratio of what Group B and Group A want to get increases monotonically with increasing truncation levels. This is accompanied by an increase in the payoff of Group B. We use this general observation to formulate a monotonicity axiom for normative bargaining solutions on economic situations with goals in Chapter 6.

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Chapter 4

Norms in Economic and Ethical Environments

4.1 Introduction

In this chapter, we describe some observations in single experiments. First, we deal with a phenomenon of agreements allocating non strongly Pareto optimal payoff pairs. We investigate the arguments the subjects use in the discussions with their group partner when they decide upon their offers. From the bargaining behavior in some of the games we can conclude that envy plays a role. The effect of envy is due to a certain power of the equal payoff principle. In some experiments this principle was not dominated by a collective efficiency principle defined by the strong Pareto efficiency in payoffs.

Second, we compare experiments with an environment where the positions of the subjects in the games are assigned according to their result in a quiz to games with randomly assigned positions. Since it was hard to recruit eight participants for an experiment at a certain date, we could only play four games with a quiz environment. Therefore, a statistical evaluation of differences in agreements, goals and other variables is not possible. We think, however, that it is interesting enough to find out and to describe the special features of these games.

Third, we present two variants of Situation $1 * 2.5$, where the payoffs of one group are connected to payments to indigents. We describe some arguments that occured in the discussions of the groups. In addition, we compare the values of the games with Situation $1 * 2.5$ in the standard environment.

4.2 Strong Pareto Optimality in Payoffs or Envy? *)

In this section we will describe a phenomenon that is implied by the conflict between the different principles the strong and the weak groups apply in the games. In two of four repetitions of Situation 5 (cf. Figure 4.1) this conflict is solved by a non–strongly Pareto optimal agreement. Here Pareto optimality is defined in terms of payoffs for the players.

We start with the description of an experiment of May 17, 1990. The payoff constellation is defined by Situation 5. The positions in the game were assigned randomly to the subjects. Group A had to make the first offer. The agreement after 22 rounds of bargaining which lasted 45 minutes was 7.50 DM for each player in Group A and 6 DM for each player in Group B.

*) The main part of this section is literally taken from Klemisch–Ahlert,M.: ***Distributive Results in Bargaining Experiments.*** In: Gaertner,W. and Klemisch–Ahlert,M. (1992)

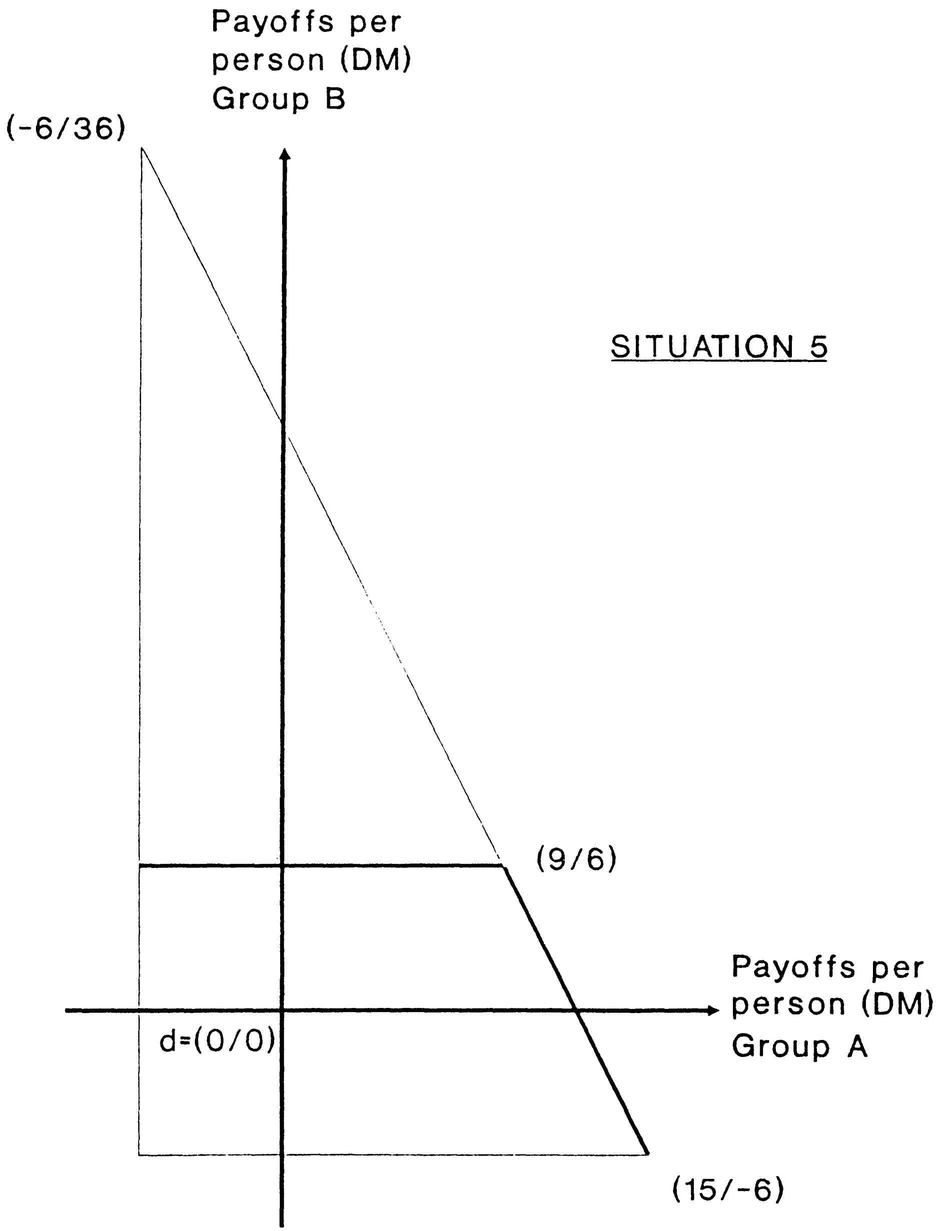

Payoffs per
person (DM)
Group B
(-6/36)
SITUATION 5
(9/6)
Payoffs per
person (DM)
Group A
d=(0/0)
(15/-6)

Figure 4.1

The planned bargaining goal of Group A was (10|4) which corresponds to the proportionality principle over the whole payoff set. The agreement that was seen as attainable and the lowest acceptable agreement by Group A was (9|6). Group A expected that their opponents were planning to receive (9|6) and that this outcome would also be their lowest agreement.

Group B's planned bargaining goal was (6|6), which is the equal payoff solution. The agreement they saw as attainable was (7|6) which was also their lowest acceptable agreement. Group B expected that their opponents were planning to get (9|6) and that A's lowest agreement was (7|6).

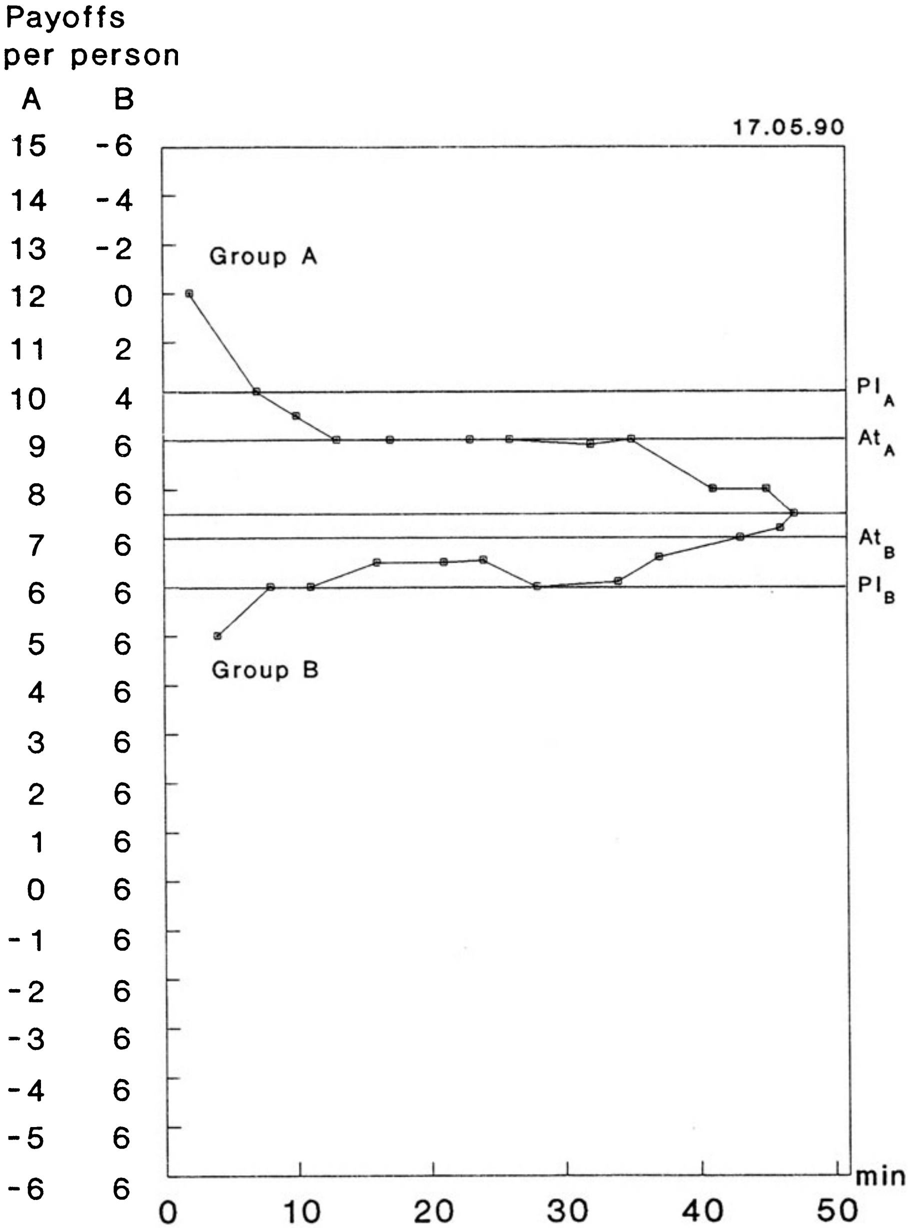
First there was a conflict between the principles the groups used in order to form their planned goals, and in addition, there was a conflict between the expectation of Group A and the aspiration levels of Group B. Because the groups could not discuss these conflicts with each other, the consequence was a long bargaining process (cf. Figure 4.2).

Group A did not want to accept non–strongly Pareto optimal outcomes, because if Group A were in the position of Group B it would grant each of its opponents 9 DM. Group B also discussed the non–strongly Pareto optimal interval between (6|6) and (9|6), saying "Actually we should be indifferent between these outcomes but we don't give any money away. (9|6) means that each of the others will receive 3 DM more than we do." In addition, they argued that indeed they could at most get 6 DM, but nearly everywhere in the payoff space.

Interestingly, they saw it as "giving money away" if the other party received more than equal payoffs. It seems to us that they wanted to express a reduction in preferences.

The positions of the groups were very unyielding. Only because of the great length of the bargaining procedure, Group A decided to make an offer lower than (9|6).

SITUATION 5



Pl_i = Planned Bargaining Goal of Group i
At_i = Agreement Seen as Attainable for Group i

Figure 4.2

In the experiment of November 14, 1990 we chose the same economic and ethical environment as in the former experiment. The agreement was (8|6) after 14 rounds in 44 minutes.

The planned bargaining goal of Group A was (10|4), the agreement that was seen as attainable was (9|6), the lowest acceptable agreement was (7|6) or (8|6). Group A expected that its opponents were planning to receive (6|6) and that their lowest agreement would be (9|6).

Group B's planned bargaining goal was (6|6). Any outcome between (6|6) and (9|6) was seen as attainable. The lowest acceptable agreement was (9|6). Group B expected that Group A was planning to get (10|4). Their expectations about the lowest acceptable agreement of Group A are not clear. Between the two players in Group B, there was a discussion how to deal with the outcomes between (6|6) and (9|6). Player 3 suggested to aspire to (6|6) and not to give the opponents more money without fighting. Player 4 said that he and his partner should be indifferent between the points in the interval from (6|6) to (9|6). He asked his partner for his reasons and he asked whether he simply wanted to be beastly. Player 3 argued that they were disadvantaged by the random assignment of the groups, and that he therefore did not perceive and did not want that the opponents should get 3 DM more than Group B.

Group A argued that (9|6) should be attainable because fundamentally it would not hurt Group B.

The bargaining process of this experiment is represented in Figure 4.3. The difference from the first experiment is that the conflict between the principles leading to the aspiration levels of the groups was not that strong in this case. Both groups discussed every bargaining step at great length, and they involved former steps of their opponents in their considerations.

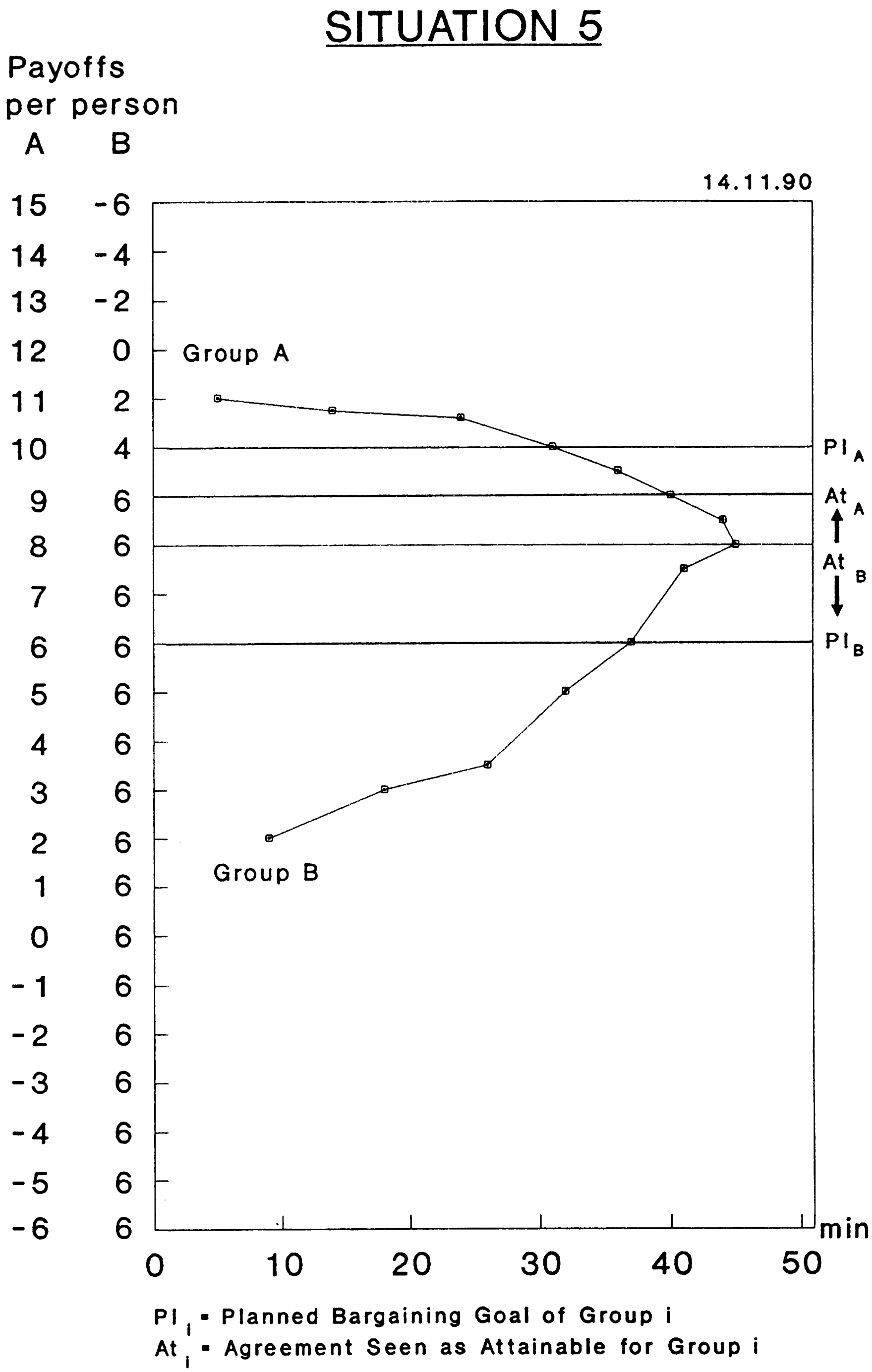

SITUATION 5
Payoffs
per person
A B
14.11.90
15 -6
14 -4
13 -2
12 0
11 2
10 4
9 6
8 6
7 6
6 6
5 6
4 6
3 6
2 6
1 6
0 6
-1 6
-2 6
-3 6
-4 6
-5 6
-6 6
Group A
Group B
Pl A
At A
At B
Pl B
0
10
20
30
40
50
min
Pl i = Planned Bargaining Goal of Group i
At i = Agreement Seen as Attainable for Group i

Figure 4.3

In the other repetitions of Situation 5 there was no conflict between the lowest acceptable agreements of the groups. Therefore the bargaining procedures were very short, at most 6 rounds in 10 minutes. The agreements were (9|6). The phenomenon of non–strongly Pareto optimal agreements also occurred in other situations. When the equal payoff outcome was dominated by a strongly Pareto optimal payoff constellation, often the weak group discussed how to deal with the principle of equal payoffs.

We can conclude from this that envy plays a role in the distribution of money in our bargaining experiments. Envy may lead to non–strongly Pareto optimal payoff agreements. Does this mean that the strong Pareto principle as a collective rationality requirement does not hold? We cannot conclude this from our observations. The Pareto principle is defined for preferences. The strong Pareto principle is compatible with our results, if we define the preferences of a person not only as dependent on her own endowment in money, but also on the set of allocations of money to all persons involved in the distributive problem. Then, for instance, in our Situation 5 envy implies that Group B strictly prefers (6|6) to (9|6). In this case, even (6|6) is a strongly Pareto optimal outcome in preferences.

In Chapter 6 we will model a normative bargaining theory on economic situations for two persons. There economic situations are defined by a set of feasible allocations of commodities, an initial allocation and utility functions of the persons. The observations we describe in this chapter lead to a definition of an economic situation, where the utility level of each person does not only depend on her own commodity bundle but may also depend on the commodity bundle of her opponent, i.e. on the whole allocation. Therefore it is possible in our normative model that an agreement allocation belonging to a strongly Pareto optimal outcome in utilty space is not strongly Pareto optimal in commodities.

4.3 Environments with Contributions

On each afternoon of May 15 and May 16, 1990 there was a set of eight subjects each participating in our experiments. The subjects were recruited from the macroeconomic lectures for students in their fourth semesters. They studied business administration or economics at the University of Osnabrück. The 16 students had never played a game in our experiments before. After the instructions, they were randomly assigned to groups of two players and played one of the nine standard situations. Then they met in a large room and had to pass a multiple choice test in microeconomic theory. They were told that each person could gain 20 points in the quiz and that, dependent on their common total number of points, the scale of payoffs for the next games would be determined. The following relationship between points (p) and factor (f) for the payoffs was given to them at the blackboard.

p	0-20	21-40	41-60	61-80	81-100	101-120	121-140	141-160
f	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0

In addition they knew that the positions in the next games would be assigned dependent on their individual numbers of points: high numbers would lead to strong positions, low numbers to weak positions. Then they had to fill in the questionaire of the quiz. The number of points was evaluated afterwards. The factor was ascertained and the subjects were assigned to groups for the following games. The number of points of each subject and the constellations of the groups were written on the blackboard.

The Experiment of May 15, 1990

The multiple choice test of May 15 had four questions with five possible answers each. An answer consisted in filling in a cross, if the subjects thought that the statement was right, or not, if they considered the statement to be wrong. There was no restriction on the number of possible right answers per question. The result of the quiz was the following:

Person	I	II	III	IV	V	VI	VII	VIII	
number of points	13	14	13	12	12	11	11	10	$\sum 97 \widehat{=}$ factor 2.5

When the quiz was announced to be a test in microeconomics, one person said that she had not heard these lectures during her studies. Usually students go to these lectures in their third semester. Though the student was told that she could pass the quiz with her knowledge from the lectures of the first two semesters of studies, she announced not to make any cross in the quiz. Since six crosses had to be made, this student achieved the maximal number of points (14)...!

The games that were played were
Situation $5 * 2.5$ Q with
Group $A = \{$Person III, Person IV$\}$, Group $B = \{$Person V, Person VI$\}$
and Situation $9 * 2.5$ Q with
Group $A = \{$Person I, Person II$\}$, Group $B = \{$Person VII, Person VIII$\}$.

The sequences of offers can be found in the graphical representations on page 58 and page 59. The letter "Q" in the headlines denotes the environments with a quiz. In the following table we compare the game $5 * 2.5$ Q to two other games with the payoff table $5 * 2.5$. "Pl" means planned bargaining goal of a group and "At" stands for an agreement seen to be attainable by a group.

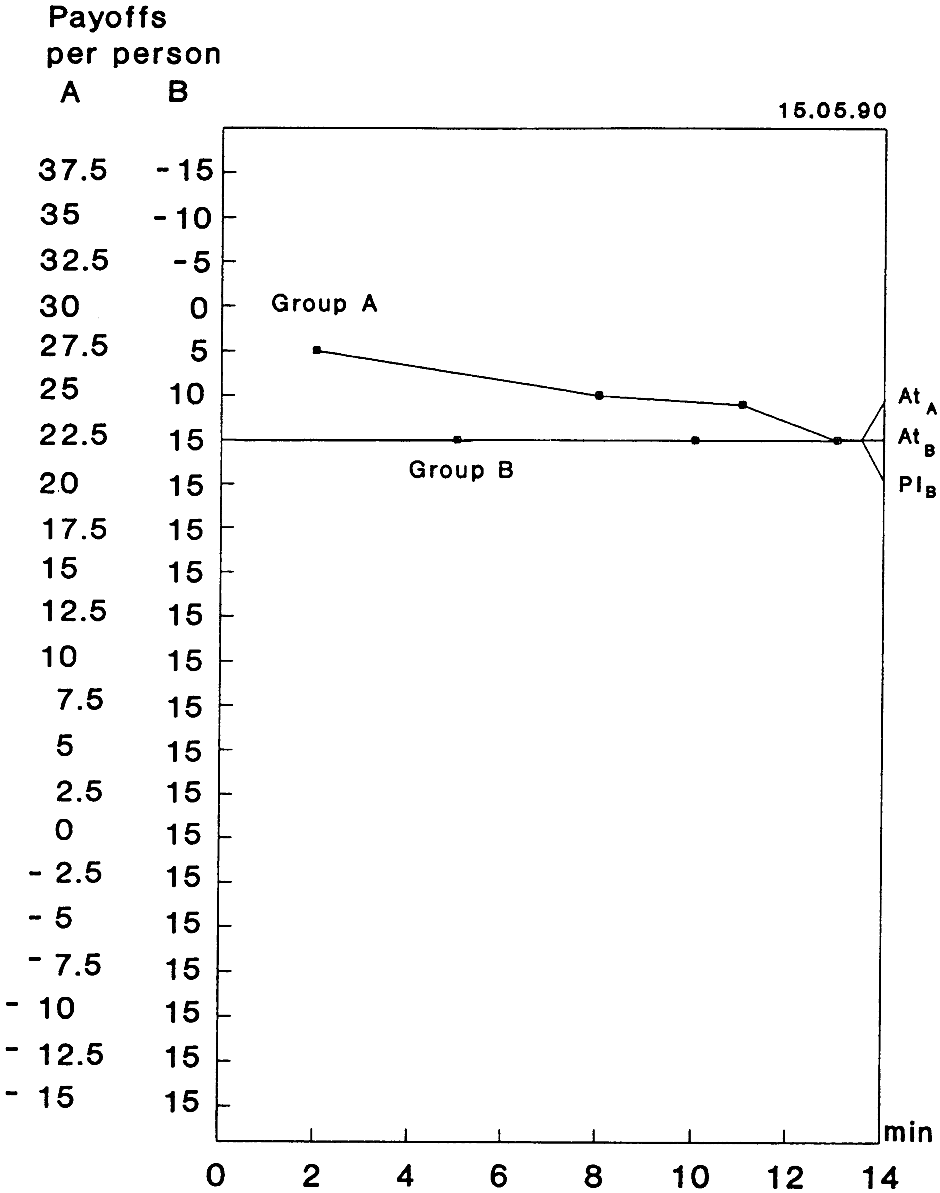
Situation	$5 * 2.5$ Q	$5 * 2.5$	$5 * 2.5$
Date	15.5.90	13.6.90	31.10.90
Result	(22.50|15)	(22.5|15)	(22.50|15)
# Rounds	6	3	2
Period of Time	13 min	12 min	4 min
Pl_A	no data	(24|12)–(25|10)	(22.50|15)
Pl_B	(22.50|15)	(22.50|15)	(22.50|15)
At_A	(22.50|15)	(22.50|15)	(22.50|15)
At_B	(22.50|15)	(22.50|15)	(22.50|15)

There is no special difference between the observed variables in the three games. In the discussions on the tapes of Situation $5 * 2.5$ Q, success or contribution of a group were not mentioned. The contributions of the groups were not very different. The group with the greater number of points had the better position and received a higher payoff in the agreement. From the data, however, we cannot conclude that "contribution" had an impact on the negotiation process.

Now we compare the data of the game $9 * 2.5$ Q and the data of the two experiments with Situation $9 * 2.5$.

Situation	$9 * 2.5$ Q	$9 * 2.5$	$9 * 2.5$
Date	15.5.90	17.5.90	10.12.91
Result	(35|12.50)	(30|15)	(45|7.50)
# Rounds	11	2	12
Period of Time	30 min	4 min	23 min
Pl_A	(50|5)	(35|12.50)–(40|10)	(50|5)
Pl_B	(30|15)	15 for *B*	(30|15)
At_A	no data	no data	(45|7.50)
At_B	(30|15)	(30|15)	(45|7.50)

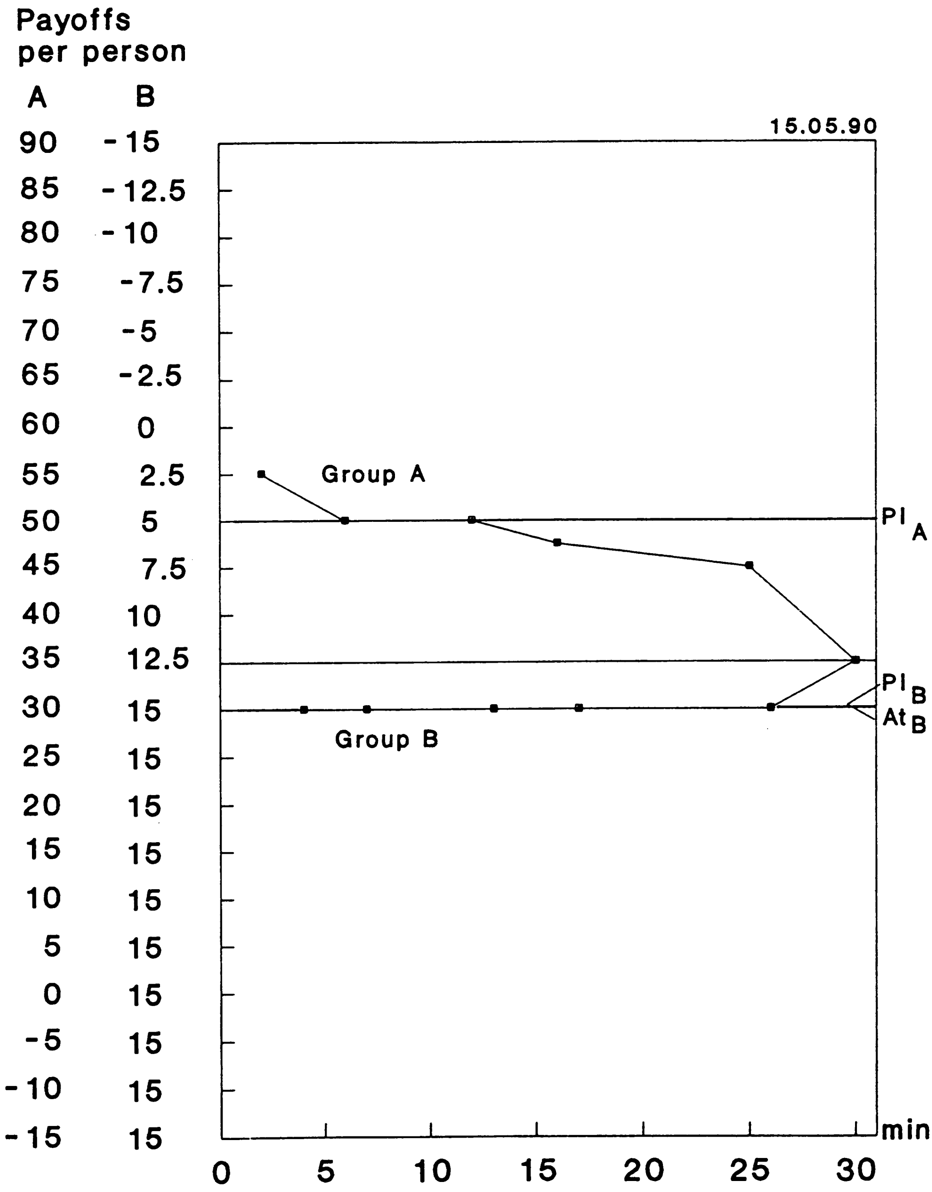
SITUATION 5 * 2.5 Q



Pl_i = Planned Bargaining Goal of Group i

At_i = Agreement Seen as Attainable for Group i

SITUATION 9 * 2.5 Q



Similar to Situation $5 * 2.5$ Q, in this case the contribution of a group to the quiz result was not mentioned. The values of variables of the experiment with quiz also occur in situations without quiz. Therefore, an influence of the quiz results on these variables cannot be assumed.

One reason for the fact that the quiz results were not mentioned in the discussions to justify claims or plans may be the following. Since one person had been successful in the test without any effort, simply by making no cross, a large number of points obviously was not necessarily the result of hard work. The intended effect of making a contribution by being good in microeconomics was disturbed. For the next experimental environment, we therefore chose another type of multiple choice test where it was not that easy to get a great number of points without knowing a considerable number of correct answers.

The Experiment of May 16, 1990

The multiple choice test of May 16 had four questions different from the questions of may 15, with five possible answers each. In the instructions of the test the participants were told that at least one and at most three answers per question were correct. Questions that would be answered by making no, four or five crosses would lead to zero points for this question. The quiz had the following result.

Person	I	II	III	IV	V	VI	VII	VIII	
number of points	14	14	13	12	12	12	10	9	$\sum 96 \hat{=}$ factor 2.5

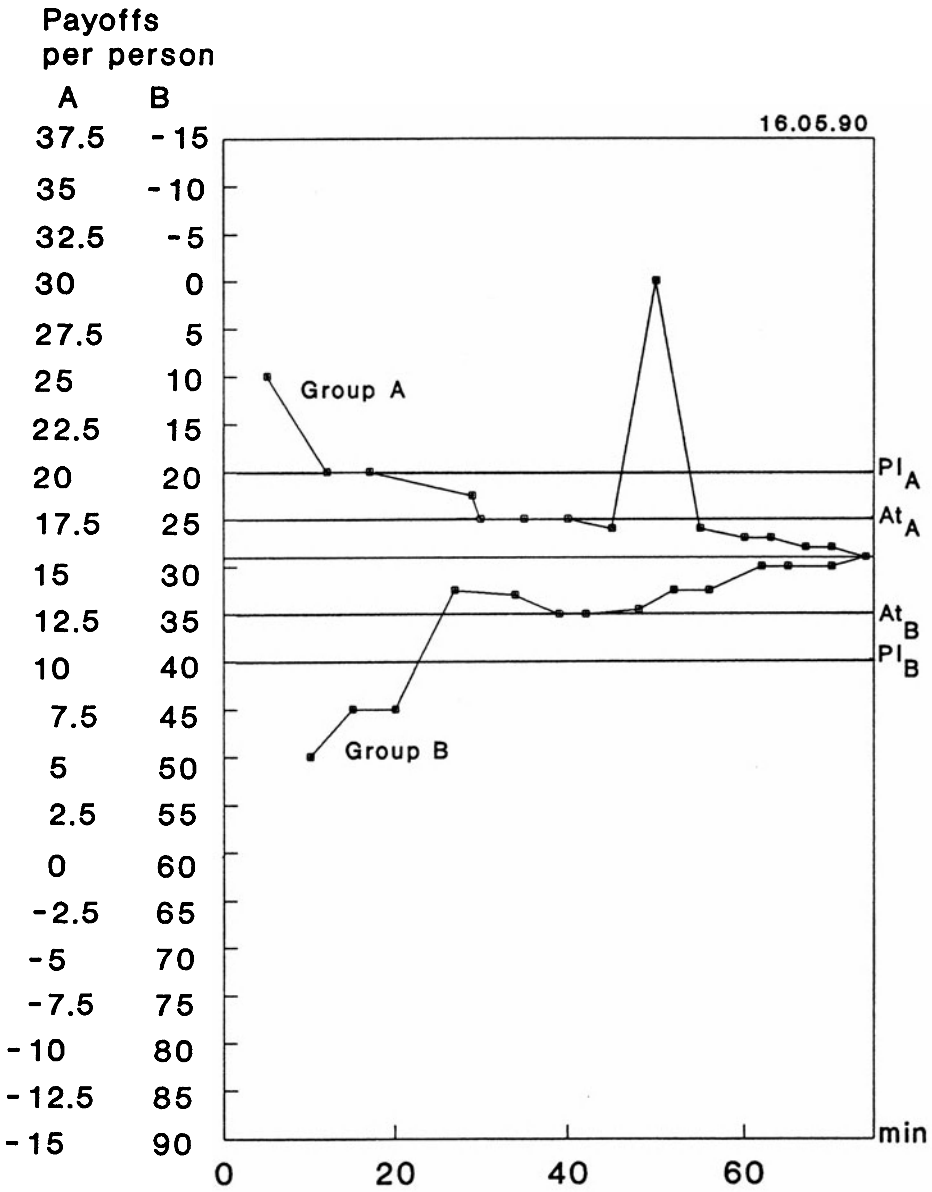
Situation $1 * 2.5$ Q was then played with
Group A = {Person V, Person VI} and Group B = {Person I, Person II}
and Situation $6 * 2.5$ Q was played with
Group A = {Person III, Person IV}, Group B = {Person VII, Person VIII}.

The sequences of offers are graphically represented. The figures also show the planned bargaining goals of the groups and the payoff pairs they assumed to be attainable. We compare the data of the situations with a quiz to the data of the multiplied situations without a quiz.

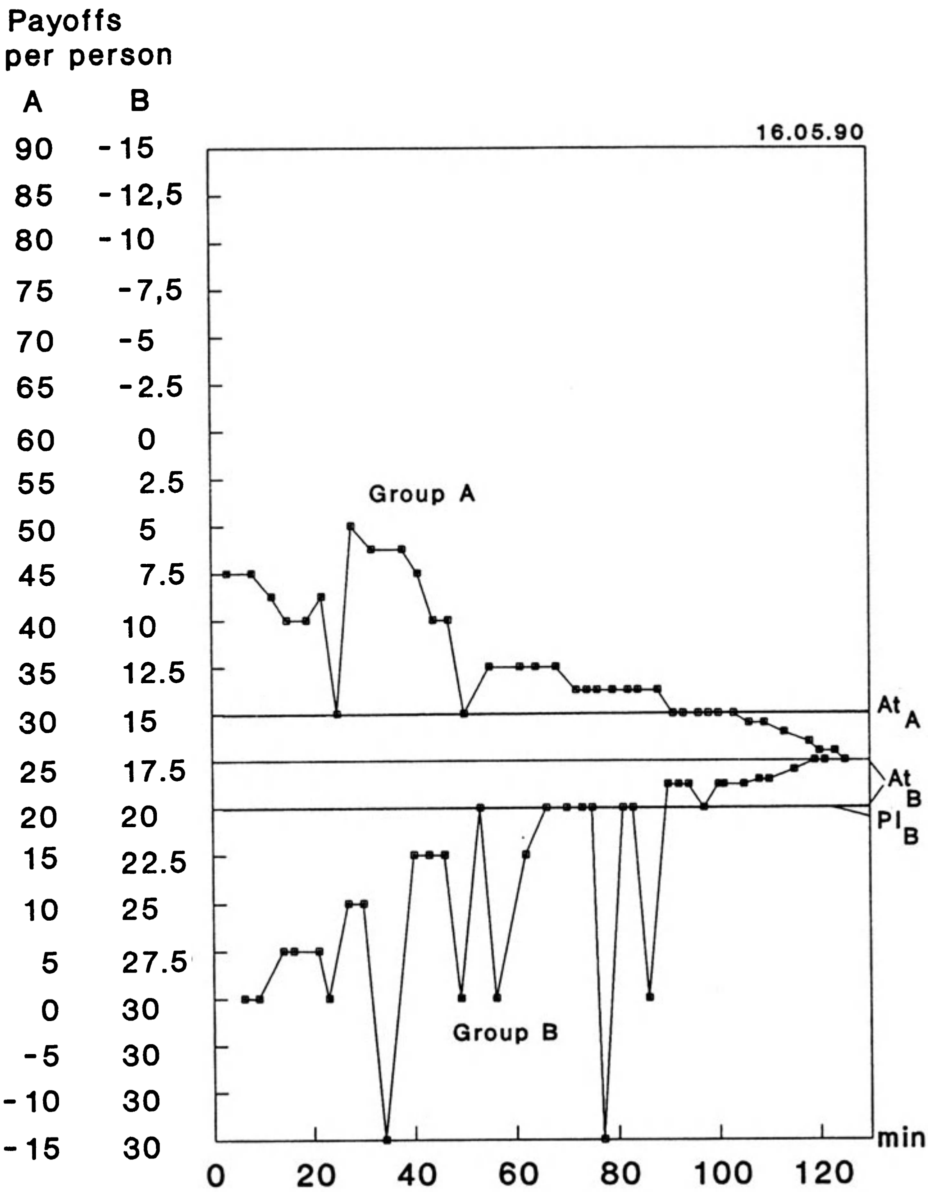
Situation	$1 * 2.5$ Q	$1 * 2.5$	$1 * 2.5$
Date	16.5.90	17.5.90	7.11.90
Result	(15.50|29)	(17.50|25)	(17|26)
# Rounds	28	33	64
Period of Time	69 min	84 min	124 min
Pl_A	(20|20)	(20|20)	(20|20)
Pl_B	(10|40)	(10|40)	(15|30)
At_A	(17.50|25)	(20|20)	no data
At_B	(12.50|35)	(15|30)	(15|30)

The agreement of Situation $1 * 2.5$ Q is better for Group B than the agreements of the Situations $1 * 2.5$. The values of what seemed to be attainable for the groups in the situation with quiz are different from the other situations. The persons in Group A thought that they could get 17.50 DM which is less than equal payoffs. The persons in Group B thought, they could get 35 DM which is more than the values in the data of the Situations $1 * 2.5$.

SITUATION 1 * 2.5 Q



SITUATION 6 * 2.5 Q



In the discussions of the groups the result of the quiz played a role. Group B mentioned that they deserved the better position. Therefore, they planned to get clearly more than Group A. Group A, too, argued with the result of the quiz. They planned to get equal payoffs, but they saw that the others would feel justified to fight for a higher payoff for them. Group A thought that their opponents wanted to get 40 DM or at least 30 DM. That is why they came up with a value inbetween their own goal and the expected goal of their opponents for what should be attainable in an agreement. Another indicator for the influence of the quiz result could be that the negotiation process of Situation $1 * 2.5$ Q was shorter than the others and consisted of less rounds. The difference, however, is not very drastic. All processes have been long in comparison to the average of 15 rounds and 31 minutes of all games.

Situation Date	$6 * 2.5$ Q 16.5.90	$6 * 2.5$ 13.6.90	$6 * 2.5$ 15.1.92
Result	(25|17.50)	(24|18)	(40|10)
Rounds	74	3	6
Time	125 min	3 min	13 min
Pl_A	no data	(25|17.50)	(50|5)–(45|7.50)
Pl_B	(20|20)	(20|20)	(35|12.50)–(40|10)
At_A	(30|15)	(20|20)–(25|17.50)	no data
At_B	(20|20)–(25|17.50)	(20|20)–(25|17.50)	(45|7.50)

In the data of Situation $6 * 2.5$ Q the equal payoff principle occured in the planned bargaining goal of group B. This also determined the planned bargaining goal of B in the game of 13.6.90. For Group A in Situation $6 * 2.5$ Q the ratio $2 : 1$ defined what they thought would be attainable. This leads to a greater payoff for A than in the game of 13.6.90. The agreement in the game of 16.5.90 is a little better for Group A than that of 13.6.90. From these data, however, an influence of the quiz on the negotiation process cannot be deduced. In contrast, the agreement in the game of 15.1.92 is extremely better for Group A than in the other games. In this game Groups A and B both had the proportionality principle over the total set in mind when they formulated their goals and payoffs of agreements seen as attainable. In this case there was no conflict between principles of the groups. The negotiation process was short. In the game of 13.6.90 the process was very short though there was a conflict between the principles. The readiness to make concessions was rather high for both groups.

The main difference between the game with a quiz to the other games of Situation 6*2.5 was that both groups did not want to make concessions that would lead to lower payoffs than what they planned to get or thought to be attainable. In the discussion of both groups their results in the quiz played a role in the formulation of their view of the situation. The way how they got their positions may be a reason for their tough bargaining behavior, but this cannot be decided uniquely from their discussions. They did not explicitely pronounce the reason for their behavior.

From these experiments we learn that the way to operationalize the environment of a "contribution to a common production" by a quiz is rather problematic. When the result of a quiz is not absorbed by the persons as a piece of work, their success may have no influence on their bargaining behavior. In cases where good quiz outcomes are seen as a type of product, there seems to be an influence on the bargaining behavior concerning goals, agreements that seem to be attainable and toughness. In addition it is remarkable that in these games the quiz results were mentioned in the discussions of the groups. They had an influence on their reasoning. From our small data set, however, a statistical evaluation of these phenomena is not possible. In a larger data set, we could not expect to observe a contribution principle in ratios of contributions like in reward allocation experiments (cf. Güth, 1989). We would, however, expect some monotonic increase of agreements, goals and agreements seen as attainable in favor of the better positions in comparison to the situation with random assignments of positions. With respect to distributive allocations in questionnaires this monotonicity has been observed by Yaari and Bar–Hillel (1984) and Schokkaert and Overlaet (1989).

4.4 Environments with Payments to Indigents

We have run two experiments where the subjects were confronted with a different type of environment. It was the time before Christmas 1991 where at each of two afternoons four subjects first played two games with standard situations with randomly assigned positions. After these games they gathered in a room. They were given a list of 24 names of institutions helping other people or animals. The four subjects got the instruction to choose one of these institutions or any other one they liked to propose and to which they would like to transmit a gift of a size up to about 200 DM (13.12.91) or 100 DM (17.12.91). They had as much time as they wanted to discuss the object and the work of the organizations and to find an agreement. The discussions of the groups lasted about 15 minutes. On 13.12.92 the subjects chose "TWER–Hilfe", an organization of the German–Soviet–Society. On 17.12.92 the group chose "Weisser Ring", an organization helping victims of crimes. Afterwards the positions were assigned randomly to the subjects.

The Experiment of Dec. 13, 1991

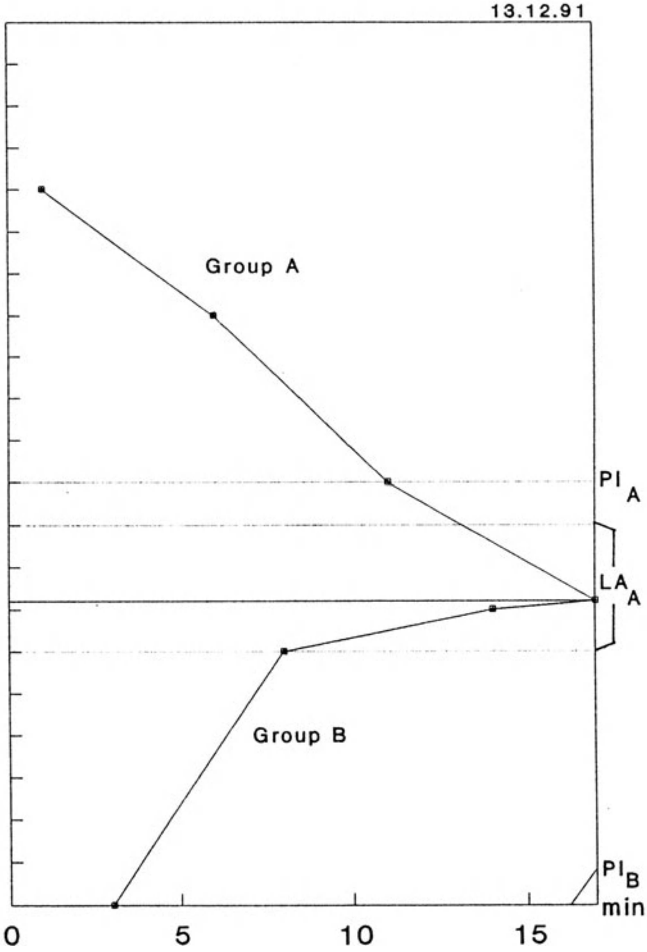
The payoffs of Group B were combined to the possible gifts to "TWER–Hilfe". The sum of the payoffs of the persons in Group B was identical to the gift. Only for negative payoffs of Group B the gift was zero. The payoffs of the persons in Group A were not connected to gifts. This group was negotiating only about its own gain. The payoff table had three columns: The payoff per person of Group A, the gift and the payoff per person of Group B. The payoff pairs of the groups were identical to Situation $1 * 2.5$. The difference between these situations is that the stronger group negotiated about its payoffs and the gift. We called this environment Situation $1 * 2.5$ B.

Group B tried to reach the agreement $(-15|180|90)$ which would have been good for the "TWER–Hilfe" and the maximal sum of payoffs for all players. They had the idea to share the payoff they would get with the members of Group A. They realized that their opponents would have to pay 15 DM each in this agreement, but hoped that they would understand that this was the best agreement for all participants as well as for the "TWER–Hilfe". When Group A did neither accept $(-15|180|90)$ nor the next offer of $(0|120|60)$, where Group A would not have to pay anything, the subjects in Group B expressed their regret that their opponents seemed not to understand their idea.

Group A, howewer, had understood the idea of Group B, but did not trust their opponents. They thought that Group B wanted to achieve a large amount of money as a gift, perhaps the maximal amount. They expected this, because it had been one of the students in Group B who had suggested "TWER–Hilfe" in the discussion. Group A wanted to have a positive payoff in an agreement. Their goal was $(10|80|40)$ and the lowest acceptable agreement (LA_A) between $(7.50|90|45)$ and $(0|120|60)$. Finally the agreement was $(3|108|54)$ after 7 rounds and 17 minutes. The sequence of offers and some aspiration levels are represented in the figure on the following page.

SITUATION 1*2.5 B

Payoff per person A	Gift	Payoff per person B
37.5	0	-15
35	0	-10
32.5	0	-5
30	0	0
27.5	10	5
25	20	10
22.5	30	15
20	40	20
17.5	50	25
15	60	30
12.5	70	35
10	80	40
7.5	90	45
5	100	50
2.5	110	55
0	120	60
-2.5	130	65
-5	140	70
-7.5	150	75
-10	160	80
-12.5	170	85
-15	180	90



LA_A = Lowest Acceptable Agreement of Group A

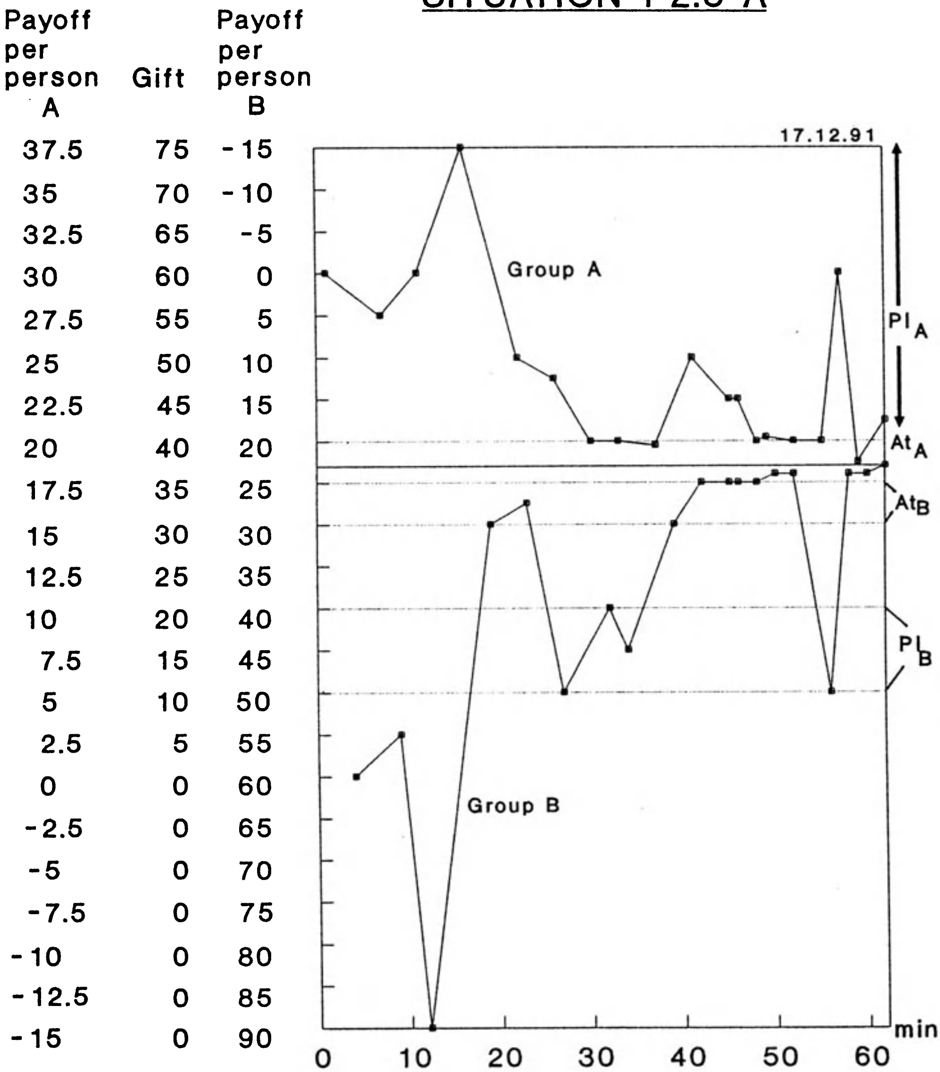
The Experiment of Dec. 17, 1991

In this game the payoffs of Group A were connected to the gifts to "Weisser Ring". Again, for negative payoffs of the persons in Group A the gift was zero. Otherwise the sum of the payoffs for the players of Group A was identical to the gift. We called this situation $1 * 2.5\ A$. In contrast to Situation $1 * 2.5\ B$, this time the weaker group negotiated about its payoff and the gift. The payoff constellations for the groups were the same as in Situation $1 * 2.5$.

Group A planned to get more than 20 DM for each person. They argued that if Group B wanted to have a clear conscience, they should agree to a gift of at least 40 DM. (20|40|20) was the point Group A saw as attainable. This allocation is defined by the equal payoff principle for all players.

Group B discussed that their opponents had the advantage of having the gift on their side. When the players in Group B thought about which proposal they should make, they always argued that, the higher the amount they claimed for themselves, the lower the gift would be. A gift of less than 10 DM was not acceptable to them. Therefore, they restricted the set of allocations coming into question for an agreement by this lower bound. Within this range they planned to get a high payoff. Their planned bargaining goal was between (10|20|40) and (5|10|50). Attainable seemed to them an allocation in the intervall (17.50|35|25) through (15|30|30). They thought that they already made a concession that was large enough when they offered (17.50|35|25). The gap between the acceptable agreements for the two groups led to a long negotiation process. After 62 minutes and 38 rounds the agreement was a point in the middle of the gap between their acceptable agreements. The sequence of offers and the aspiration levels are to be found on the following page.

SITUATION 1*2.5 A



In the following table we compare the data of the experiments with gifts to games with Situation $1 * 2.5$. This means that the payoff tables for the groups are identical in all four games, but the ethical environment is changed. We only denote the variables for payoffs of the persons in each group and not the size of the gift.

Situation	$1 * 2.5\ B$	$1 * 2.5\ A$	$1 * 2.5$	$1 * 2.5$
Date	13.12.91	17.12.91	17.5.90	7.11.90
Result	(3|54)	(18.50|23)	(17.50|25)	(17|26)
# Rounds	7	38	33	64
Period of Time	17 min	62 min	84 min	124 min
Pl_A	(10|40)	> 20 for A	(20|20)	(20|20)
Pl_B	(−15|90)	(10|40)–(5|50)	(10|40)	(15|30)
At_A	no data	(20|20)	(20|20)	no data
At_B	no data	(17.50|25)–(15|30)	(15|30)	(15|30)

The values of the variables of Situation $1 * 2.5\ A$ are very similar to those of the Situations $1 * 2.5$. In Situation $1 * 2.5\ A$, both groups had planned bargaining goals that are a little more demanding than the goals in the situations with the standard environment. The reason for Group A to plan more than the equal payoff principle would grant them is that they were agents for "Weisser Ring". After Group B had defined the miminal gift to be 10 DM they felt justified to want a payoff close to the best result for them respecting this condition. In the Situations $1 * 2.5$ the Groups A used the equal payoff principle and the Groups B used the proportionality principle $1 : 2$ for the individually rational payoff set or $1 : 4$, a rounded proportionality principle over the total set in favor of group B.

Situation $1 * 2.5\ B$ is very different from the other three data sets. Here the aim of Group B to exploit the payoff table for all persons together dominated the negotiation process. In addition, Group A was not very demanding in their plans. One has the impression that they were discouraged to fight for greater payoffs, when they were faced with the high amounts the other group could get and the high amounts of the gift.

The only thing they wanted to make sure was to get a positive payoff.

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Part II

Normative Bargaining Theories on Economic and Ethical Environments

Chapter 5

Roemer's Bargaining Theory on Economic Environments

In a sequence of articles Roemer criticizes the traditional welfaristic approach and offers an axiomatic bargaining theory on economic environments instead. Concerning the application of bargaining theory to problems of distributive justice he argues: "Bargaining theory admits information only with respect to utilities of the agents once the threat point has been determined, while distributive justice is concerned with issues of rights, needs, and preferences as well" (1986a). However, bargaining models have the possibility to take these features of bargaining situations into account, if the informational background of the model is enlarged. Roemer (1985, 1986a, 1986b, 1988) proposes to consider bargaining solutions that are described as distributive mechanisms on a space of economic environments. We give a short overview on Roemer's results and we concentrate our description on the comparison between classical bargaining theory and bargaining theory on economic environments. Notations, definitions and theorems of this chapter are modified versions of those Roemer (1988) proposes.

First, Roemer shows that classical bargaining solutions can be axiomatically characterized as mechanisms in an enlarged model.

Definition 5.1

Let $\mathcal{U}^{(m)}$ be the set of all monotone, concave, continous functions u on $\mathbb{R}_0^{+m}$, such that $u(0) = 0$.
An *economic environment* is a vector $\mathcal{E} = < m; \bar{x}; u, v >$ with $m \in \mathbb{N}$, $\bar{x} \in \mathbb{R}_0^{+m}$, $u, v \in \mathcal{U}^{(m)}$.

In this definition $\bar{x}$ means a fixed endowment vector of m commodities that is to be distributed between two persons. They have utility functions u and v.

$\sum^{(m)}$ is the class of all economic environments of dimension $m \in \mathbb{N}$ and $\sum = \bigcup\limits_m \sum^{(m)}$.

The set of feasible utility pairs for a given situation $\mathcal{E} \in \sum$ is
$\mathcal{A}(\mathcal{E}) = \{(\bar{u}, \bar{v}) \in \mathbb{R}_0^{+2} \mid \exists \bar{x}^1, \bar{x}^2 \in \mathbb{R}_0^{+m},\ \bar{x}^1 + \bar{x}^2 \leq \bar{x},\ u(\bar{x}^1) = \bar{u}, v(\bar{x}^2) = \bar{v}\}$.
Here, also allocations are feasible such that not the whole vector $\bar{x}$ is distributed. Under these assumptions, $\mathcal{A}(\mathcal{E})$ is closed, comprehensive and convex for all $\mathcal{E} \in \sum$. If in addition strict comprehensiveness is required, we reduce the sets $\sum^{(m)}$ to $\Gamma^{(m)}$ and the domain $\sum$ to $\Gamma = \bigcup\limits_m \Gamma^{(m)}$.

Definition 5.2

An *allocation mechanism* F on $\sum$ (or Γ) is a correspondence that assigns to every economic environment $\mathcal{E} \in \sum$ (or Γ) a set of feasible allocations. It is assumed that F induces a function μ_F in utility space with
$\mu_F(\mathcal{E}) = (u(F^1(\mathcal{E}), v(F^2(\mathcal{E}))$. F is called *essentially a function.*
$F^1(\mathcal{E})$, $F^2(\mathcal{E})$ are the sets of bundles that F assigns to person 1 or 2 resp. .
The utility values $u(F^1(\mathcal{E}))$ and $u(F^2(\mathcal{E}))$ are uniquely defined. It is also required that F chooses all the allocations that are mapped onto the same given point in utility space. Then F is called a *full correspondence.*

Roemer considers allocation mechanisms F that are a full correspondence and essentially a function and are defined on the class of economic environments $\sum$ or on Γ.

Axiom 5.1: Domain $\sum$
The allocation mechanisms are defined on $\sum$.

Axiom 5.2: Domain Γ
The allocation mechanisms are defined on Γ.

The property of bargaining solutions in the Nash model to depend only on utility information is formulated in a welfarism axiom. Let F be a mechanism on $\sum$.

Axiom 5.3: Welfarism
$\forall \mathcal{E}, \mathcal{E}' \in \sum$ with $\mathcal{A}(\mathcal{E}) = \mathcal{A}(\mathcal{E}')$,
$\mu_F(\mathcal{E}) = \mu_F(\mathcal{E}')$.

If the utility possibility sets of two economic situations in $\sum$ are identical, then the utility values of the allocations the mechanism F assigns to the situations are also identical. Since the economic situations have been normalized to have status quo 0 in utility space, this axiom requires the outcome of a mechanism F in utility space for a given situation $\mathcal{E}$ to depend only on the utility possibility set $\mathcal{A}(\mathcal{E})$. This models a variant of what Roemer (1990) calls universal welfarism.

It is possible to formulate Nash's classical axioms for a bargaining solution in Roemer's model for an allocation mechanism F.

Axiom 5.4: Pareto Optimality
For a given economic environment $\mathcal{E} \in \sum$, the utility pair $\mu_F(\mathcal{E})$ is a Pareto optimal point in $\mathcal{A}(\mathcal{E})$.

Axiom 5.5: Symmetry
If $\mathcal{E} \in \sum$ is an economic environment with a symmetric set $\mathcal{A}(\mathcal{E})$, then $\mu_F^1(\mathcal{E}) = \mu_F^2(\mathcal{E})$.

Axiom 5.6: Independence of Irrelevant Alternatives
Let $\mathcal{E}, \mathcal{E}' \in \sum$ be economic environments with
$\mathcal{A}(\mathcal{E}) \subseteq \mathcal{A}(\mathcal{E}')$ and $\mu_F(\mathcal{E}') \in \mathcal{A}(\mathcal{E})$. Then $\mu_F(\mathcal{E}') = \mu_F(\mathcal{E})$.

Axiom 5.7: Scale Invariance
Let $\mathcal{E}$, $\mathcal{E}' \in \sum$ be economic situations such that
$\exists\, a^1, a^2 \in \mathbb{R}_+$, $(\bar{u},\, \bar{v}) \in \mathcal{A}(\mathcal{E}) \;\Leftrightarrow\; (a^1\bar{u},\, a^2\bar{v}) \in \mathcal{A}(\mathcal{E}')$.
Then $\mu_F^i(\mathcal{E}') = a^i \mu_F^i(\mathcal{E})$ for $i = 1, 2$.

The transpositions of the axioms of Pareto Optimality, Symmetry and Independence of Irrelevant Alternatives from Nash's model into Roemer's model are straightforward. In the formulation of the axiom of Scale Invariance it is not necessary to deal with the whole class of positive affine transformations, because the status quo in utility space is assumed to be 0 in all economic environments. Roemer (1988) shows that these axioms characterize a mechanism on the domain $\sum$ which essentially is the Nash solution.

Theorem 5.1
There is a unique allocation mechanism N satisfying the axioms Domain $\sum$, Welfarism, Scale Invariance, Pareto Optimality, Symmetry and Independence of Irrelevant Alternatives.
N chooses the set of allocations in $\mathcal{E} = < m; \bar{x}; u, v >$ such that each allocation maximizes $\bar{u} \cdot \bar{v}$ for $(\bar{u},\, \bar{v}) \in \mathcal{A}(\mathcal{E})$.

Billera and Bixby (1973) show that any convex, closed and comprehensive set in utility space (for n players, $n \geq 2$) can be derived from some economic environment with many commodities. Therefore Theorem 5.1 is an implication of Nash's theorem.

In the characterization above, the axiom of Welfarism is not needed because it is implied by the axiom of Scale Invariance and also by the axiom of Independence of Irrelevant Alternatives.

Roemer also shows that several other solution concepts that have been formulated on the traditional domain of bargaining situations can be characterized by axioms on economic environments. Among these solutions are the egalitarian solution, the proportional solution and the Kalai–Smorodinsky solution. It is possible to omit the Welfarism axiom in these characterizations too, because it is implied by one or more of the other axioms. The reason for this implication is that some axioms describe properties of the mechanisms that are derived from the comparison of two economic environments that are related in a certain way. In these axioms, like e.g. in Independence of Irrelevant Alternatives, the comparison is based on conditions dependent only on the images of the environments in utility space. This means that only utility information is relevant for the solution. This idea is the background of the proofs that the axiom of Welfarism is not needed in the theorems Roemer (1988) states. We stress this aspect in order to point out that if we want to develop a non–welfaristic bargaining solution, it is necessary to formulate axioms that do not in the end depend on the images of the situations in utility space.

Now we describe Roemer's approach to formulate axioms involving more economic information for the example of the Nash solution.

Axiom 5.8: Economic Symmetry
For any economic environment $\mathcal{E} = < m; \bar{x}; u, u > \in \sum$,
$\left(\frac{\bar{x}}{2}, \frac{\bar{x}}{2}\right) \in F(\mathcal{E})$.

If the economic environment $\mathcal{E}$ itself is symmetric, i.e. if the persons have the same utility functions, then equal division of the resources is an element of $F(\mathcal{E})$. Since the situation does not provide a distinction between the persons, the allocation that treats the persons identically should belong to the set of allocations forming the solution.

Axiom 5.9: Cardinal Non–comparability
Let $\mathcal{E} = < m; \bar{x};; u, v >$ be an economic situation in $\sum$ and $a, b \in \mathbb{R}^+$. Then for the situation $\mathcal{E}' = < m; \bar{x}; au, bv > \quad F(\mathcal{E}') = F(\mathcal{E})$.

Cardinal Non–comparability is an axiom that is implied by Scale Invariance. In the axiom of Scale Invariance, e.g. the dimensions of the allocations of the compared economic environment may be different, only the utility possibility sets matter. In the axiom of Cardinal–Non–comparability, the situation $\mathcal{E}'$ is generated from $\mathcal{E}$ by applying scale transformation to each utility function. The other components of the environment remain the same.

Roemer formulates a further axiom that replaces the axiom of Independence of Irrelevant Alternatives. This axiom requires a type of consistency of the mechanism F.

Axiom 5.10: Strong Consistency of Resource Allocation across Dimension (CONRAD*)
Let $\mathcal{E} = < m + \ell; (\bar{x}, \bar{y}); u(x,y), v(x,y) > \in \sum$ be an economic environment with $\bar{x} \in \mathbb{R}_0^{+m}$, $\bar{y} \in \mathbb{R}_0^{+\ell}$, $u, v \in \mathcal{U}^{(m+\ell)}$, such that each of the goods with an index $j = m+1, \ldots, m+\ell$ is liked by at most one of the agents.
Let $((\hat{x}^1, \hat{y}^1), (\hat{x}^2, \hat{y}^2))$ be an allocation in $F(\mathcal{E})$.
Define utility functions u^*, v^* on $\mathbb{R}_0^{+m}$ by
$u^*(x) = u(x, \hat{y}^1)$,
$v^*(x) = v(x, \hat{y}^2) \quad \forall x \in \mathbb{R}_0^{+m}$.
In the case $u^*(0) = v^*(0) = 0 \quad u^*, v^*$ are in $\mathcal{U}^{(m)}$
and $\mathcal{E}^* = < m; \bar{x}; u^*, v^* >$ is an economic environment.
Then $(\hat{x}^1, \hat{x}^2) \in F(\mathcal{E}^*)$.

In CONRAD*, an economic environment of dimension m is constructed from an economic environment of dimension $m + \ell$, if this fulfills some conditions. The most important condition is that the ℓ goods belonging to the vector $\bar{y}$ are each only wanted by at most one of the individuals. If we have an allocation $((\hat{x}^1, \hat{y}^1), (\hat{x}^2, \hat{y}^2))$ that is assigned by F to the $m + \ell$–dimensional situation and if we define the allocation of the last ℓ dimensions to be $(\hat{y}^1, \hat{y}^2)$, we can consider the distribution problem of the first m dimensions. A mechanism F fulfills CONRAD*, if and only if the allocation $(\hat{x}^1, \hat{x}^2)$ belongs to the set of allocations chosen by the mechanism F for the m–dimensional environment.

Roemer shows that Domain $\sum$ and Independence of Irrelevant Alternatives imply CONRAD*. This means that CONRAD* is a weak version of Independence of Irrelevant Alternatives on the domain $\sum$. Irrelevant in this case are changes in the allocation of the last ℓ goods, because this would not lead to points outside the former utility possibility set.

Roemer proves the following theorem.

Theorem 5.2
There is a unique allocation mechanism N on $\sum$ fulfilling the axioms CONRAD*, Pareto Optimality, Economic Symmetry and Cardinal Non–Comparability. N is the mechanism that assigns to an economic environment $\mathcal{E} = < m; \bar{x}; u, v >$ the set of allocations that maximize $u(\bar{x}^1) \cdot v(\bar{x}^2)$.

This theorem characterizes the Nash solution on economic environments. Roemer (1988) also gives axiomatic characterizations of the monotone utility path solution, the egalitarian solution, the proportional solution and the Kalai–Smorodinsky solution with axioms on economic environments and another version of the CONRAD*–axiom. We remark that Roemer proves the theorems for a slightly smaller domain than $\sum$. He requires the utility function u in $\mathcal{U}^{(m)}$ to fulfill the condition

$$\lim_{t \to \infty} \frac{1}{t} u(tx) = 0 \quad \forall x \in \mathbb{R}_0^{+m}.$$

One of the main technical tools used in the proofs is a lemma which is an implication of Howe's theorem (1987). In order to apply this theorem, this additional assumption on the utility functions is needed.

Since Symmetry implies Economic Symmetry, Scale Invariance implies Cardinal Non–comparability, and Domain $\sum$ and Independence of Irrelevant Alternatives imply CONRAD*, the theorem characterizing the Nash mechanism on economic environments is stronger than the theorem using the welfaristic axioms. The same holds for the theorems dealing with the other mechanisms.

However, if one would have had the hope to find non–welfaristic mechanisms by requiring properties formulated on economic environments, the results would be disappointing. The mechanisms that are characterized are welfaristic. This observation was already made by Roemer (1986b) in the case of the egalitarian solution. The title of the paper displays the result: "Equality of Resources implies Equality of Welfare". The attempt to formulate egalitarian distributive norms in the economic space and assuming interpersonal comparability of utilities leads to the solution that equalizes the utilities of the persons.

If the CONRAD*–axiom is ommitted in Theorem 5.2 and is replaced by an economic axiom of Independence of Irrelevant Alternatives, the remaining axioms are fulfilled by a number of mechanisms on $\sum$. Roemer (1986c) describes a class of non–welfaristic mechanisms which have the required properties on economic environments. Assume there is a set of economic environments $\mathcal{E}^1, \ldots, \mathcal{E}^k$ that fulfill certain requirements, especially they have to be different enough and non–symmetric. For each environment $\mathcal{E}^i$ we fix a Pareto optimal, individually rational outcome $\sigma^i \in \mathcal{A}(\mathcal{E}^i)$. Then there is a mechanism F having all the required properties on economic environments and leading to the outcomes σ^i in utility space when it is applied to $\mathcal{E}^i$ for all $i = 1, \ldots, k$.

Therefore, the question arises whether it is possible to formulate reasonable properties of an allocation mechanism for bargaining problems that lead to a unique characterization of a non–welfaristic solution concept. From the results described above it is obvious that such a theory has to have non–welfaristic parts. CONRAD* leads to a unique characterization of a solution that is welfaristic, i.e. together with the other axioms on economic environments it implies welfarism. One possible idea is to do without some of the non–welfaristic axioms on economic environments and to replace them by more welfaristic formulations, but to bring some new non–welfaristic information into the model. We will proceed this way in the following chapter for a more general type of economic environments. In addition we will not assume a fixed commodity bundle to be distributed, and we will not formulate requirements on the shapes of the utility functions of the persons.

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Chapter 6

Bargaining Solutions with Goals

6.1 Introduction

One of the aspiration levels we observe in our experiments is the goal a person has in mind at the beginning of the negotiation process. A persons formulation of the goal emerges from the consideration of the economic features of the situation and the application of certain distributive rules which depend on the economic and ethical environment of the bargaining problem. In this chapter we present a two–person model for bargaining problems in which the bargaining situation is described by economic terms. The goals the persons develop have an influence on the bargaining agreement. We model this by defining a goal for each person. These goals depend on economic and on ethical aspects of the situation. For given ethical and economic features of an environment, we define a goal function on the set of all situations that may occur in this kind of environment. We require goal functions to have certain reasonable properties. From the variety of goal functions that are admissible in our model let us give a well known example here. If, for instance, the goal of each person is defined by her ideal utility value in the bargaining situation, the ideal point in the definition of Kalai and Smorodinsky

(1975) represents the goal point for a given situation. This definition of goals uses only utility information and therefore leads to a welfaristic goal function. We also discuss the possibilities to define non–welfaristic goal functions.

Each goal function represents a specific way the persons find their goals. Each way is the result of a certain ethical and economic background of the economic situation that is described in the model. We distinguish between ethical and economic environments and economic situations. Economic situations are embedded in a certain ethical and economic environment. The same economic situation can occur in different environments. The goals of the persons that are faced with a given situation may depend on the ethical and economic environment of the situation, because it has an influence on the distributive rules the persons consider to be applicable and justifiable.

In the example of the ideal point being the goal point, the Kalai–Smorodinsky solution can be derived as a proportional solution with respect to the status quo and the ideal point. In our theory we use the concept of proportional solutions to define classes of solutions. We give two types of axiomatic characterizations for these bargaining solutions. The first one uses a monotonicity relation between relative goals and bargaining outcomes. In monetary terms this monotonicity property is observed in our experiments. The second characterization uses an equity axiom in concession. This axiom generalizes Gauthier's idea that persons look at their ideal utility gain when they measure the size of a concession. We assume that they calculate concessions relative to the utility gain which is defined by the difference between their goal and the status quo.

Furtheron, we discuss how in our model non–welfaristic goal functions lead to non–welfaristic bargaining solutions.

6.2 Economic Situations and Goals

We consider bargaining situations with two persons. The set of *feasible alternatives* X is assumed to be representable in an Euclidean Space. We can interpret X e.g. as a set of feasible allocations of m commodities. Then X is a subset of $\mathbb{R}^{2m}$ with $m \in \mathbb{N}$ and an element $x \in X$ is denoted by $x = (x_1^1, \ldots, x_1^m; x_2^1, \ldots, x_2^m)$ where the first part of the vector x describes the quantities of the commodities of person 1 and the second part the allocation of person 2. The choice of subsets of $\mathbb{R}^{2m}$ enables us to consider the distribution of resources and of losses.

The *initial endowment* of the persons in the m commodities is denoted by $x_0 \in X$. x_0 is the *status quo* of the bargaining problem in commodity space.

We assume the persons to have cardinal v.Neumann–Morgenstern utility functions u_1 resp. u_2 on $\mathbb{R}^{2m}$. Notice that we allow the utility functions of a person not only to depend on her own commodity bundle but also on the commodity bundle of the other person. Therefore, we can model for instance the phenomenon of envy.

We denote the set of lotteries on X by $\bar{X}$.

Definition 6.1
Let $X \subseteq \mathbb{R}^{2m}$ be a set of feasible alternatives with status quo $x_0 \in X$ and $u = (u_1, u_2)$ a pair of v.Neumann–Morgenstern utility functions on X. If $u(\bar{X})$ is convex and compact, and if there is a $y \in \bar{X}$ such that

$u(y) > u(x_0)$ (existence of a bargaining incentive),

then (X, x_0, u) is an *economic situation* with m commodities. We denote the set of all economic situations with m commodities by $\mathcal{E}^m$ and the set of all economic situations $\bigcup_{m \in \mathbb{N}} \mathcal{E}^m$ by $\mathcal{E}$.

Notice that we do not assume restrictions on the shape of X. The restrictions in Definition 6.1 are formulated for $u(\bar{X})$. There convexity is implied by the properties of v.Neumann–Morgenstern utility functions on a set of lotteries.

The set $\mathcal{E}$ is one part of the domain for the bargaining solutions we are going to introduce. Consider two persons that are faced with a certain economic situation. We assume that they have bargaining goals in mind when they enter the negotiation. A bargaining goal is defined by the utility value a person really wants to achieve in an agreement. The goal each person constructs for herself depends on the economic situation. It may also depend on further economic or ethical aspects of the environment of the situation. These exterior aspects that are not described by the definition of an economic situation lead to different types of goal functions.

Definition 6.2

A *goal function* g is a mapping $g : \mathcal{E} \longrightarrow \mathbb{R}^2$, where $g = (g_1, g_2)$.
For any $(X, x_0, u) \in \mathcal{E}$, $g_1(X, x_0, u) \in \mathbb{R}$ and $g_2(X, x_0, u) \in \mathbb{R}$ define the goal of person 1 and person 2 resp. in utilities such that properties 6.1 through 6.4 hold.

Property 6.1: Individual Rationality

$\forall (X, x_0, u) \in \mathcal{E} \qquad g_i(X, x_0, u) \geq u_i(x_0)$ for $i = 1, 2$.

The property of Individual Rationality means that each person formulates a goal that is at least as high as her utility value of the status quo.

Property 6.2: Individual Feasibility

$\forall (X, x_0, u) \in \mathcal{E} \qquad g_i(X, x_0, u) \in u_i(\bar{X})$ for $i = 1, 2$.

The property of Individual Feasibility requires the goal of each person to lie in the set of feasible utility values of that person. This set is equal to the image of $\bar{X}$ under the utility function of the person.

Property 6.3: Transformation Invariance

$\forall (X, x_0, u) \in \mathcal{E} \qquad g(X, x_0, \lambda u) = \lambda g(X, x_0, u)$
for all pairs of positive affine transformations λ of the utility functions.

Since we assume in all our models interpersonally non–comparable utilities, we have to allow the application of pairs of positive affine transformations to the utility functions. The goals of the transformed economic situation are the transformed goals of the original situation.

Property 6.4: Existence of a Conflict between the Goals of the Persons
$\forall (X, x_0, u) \in \mathcal{E} \quad g(X, x_0, u)$ is not Pareto dominated by a point in $u(\bar{X})$.

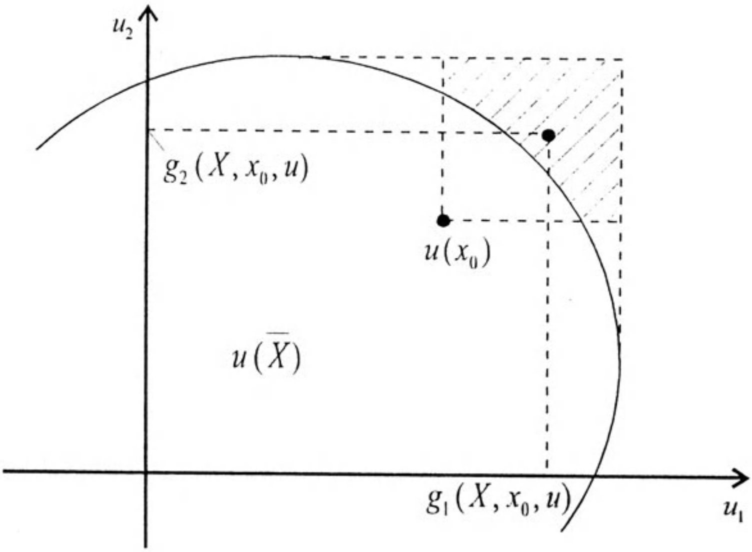

Figure 6.1

This property requires that the goals are demanding enough. If there would be a Pareto dominating point in $u(\bar{X})$, then at least one of the persons could enlarge her goal such that the new pair of goals would still be feasible. There are two possibilities for a pair of goals. First, if it is not feasible, then there is a conflict between the goals of the persons. Second, if it is feasible, Property 6.4 implies that the pair of goals is a strongly Pareto optimal point of $u(\bar{X})$.

Now we define a property that not every goal function has to have. This property describes economic and ethical environments where the two persons formulate their goals in the same way, if it is not possible to distinguish between their positions in a given economic situation.

Property 6.5: Symmetry

Let π denote the permutation of the numbers of the persons. A goal function on $\mathcal{E}$ is *symmetric*, if for all economic situations $(X, x_0, u) \in \mathcal{E}$ with

$\pi X = X$,

$\pi x_0 = x_0$,

$u_1(x) = u_2(\pi x) \qquad \forall x \in X$ and

$u_2(x) = u_1(\pi x) \qquad \forall x \in X$

$g_1(X, x_0, u) = g_2(X, x_0, u)$ holds.

The economic situation described in the definition is symmetric in the following way: X is symmetric, x_0 is symmetric and the evaluation of the alternatives in X by the utility functions of the persons is the same, when we permute the allocations. If $x = (x_1^1, \ldots, x_1^m; x_2^1, \ldots, x_2^m)$ then $\pi x = (x_2^1, \ldots, x_2^m; x_1^1, \ldots, x_1^m)$. In this case we cannot distinguish between the two persons in the economic situation. Then a symmetric goal function assigns a symmetric utility pair of goals to the situation.

If there would be an ethical and economic environment implying differences between the positions of the persons, even for symmetric economic situations, the goal function might be non–symmetric.

We like to stress that it is possible that the bargaining situation in utility space $(u(\bar{X}), u(x_0))$ is symmetric, but $g_1(X, x_0, u) \neq g_2(X, x_0, u)$. We illustrate this in Figure 6.2. There the economic situation (X, x_0, u) is not symmetric, but can be mapped by appropriate functions u_1, u_2 onto a symmetric situation in utility space. The goals in the situation (X, x_0, u) are defined by "equal amounts" for person 1 and "proportional amounts" for person 2. These goals are images of a symmetric goal function, because for a symmetric economic situation both goals are identical. The goal point in the given example is not symmetric in utility space.

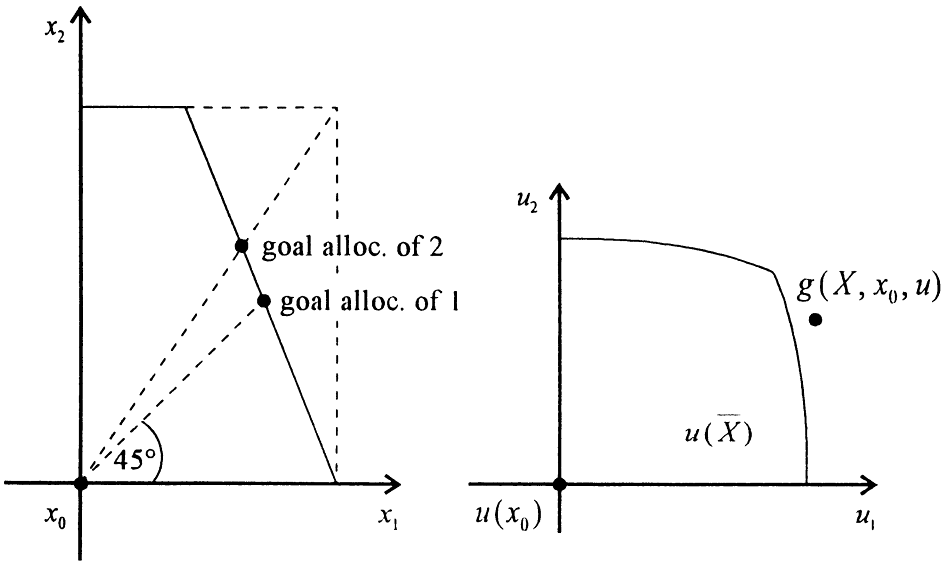

Figure 6.2

Therefore the notion of "symmetry" of a goal function is different from the notion of "symmetry" of a bargaining solution.

Since the arguments of goal functions are economic situations, and since they are not defined on sets of traditional bargaining situations, they can be non–welfaristic. If we consider economic situations like the payoff sets in our experiments, a goal of a person, that is defined by principles like "equal payoffs" or "proportional payoffs" does not use utility information. These goals are formulated dependent on economically defined norms and can be mapped into the utility space afterwards. Of course there are welfaristic goal functions, like the ideal utility values of the persons in the individual rational part of the set of feasible utility pairs. We will describe welfaristic and non–welfaristic goal functions in section 6.6 of this chapter.

6.3 Bargaining Solutions

In the following we formulate the concept of bargaining solutions that depend on a given set of possible goal functions.

Definition 6.3
Let G be a set of goal functions on $\mathcal{E}$.
A *bargaining solution* is a mapping
$f : \mathcal{E} \times G \longrightarrow \mathbb{R}^2$
such that $f(X, x_0, u; g) \in u(\bar{X})$ for all $(X, x_0, u) \in \mathcal{E}, g \in G$.

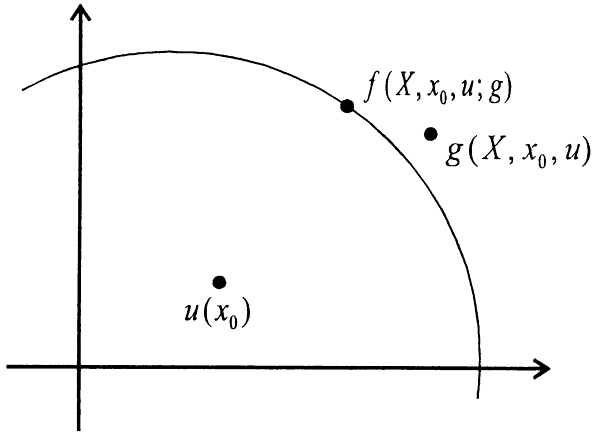

Figure 6.3

A bargaining solution is applied to an economic situation and to a given goal function. The solution chooses a point in utility space that is feasible, i.e. it lies in $u(\bar{X})$. The main difference to other solution concepts is that the outcome depends on the goal that itself depends on the economic situation. In the Nash model (1950) the solution would be defined dependent only on (X, x_0, u). In the theory of bargaining with claims by Thomson and Chun (1992), the claim point does not depend on the economic situation. It is exogenous.

We now formulate a collection of axioms for a bargaining solution f on economic situations with goals.

Axiom 6.1: Weak Pareto Optimality
$\forall (X, x_0, u) \in \mathcal{E}, g \in G \qquad f(X, x_0, u; g) \in \mathrm{WPO}(u(\bar{X}))$.

This axiom restricts the possible outcomes of a bargaining situation to the set of weakly Pareto optimal points in the set of feasible utility points $u(\bar{X})$.

We would like to mention at this point that in our model the utility evaluation of each person may depend on the allocation of commodities to both persons. Therefore, even if we would require strong Pareto optimality in our model, this would be no contradiction to the observation of non–strongly Pareto optimal outcomes in payoffs in our experiments.

Axiom 6.2: Symmetry
For all $(X, x_0, u) \in \mathcal{E}$ and $g \in G$, if $u(\bar{X})$ is symmetric, $u_1(x_0) = u_2(x_0)$ and $g_1(X, x_0, u) = g_2(X, x_0, u)$, then $f_1(X, x_0, u; g) = f_2(X, x_0, u; g)$.

In this context, symmetry means that if the image of the bargaining problem in utility space is symmetric, then the outcome should also be symmetric.

Axiom 6.3: Transformation Invariance
Let λ be a pair of positive affine transformations on $\mathbb{R}$. Then for all $(X, x_0, u) \in \mathcal{E}$ and $g \in G$,
$f(X, x_0, \lambda u; g) = \lambda f(X, x_0, u; g)$.

This type of transformation invariance is a necessary requirement because we assume that the persons have non–interpersonally comparable v.Neumann–Morgenstern utility functions.

Now we formulate an axiom that models a monotonicity property we observed in our experiments. It describes the monotonicity relation between the normalized ratios of the goals of the persons and the utility value of the outcome for one person.

Axiom 6.4: Individual Monotonicity
Let $E = (X, x_0, u)$ and $E' = (X', x_0', u')$ be economic situations in $\mathcal{E}$ and $g \in G$ be a goal function.
If $u(x_0) = u'(x_0')$, $u(\bar{X}) \subseteq u(\bar{X}')$

and $\dfrac{g_i(E) - u_i(x_0)}{g_j(E) - u_j(x_0)} \leq \dfrac{g_i(E') - u_i'(x_0')}{g_j(E') - u_j(x_0')}$ for $i \neq j$,

then $f_i(E, g) \leq f_i(E', g)$.

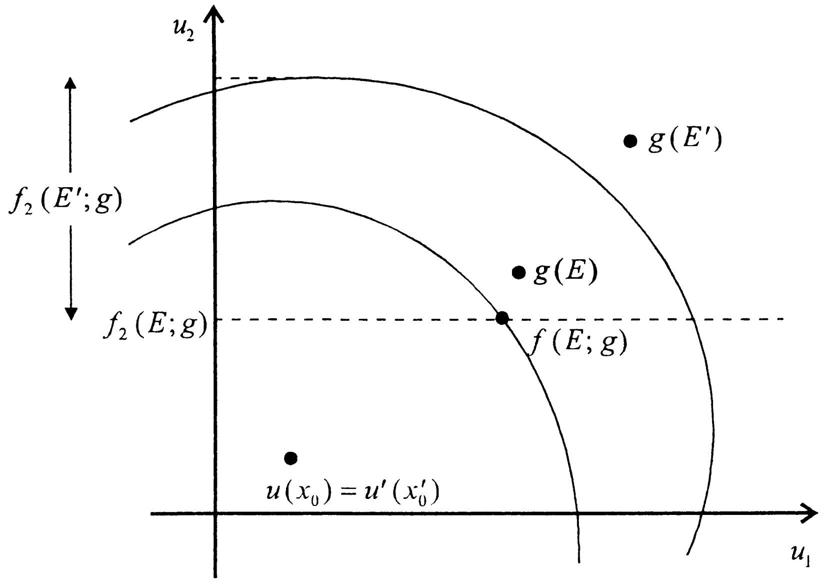

Figure 6.4

In this axiom we compare two economic situations E and E'. The utility values of the status quos are identical and the feasible set of utility pairs of situation E' includes the feasible set of E. If we apply a goal function g to the situations and observe that the ratio of the utility differences between goal and status quo increases in favor of person i replacing situation E by situation E', then this leads to an improvement of the utility value of person i in the bargaining outcome.

We define a bargaining solution on economic environments with goals that generalizes the concept of the proportional solution with claims in Thomson and Chun (1992).

Definition 6.4

Let G be a set of goal functions on $\mathcal{E}$.

The *proportional solution* is a mapping

$P : \mathcal{E} \times G \longrightarrow \mathbb{R}^2$

such that $P(X, x_0, u; g)$ is the maximal point in $u(\bar{X})$ on the segment connecting $u(x_0)$ and $g(X, x_0, u)$.

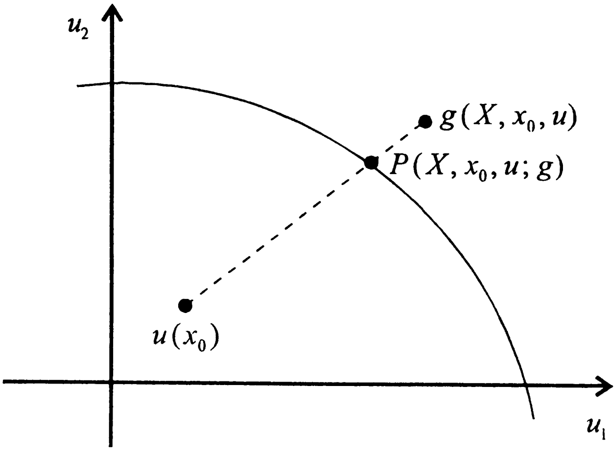

Figure 6.5

The proportional solution chooses a point on the boundary of $u(\bar{X})$ such that the ratio of the utility gains of the persons is identical to the ratio of the utility differences between the goals and the status quo and such that the outcome is individually rational. We can also formulate this in terms of utility differences to the goal point of the situation. The proportional solution is a point of the individually rational part of the boundary of $u(\bar{X})$ such that the ratio of the utility differences between the goals and the solution is identical to the ratio of the utility differences between the goals and the status quo. The properties of Individual Rationality, Individual Feasibility and Existence of a Conflict between the Goals of the Persons, i.e. properties of the goal function, imply the strong Pareto optimality of P. This means

$$\forall (X, x_0, u) \in \mathcal{E}, g \in G \qquad P(X, x_0, u; g) \in \mathrm{PO}(u(\bar{X}))\,.$$

6.4 A Characterization of the Proportional Solution by a Monotonicity Axiom

We now assume that the set of goal functions has a single element g. This means that the ethical and economic environment in which the economic situations in $\mathcal{E}$ are embedded is uniquely determined. Especially we assume that g is a symmetric goal function. Then the following theorem holds.

Theorem 6.1
Let g be a symmetric goal function on $\mathcal{E}$.
The proportional solution P is the only solution on $\mathcal{E} \times \{g\}$ fulfilling the axioms of Weak Pareto Optimality, Symmetry, Transformation Invariance and Individual Monotonicity.

Proof: Let f be a solution on $\mathcal{E} \times \{g\}$ fulfilling the four axioms and let (X, x_0, u) be a situation in $\mathcal{E}$. Because P fulfills the axiom of Transformation Invariance, we can assume that $u(x_0) = (0,0)$ and $g(X, x_0, u) = (1,1)$. This implies $P(X, x_0, u; g) = (a, a)$ for some $a \in (0, 1]$. Let π be the permutation of the player numbers. For $S = u(\bar{X})$ we define a symmetric set $S' = S \cap \pi S$.
Now we construct an economic situation (X', x_0', u') in $\mathcal{E}$ by defining $X' = S'$; $x_0' = (0,0)$; $u_1'(x_1, x_2) = x_1, u_2'(x_1, x_2) = x_2 \quad \forall\, (x_1, x_2) \in \mathbb{R}^2$.
Since S is symmetric, $g(S', x_0', u') = (b, b) \in \mathbb{R}^2$ with $b \geq a$.
The axioms of Weak Pareto Optimality and Symmetry applied to f imply $f(X', x_0', u') = (a, a)$.

We observe that $\dfrac{g_1(X, x_0, u)}{g_2(X, x_0, u)} = \dfrac{g_1(X', x_0', u')}{g_2(X', x_0', u')} = 1$,

$u(x_0) = u'(x_0') = (0,0)$ and $u'(\bar{X}') \subseteq u(\bar{X})$.
We apply the axiom of Individual Monotonicity in both directions to $f(X, x_0, u; g)$ and $f(X', x_0', u'; g)$.
This implies

$$f_1(X, x_0, u; g) \geq f_1(X', x_0', u'; g) = a$$
$$f_2(X, x_0, u; g) \geq f_2(X', x_0', u'; g) = a.$$

Since (a, a) is strongly Pareto optimal in $u(\bar{X})$,
$f(X, x_0, u; g) = (a, a) = P(X, x_0, u; g)$. □

6.5 A Characterization of the Proportional Solution by a Concession Axiom

For the Kalai–Smorodinsky solution there exists an axiomatic characterization that uses an equity axiom in relative utility gains or in concessions of the persons (cf. Klemisch–Ahlert (1992)). The question arises whether it is possible to characterize the proportional solution on the domain of economic situations with goals in a similar way. If an economic situation (X, x_0, u) in $\mathcal{E}$ and a goal function g are given, we know the status quo in utility space $u(x_0) = (d_1, d_2)$ and the goal point $g(X, x_0, u) = (c_1, c_2)$. We assume $(c_1, c_2) > (d_1, d_2)$. If a point $y = (y_1, y_2) \in u(\bar{X})^+$ is proposed as a possible agreement, each person i faces the difference $c_i - y_i$. If this utility difference is positive, it is the concession person i has to make in comparison to her goal, if she accepts y. If the difference is negative, it describes a utility gain in comparison to the goal. The maximal concession person i can make is $c_i - d_i$. If $c_i - y_i$ is positive, $\frac{c_i - y_i}{c_i - d_i}$ measures the utility loss person i has to concede relative to the wanted gain $c_i - d_i$, if y is the bargaining outcome, $\frac{y_i - d_i}{c_i - d_i} = 1 - \frac{c_i - y_i}{c_i - d_i}$ measures the relative utility gain of person i in y.

In order to characterize the proportional solution on $\mathcal{E} \times G$ for sets of goal functions G we have to introduce a further property of a goal function g and two axioms for bargaining solutions f.

Property 6.6: Strong Individual Rationality
$\forall (X, x_0, u) \in \mathcal{E}\ , g_i(X, x_0, u) > u_i(x_0)$ for $i = 1, 2$.

The property of Strong Individual Rationality means that the goal of each person has a utility value which is greater than the utility value of the status quo. This requirement is slightly stronger than the property of Individual Rationality, but it is not too demanding. If a person has a goal that leads to the same utility level as the status quo, then there is no reason why this person should negotiate. We need the property of Strong Individual Rationality in the definition of a relative utility loss or relative utility gain to ensure $c_i - d_i > 0$ for $i = 1, 2$.

Axiom 6.5: Individual Rationality
$\forall (X, x_0, u) \in \mathcal{E}, g \in G \qquad f(X, x_0, u; g) \geqq u(x_0)$.

Individual Rationality of a bargaining solution requires the agreement to assign a utility value to each person that is at least as high as the utility value of the status quo. This property of the outcome of the bargaining situation ensures that no person has an incentive to improve her utility by disagreeing.

Axiom 6.6: Equity in Concessions
Let (X, x_0, u) be an economic situation in $\mathcal{E}$ and g a strongly individual rational goal function. We define $u(x_0) = (d_1, d_2)$ and $g(X, x_0, u) = (c_1, c_2)$. Let y be a point in $u(\bar{X})^+$.
If there is a person $i \in \{1, 2\}$ such that $y_i - c_i > 0$, then $y \neq f(X, x_0, u; g)$.
If $c_i - y_i \geq 0$ for all $i \in \{1, 2\}$ and if there is a point $x \in u(\bar{X})^+$ and a person $j \in \{1, 2\}$ such that for $i \neq j$

$$\frac{c_i - y_i}{c_i - d_i} < \frac{c_i - x_i}{c_i - d_i} < \frac{c_j - x_j}{c_j - d_j} < \frac{c_j - y_j}{c_j - d_j},$$

then $y \neq f(X, x_0, u; g)$.

The axiom of Equity in Concessions consists of two parts. If an individually rational point y is proposed such that a person i receives a larger utility value than her goal would be, this means that person $j \neq i$ receives less than her goal. This person will reject the proposal y, because it gives person i more than she really wants to get at the cost of a small utility value for person j. Person j's utility value could be improved by proposing a Pareto optimal point in $u(\bar{X})^+$ that guarantees to person i her goal. In the second part of the axiom it is assumed that a point y is proposed where both persons make concessions with a value of at least 0 in comparison to their goals. If there is a point $x \in u(\bar{X})^+$ such that a person j could reduce her relative concession by proposing x. The relative concession of person $i \neq j$ might increase, if x is the outcome, in comparison to y, but in a way that she still has to make a smaller relative concession than person j. In this case y will not be the agreement outcome. The axiom does not state that x will be accepted by both persons. It is possible that x can be rejected by finding a point $z \in u(\bar{X})^+$ with the property that is required in the axiom and so on. The question that arises is to describe the set of outcomes that cannot be rejected by this axiom and that fulfills some other desirable axioms.

The idea to compare relative concessions traces back to Zeuthen (1930). The type of concession we use in this model is similar to the type Gauthier (1985, 1986) uses to develop his theory of distributive justice by bargaining. Gauthier's model is a special case of our model. Gauthier uses a special goal function that is defined by the ideal utility values of the persons, whereas we allow arbitrary sets of strongly individual rational goal functions. We generalize a theorem dealing with concessions of Gauthier's type (cf. Klemisch–Ahlert (1992)) and prove the following result.

Theorem 6.2
Let G be a set of strongly individual rational goal functions on $\mathcal{E}$. Then the proportional solution P is the only solution on $\mathcal{E} \times G$ satisfying the axioms of Weak Pareto Optimality, Transformation Invariance, Individual Rationality and Equity in Concessions.

Proof: It is easy to show that P fulfills the axioms.

Let f be a solution on $\mathcal{E} \times G$ fulfilling the axioms of the theorem and g be a strongly individual goal function in G. Let (X, x_0, u) be a situation in $\mathcal{E}$, $u(x_0) = (d_1, d_2)$ and $g(X, x_0, u) = (c_1, c_2)$. Because f is individually rational, $y = f(X, x_0, u; g)$ is a point in $u(\bar{X})^+$. y is strongly Pareto optimal in $u(\bar{X})$ because of the definitions of goal functions and bargaining solutions. We also know that $c_i - y_i \geq 0$ for all $i \in \{1, 2\}$. Let us assume that

$$\frac{c_i - y_i}{c_i - d_i} < \frac{c_j - y_j}{c_j - d_j} \text{ for } i \neq j \in \{1, 2\}.$$

We choose $x = \frac{1}{2}(y + P(X, x_0, u; g))$. Then x is in $u(\bar{X})^+$.

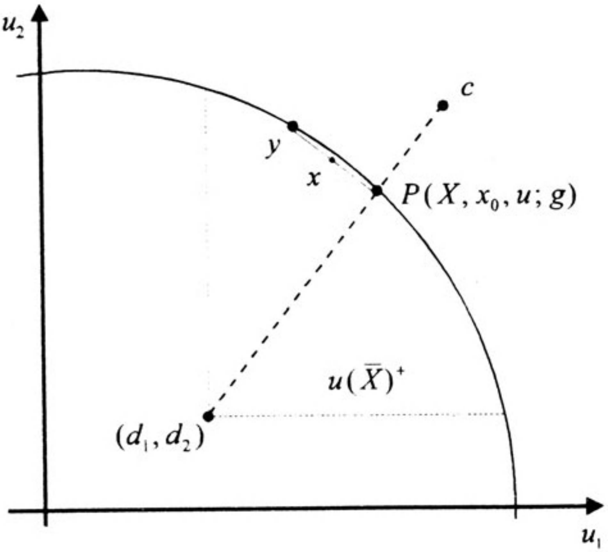

Figure 6.6

$$\frac{P_1(X, x_0, u; g) - d_1}{c_1 - d_1} = \frac{P_2(X, x_0, u; g) - d_2}{c_2 - d_2}$$

and the strong Pareto optimality of P imply

$$\frac{c_i - y_i}{c_i - d_i} < \frac{c_i - P_i(X, x_0, u; g)}{c_i - d_i} \qquad \text{and}$$

$$\frac{c_j - y_j}{c_j - d_j} > \frac{c_j - P_j(X, x_0, u; g)}{c_j - d_j} .$$

Therefore $\quad \dfrac{c_i - y_i}{c_i - d_i} < \dfrac{c_i - x_i}{c_i - d_i} < \dfrac{c_j - x_j}{c_j - d_j} < \dfrac{c_j - y_j}{c_j - d_j} .$

The axiom of Equity in Concessions implies $y \neq f(X, x_0, u; g)$, a contradiction to our assumption. □

The axiom of Equity in Concessions can equivalently be formulated in terms of relative utility gains. The difference between the two types of axioms is the point of view of the persons, whether they look at what they get in comparison to the status quo or what they have to give up in comparison to their goals.

In this chapter we characterized the proportional solution P on the domain $\mathcal{E} \times G$ where G is a set of strongly individual rational goal functions. Since G may contain more than one element in this case, more than one ethical or economic environment of economic situations can be modelled in this theory. This is possible because the equity requirement in relative concessions does only depend on the utility information after the goals for a given situation in $\mathcal{E}$ have been fixed. The equity norm in the axiom does not vary with different ethical or economic environments of the situation in $\mathcal{E}$. If we would allow this variation, this would lead to a non–welfaristic concession axiom, what is a question for further research. In our model, different ethical or economic environments are taken into account by assuming that the way persons formulate their goals may depend on the environment. This means that every considered ethical or economic environment is represented by a certain goal function in G. This representation is not necessarily unique. Different environments may lead to identical goal functions. The set G of goal functions is the instrument to describe different information beyond the economic situation.

6.6 Goal Functions and Proportional Solutions

In this chapter we give some examples of goal functions g that lead to bargaining solutions on $\mathcal{E} \times \{g\}$. Some of the bargaining solutions are welfaristic solutions that are known as solutions on traditional sets of bargaining situations. We will also present a new type of goal function that uses more than utility information and leads to a non–welfaristic bargaining solution.

Definition 6.5
A goal function g on $\mathcal{E}$ is *universally welfaristic*, if
$g(E) = g(E')$ for all $E = (X, x_0, u) \in \mathcal{E}, E' = (X', x_0', u') \in \mathcal{E}$
with $u(x_0) = u'(x_0')$ and $u(X) = u'(X')$.

A universally welfaristic goal function only depends on the image of the economic situation in utility space. If the images in utility space of two economic situations are identical, then the goal points in utility space are identical.

Definition 6.6
A goal function g on $\mathcal{E}$ is *simply welfaristic*, if
$g(E) = g(E')$ for all $E = (X, x_0, u) \in \mathcal{E}, E' = (X', x'_0, u') \in \mathcal{E}$
with $X = X', x_0 = x'_0, u(x_0) = u'(x'_0)$ and $u(X) = u'(X')$.

If the sets of feasible alternatives, the status quos and the images in utility space of two economic situations are identical, then the goal points of a simply welfaristic goal function are identical, too.

Definition 6.7
A bargaining solution f on $\mathcal{E} \times G$ is *universally welfaristic*, if
$f(E; g) = f(E'; g)$ for all $E = (X, x_0, u) \in \mathcal{E}$, $E' = (X', x'_0, u') \in \mathcal{E}$
with $u(x_0) = u'(x'_0), u(X) = u(X')$ and for all $g \in G$.

Definition 6.8
A bargaining solution f on $\mathcal{E} \times G$ is *simply welfaristic*, if
$f(E; g) = f(E'; g)$ for all $E = (X, x_0, u) \in \mathcal{E}$, $E' = (X', x'_0, u') \in \mathcal{E}$
with $X = X', x_0 = x'_0, u(x_0) = u'(x'_0), u(X) = u'(X')$ and for all $g \in G$.

Let g be a goal function in G. Then the definition of a universally welfaristic bargaining solution requires the following: Whenever the status quos in utility space and the sets of feasible utility pairs of two economic situations are identical, the outcomes of the problems defined by the bargaining solution in utility space are the same. The definition of a simply welfaristic bargaining solution uses the additional assumption that the sets of feasible alternatives and the status quos are identical.

These four definitions capture Roemer's concepts of universal and simple welfarism.

Remark If g is a goal function of one of the two welfarism types, then P is a welfaristic bargaining solution on $\mathcal{E} \times \{g\}$ of the corresponding type. In general, if G is a set of goal functions being of the same type of welfarism, then P is a welfaristic bargaining solution on $\mathcal{E} \times G$ of that type. This means, if P is non–welfaristic on a domain $\mathcal{E} \times G$, then there has to be at least one non–welfaristic goal function g in G.

We now give some examples of welfaristic goal functions leading to well known welfaristic bargaining solutions.

Example 6.1: Kalai–Smorodinsky Solution

We define a function $a : \mathcal{E} \rightarrow \mathbb{R}^2$ by

$a_1(X, x_0, u) = \max \{x_1 \mid (x_1, x_2) \in u(\bar{X}), (x_1, x_2) \geqq u(x_0)\}$

$a_2(X, x_0, u) = \max \{x_2 \mid (x_1, x_2) \in u(\bar{X}), (x_1, x_2) \geqq u(x_0)\}$

for all $(X, x_0, u) \in \mathcal{E}$.

It is easy to see that a satisfies the properties of Individual Rationality, Individual Feasibility, Transformation Invariance and Existence of a Conflict between the Goals of the Persons, i.e. a is a goal function.

a is symmetric. This can easily be proved formally. The symmetry of a is intuitively clear, because both persons use analogous procedures to choose their goals. Their goal is their ideal utility value in the individual rational part of the set of feasible utility pairs.

Obviously, a is a universally welfaristic goal function on $\mathcal{E}$, since only utility information is used to determine a. Therefore, a is also simply welfaristic.

Now we consider the proportional solution P on $\mathcal{E} \times \{a\}$. Then P is a universally and simply welfaristic bargaining solution on $\mathcal{E} \times \{a\}$. For a given economic situation $(X, x_0, u) \in \mathcal{E}$ this solution chooses the point $P(X, x_0, u; a)$, which is the intersection of the line connecting $u(x_0)$ and $a(X, x_0, u)$ with the Pareto optimal boundary of $u(\bar{X})$. This point is the Kalai–Smorodinsky solution (1975) of the bargaining situation $(u(\bar{X}), u(x_0))$ in the traditional formulation of the Nash model (cf. Figure 6.7).

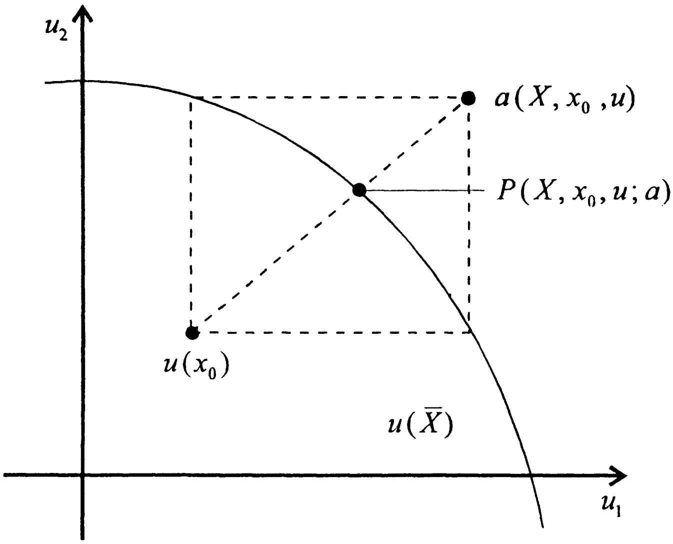

Figure 6.7

Example 6.2: Kalai–Rosenthal Solution
We define a function $b : \mathcal{E} \to \mathbb{R}^2$ by
$b_1(X, x_0, u) = \max \{x_1 \mid (x_1, x_2) \in u(\bar{X})\}$
$b_2(X, x_0, u) = \max \{x_2 \mid (x_1, x_2) \in u(\bar{X})\}$
for all $(X, x_0, u) \in \mathcal{E}$.

Again like in the example above, b is a symmetric goal function, and b is universally and simply welfaristic. Therefore, the proportional solution P on $\mathcal{E} \times \{b\}$ is also welfaristic of both types. If an economic situation $(X, x_o, u) \in \mathcal{E}$ is given, P chooses the point $P(X, x_0, u; b)$. This point is the intersection of the segment between $u(x_0)$ and $b(X, x_0, u)$ with the boundary of $u(\bar{X})$.

This point is the Kalai–Rosenthal solution (1978) of the bargaining situation $(u(\bar{X}), u(x_0))$ (c.f. Figure 6.8).

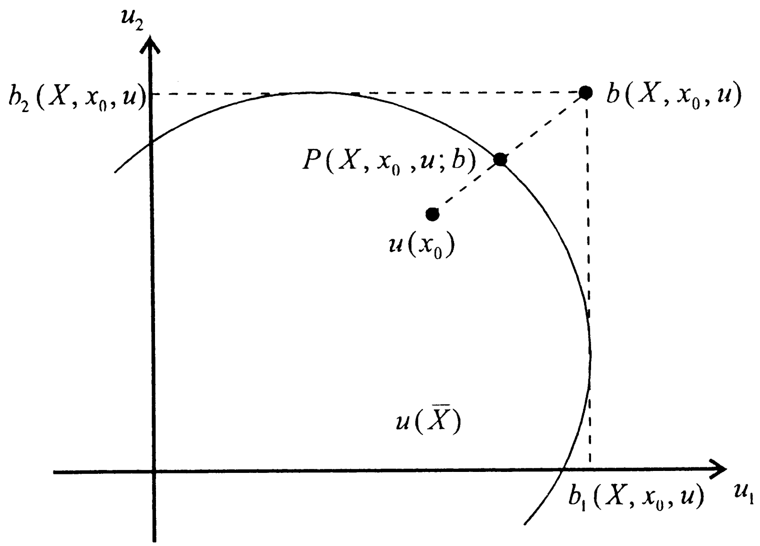

Figure 6.8

The Theorems 6.1 and 6.2 lead to new axiomatic characterizations of the Kalai–Smorodinsky solution and the Kalai–Rosenthal solution in the framework of our model of bargaining with goal functions.

Example 6.3
Let us briefly give an example of a goal function being simply welfaristic but not universally welfaristic: Define $g(X, x_0, u) = b(X, x_0, u)$ for all situations in $\mathcal{E}$ where X is a symmetric set, and $g(X, x_0, u) = a(X, x_0, u)$ for all other situations. This goal function leads to a proportional solution that is simply welfaristic but not universally welfaristic.

Example 6.4
In this example we reduce the domain of economic environments for the bargaining solution, we are going to construct, to a subset of $\mathcal{E}^1$. These economic situations have sets of feasible alternatives that can be interpreted as sets of allocations of one commodity to the two persons. In addition we assume that the convex hull of X is compact and that the boundary of the convex hull of X is a subset of X. These requirements are e.g. met if X is convex and compact. They are also fulfilled by

the payoff sets of our experiments. We also assume that the status quo $x_0 \in X$ is equal to $(0,0)$. We denote the reduced domain of economic environments by $\mathcal{E}_0^1$.

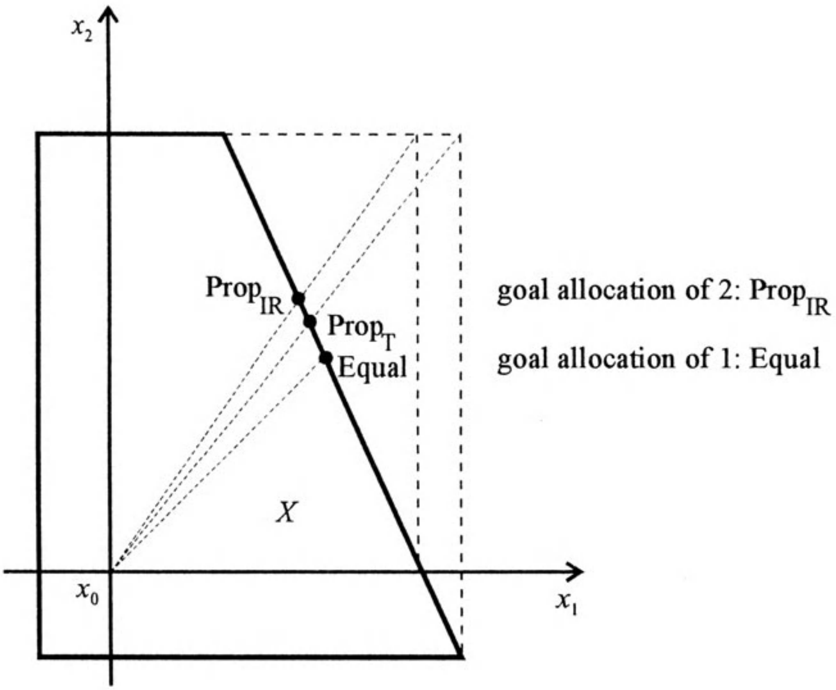

Figure 6.9

We consider three distributive principles in the allocation set:

(i) equal amounts (Equal)

(ii) proportional allocation with respect to the individually rational part
of X (Prop_{IR})

(iii) proportional allocation with respect to the total set X (Prop_{T}).

These are some of the principles that are applied in our experiments. We assume that both players think that these three principles are applicable and justifiable in the context of the given bargaining problem. Each principle leads to a unique point on the boundary of the convex hull of X which is a subset of X.

Each person calculates the allocations that are implied by each of these principles. Then each person chooses a principle that leads to her most preferred allocation among these allocations. This leads to a *conflict between the principles* the persons have in mind when they formulate their goals.

The goal $h_i(X,(0,0),u)$ of person i for $i = 1, 2$ is defined by the utility value of the allocation that belongs to her choosen principle. The pair of goals defines the outcome of the goal function for the given situation $(X,(0,0),u)$.

This means that in this model the conflict of principles is represented by the pair of utility values of the goal function $(h_1(X,(0,0),u), h_2(X,(0,0),u))$.

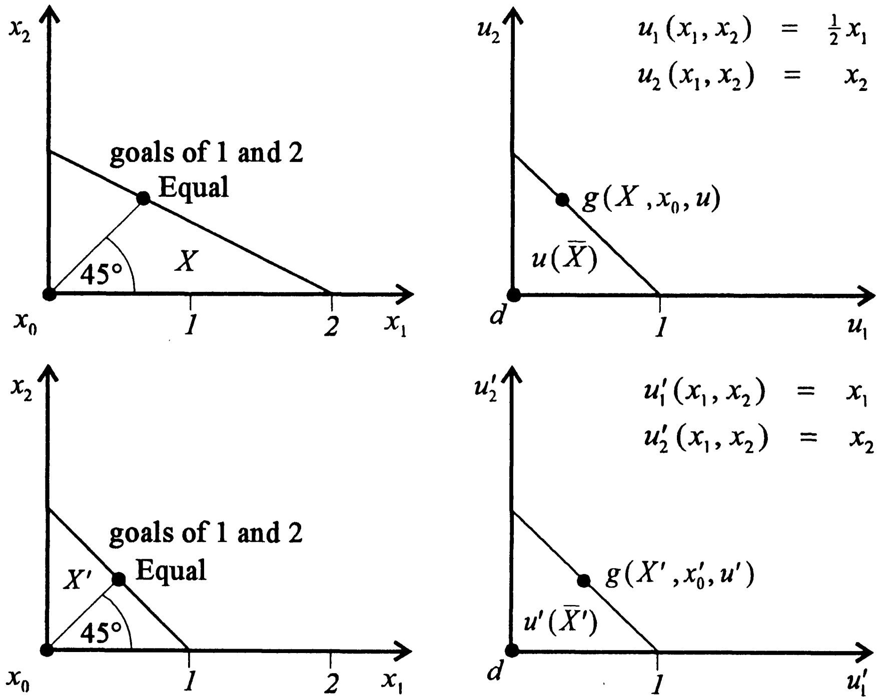

Figure 6.10

In Figure 6.10 we give an example of two situations in order to show that h is non–welfaristic. Since both persons use the same procedure to define their goals, h is symmetric.

The goal function h leads to a bargaining solution on $\mathcal{E}_0^1 \times \{h\}$, that is defined by the proportional solution P on $\mathcal{E}_0^1 \times \{h\}$. The example in Figure 6.10 shows that P is non–welfaristic on $\mathcal{E}_0^1 \times \{h\}$. Though P uses only utility information to define the bargaining outcome, it is non–welfaristic in combination with the non–welfaristic goal function h.

6.7 An Outlook on a Bargaining Theory on Ethical and Economic Environments

A bargaining problem consists of an economic situation which is described by a triple (X, x_0, u) and an ethical and economic environment of this situation. The environment may be characterized by needs of the persons involved into the bargaining problem, by their productivity or contribution to the cooperative product described by X, by their rights or legitimate claims or by further aspects that are not represented in (X, x_0, u).

Facing the bargaining problem, each person has a set of distributive norms in mind. These norms are defined on (X, x_0) and use economic information. The person checks whether these norms are applicable and justifiable. The person may decide upon the applicability and justifiability of distributive rules dependent on her education, experience in this type of situations or personal characteristics. Of course, the ethical and economic environment has an influence on this decision. For instance, if contributions to a cooperative surplus play a role, a proportionality principle with respect to the proportions of the contributions may be judged to be applicable. Different needs of persons may lead to a principle that supplies all persons with certain basic amounts of some goods and divides the rest according to some proportionality rule or another principle.

When a person has defined her set of distributive norms that are applicable and justifiable in her opinion, she chooses her goal for the bargaining process in the following way: She compares the utility values of all the points in X that would result from the distributive principles. Then she chooses a principle that leads to her most preferred point. This principle defines the goal. The value of the goal of the person is her utility value of this "best" point. Under some regularity assumptions on (X, x_0, u), this utility value is uniquely defined.

The bargaining procedure is influenced by the goals the persons have in mind. Since these goals are defined by a process of selecting a certain distributive principle, the bargaining problem can be described by a new problem that is generated by the conflict between the different principles the persons choose. Solving the bargaining problem can be interpreted as solving the conflict of principles. A more general bargaining theory on ethical and economic environments first has to develop criteria to define types of environments such that the sets of norms the persons apply can be characterized.

The second part of the theory is related to the negotiation process. The solution we propose and characterize in the section above is a variant of the proportional solution. After the goal point for a given bargaining situation is found, this solution uses only the utility information of $u(\bar{X})$, $u(x_0)$ and $g(X, x_0, u)$ to solve the conflict of principles. The non–welfaristic part of our concept is the formulation of the goal function. This leads to the result that the variant of the proportional solution in our model is non–welfaristic. It is also desirable to develop a theory for the agreement after the goals have been defined, which uses not only utility information. One possibility would be to describe norms for concessions by terms that define the sizes of concessions dependent on economic information. These norms, too, may depend on the environment of the bargaining problem. Preferences of the persons then would play the role to select the distributive norms for goals and to select the principles for making concessions.

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Chapter 7

An Axiomatic Characterization of the Normalized Utilitarian Bargaining Solution

7.1 Introduction

In this chapter we provide an axiomatic characterization of a solution for two–person bargaining problems that was proposed by Cao (1981). This solution reflects the utilitarian idea to maximize sums of utilities of the two persons, but in this case without using the assumption of interpersonal comparability of utilities. This solution can therefore be interpreted as a normalized version of the utilitarian solution (c.f. Thomson, 1992). The theoretical rationality principle of maximizing weighted sums of utilities or sums of payoffs can also be observed in bargaining experiments with non–constant sums of payoffs when subjects formulate their bargaining goals. But often the subjects reject this principle as a basis for their bargaining behaviour, because they find it hard to justify. This occurs in situations when maximizing the sums of payoffs for all players would lead to extremely unequal payoffs for the single players. The property of the utilitarian solution not to respect distributive consequences is also a property of the bargaining solution that is the normalized version of the utilitarian solution. This non–attractive feature of the considered solution is the background for one of the axioms we use in our characterization.

7.2 Definitions and Notations

In this chapter, we consider the domain of bargaining problems with two persons, but we formulate the definitions and axioms as far as possible for the more general case of n persons.

Definition 7.1
A pair (S, d) where S is a subset of $\mathbf{R}$ and d is an element of $\mathbb{R}^n$ such that

(i) S is convex and compact,
(ii) $d \in S$,
(iii) there exists a point $x \in S$ such that $x > d$,

is called a *n–person bargaining situation.* $\sum^n$ is the set of all n–person bargaining situations.

S is interpreted as the set of feasible utility n–tupels. d defines the utilities for the persons if they do not reach an agreement. The point d is called disagreement point or status quo. Condition (iii) means that there exists a bargaining incentive for all persons.

In order to define the normalized utilitarian solution we need a reduced domain of bargaining solutions.

Definition 7.2
$\sum_0^n$ is the set of bargaining situations in $\sum^n$ with the following additional properties

(i) $d = 0$,

(ii) $x \geqq 0$ for all $x \in S$,

(iii) if $x \in S$ and $y \in \mathbb{R}^n$ such that $0 \leqq y \leqq x$, then $y \in S$ (comprehensiveness).

In (i) it is assumed that the status quo is 0. If we assume the persons to have cardinal v.Neumann–Morgenstern utility functions without assuming interpersonal comparability of the utilities, this condition is no restriction on the set of problems that can be solved. Condition (ii) requires, that every feasible point is individually rational. Condition (iii) allows free disposals of utilities in the individually rational part of the utility space.

Since in $\sum_0^n$ the status quo is fixed, we will denote a situation $(S, d) = (S, 0) \in \sum_0^n$ only by the set of feasible outcomes S.

Definition 7.3

For a situation S in $\sum_0^n$ we define the *ideal point* $a(S)$ by
$a_i(S) = \max\{x_i \mid (x_1, \ldots, x_n) \in (S, d)\}$ for all $i \in \{1, \ldots, n\}$.

The ideal point consists of the highest feasible utility values of each person in the given situation. In general, the ideal point is not feasible. If it were feasible, then there would be no conflict of interest and the ideal point would be the agreement.

Definition 7.4

A *bargaining solution* F on the domain $\sum^n(\sum_0^n)$ is a mapping $\sum^n(\sum_0^n) \rightarrow \mathbb{R}^n$ that assigns to every situation $(S, d) \in \sum^n(S \in \sum_0^n)$ a point in S.

$F(S, d)$ for $(S, d) \in \sum^n$ can be interpreted as the agreement point in utility space which is chosen, if the situation (S, d) is given. Of course the agreement should be feasible. A bargaining solution "solves" every bargaining problem in the given domain.

Now we consider the two–person case.

Definition 7.5

For each situation S in $\sum_0^2$ the *normalized utilitarian solution* $U(S)$

chooses the point $x \in S$ that maximizes

$$\frac{x_1}{a_1(S)} + \frac{x_2}{a_2(S)} \quad ,$$

if this maximizer is unique, and the middle of the segment of maximizers otherwise.

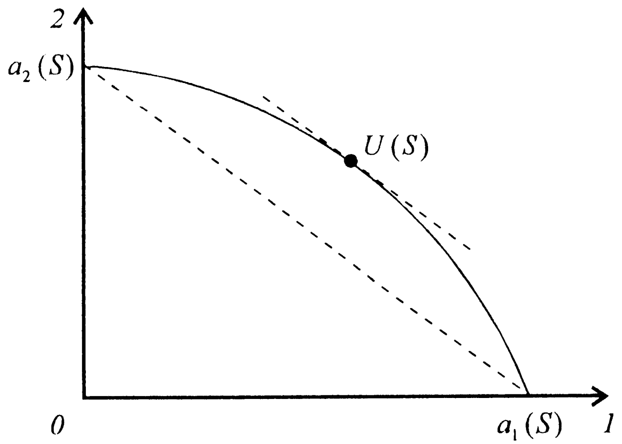

Figure 7.1

7.3 Axioms

We will characterize the normalized utilitarian solution on a slightly restricted domain. We define $\sum_1^2$ to be the set of all bargaining situations $S \in \sum_0^2$ with the additional property that the Pareto optimal boundary of S is strictly concave. In these cases the maximizer in the definition of U is uniquely determined. There cannot be a segment of maximizers. We show the necessity of this restriction later on.

For every pair of positive real numbers $\lambda = (\lambda_1, \lambda_2)$
we define for a given $x \in \mathbb{R}^2$ $\quad \lambda x = (\lambda_1 \cdot x_1, \lambda_2 \cdot x_2)$
and for a given $S \subset \mathbb{R}^2$ $\quad \lambda S = \{y \in \mathbb{R}^2 \mid \exists x \in S \text{ with } y = \lambda x\}$.

Axiom 7.1: Scale Invariance
For all $S \in \sum_1^2$ and all pairs of positive numbers $\lambda = (\lambda_1, \lambda_2)$
$F(\lambda S) = \lambda F(S)$.

The axiom of Scale Invariance means that the chosen agreement of the bargaining problem does not depend on the representation of the v.Neumann–Morgenstern utility functions of the persons. These utility functions are uniquely defined up to positive affine transformations. Since we assume the status quo always to be 0, a positive affine transformation of a utility function in this model is a multiplication by a positive real number.

Axiom 7.2: Weak Pareto Optimality
For all $S \in \sum_1^2$ and for all $x \in \mathbb{R}$,
if $x > F(S)$, then $x \notin S$.

Weak Pareto Optimality requires the solution of a situation S to be an element on the weak Pareto optimal boundary of S.

There are two permutations of the set $\{1,2\}$. One of these is the identical mapping. For every permutation π of $\{1,2\}$ we define
for a given $x \in \mathbb{R}^2$ $\quad \pi x = (x_{\pi(1)}, x_{\pi(2)})$ and
for a given $S \subseteq \mathbb{R}^2$ $\quad \pi S = \{y \in \mathbb{R}^2 \mid \exists x \in S \text{ with } y = \pi x\}$.

Axiom 7.3: Anonymity
For all $S \in \sum_1^2$ and for all permutations π of $\{1,2\}$
$F(\pi S) = \pi F(S)$.

If we change a bargaining situation by renumbering the persons, the outcome of the new situation can be derived from the former outcome by permutating the coordinates in the same way.

It is easy to prove that the axiom of Anonymity implies the axiom of Symmetry.

Axiom 7.4: Symmetry
For all $S \in \sum_1^2$, if S is symmetric, then $F(S)$ is symmetric.

Symmetry of a bargaining solution means that, if it is impossible to distinguish between the persons in a given bargaining situation, then they are treated identically by the outcome.

For a closed set $T \subseteq \mathbb{R}^n$, ∂T denotes the boundary of T.

Axiom 7.5: Weak Perverse Adding
For all $S, T \in \sum_1^2$ with $S \subseteq T$ and $F(S) \in \partial T$,
if $\exists\, i \in \{1, 2\}$ such that $\forall x \in T \setminus S \quad x_i < F_i(S)$, then $F_i(T) \geq F_i(S)$.

On the domain $\sum_1^2$, Weak Perverse Adding is a slightly weaker version of the axiom of Perverse Adding, Thomson and Myerson (1980) consider.

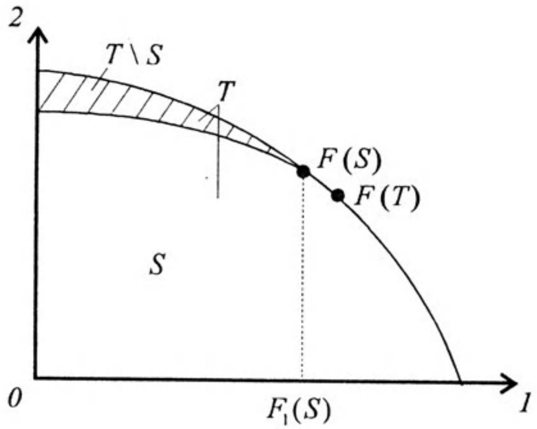

Figure 7.2

Comparing the situations S and T, we see that person i's utility values of the new points are smaller than her utility value of the outcome $F(S)$. The perversion in the axiom of Weak Perverse Adding can be described by the following observation. Though only utility pairs are added to S that have greater utilities for person $j \neq i$ than the agreement in situation S, this will increase the utility of the outcome for person i. Since we also require the solution to fulfill Weak Pareto Optimality, this implies that person j's utility of the agreement decreases in spite of her new possibilities.

7.4 Result

Theorem 7.1
The normalized utilitarian solution is the only solution on $\sum_1^2$ satisfying the axioms of Scale Invariance, Weak Pareto Optimality, Anonymity, and Weak Perverse Adding.

Proof: First we prove that U fulfills the four axioms.

U is scale invariant. If we apply a scale transformation λ to S, the maximization problem in the definition of $U(S)$ on S is equivalent to the problem on λS: x maximizes on S if and only if λx maximizes on λS.

U is weakly Pareto optimal. If there would be a point $x \in S$ with $x > U(S)$, then $U(S)$ would not be a maximizer of the sum in the definition of U.

U fulfills anonymity. Let π be a permutation of the numbers of the persons. Then $x \in S$ maximizes the sum in the definition of $U(S)$ if and only if $\pi x \in \pi S$ maximizes the sum in the definition of $U(\pi S)$.

U fulfills Weak Perverse Adding. If we assume two situations S and T in $\sum_1^2$ with $S \subseteq T$, $F(S) \in \partial T$ and w.l.o.g. $x_1 < U_1(S)$ for all $x \in T \setminus S$, then $a_1(S) = a_1(T)$ and $a_2(S) \leq a_2(T)$. Assume that y is a maximizer of the sum in the definition of $U(T)$. Then we know , that

$$\frac{y_1}{a_1(S)} + \frac{y_2}{a_2(S)} \leq \frac{U_1(S)}{a_1(S)} + \frac{U_2(S)}{a_2(S)}$$

and that

$$\frac{y_1}{a_1(T)} + \frac{y_2}{a_2(T)} \geq \frac{U_1(S)}{a_1(T)} + \frac{U_2(S)}{a_2(T)} .$$

These two inequalities imply

$$y_2 \left(\frac{1}{a_2(S)} - \frac{1}{a_2(T)} \right) \leq U_2(S) \left(\frac{1}{a_2(S)} - \frac{1}{a_2(T)} \right) .$$

If $\frac{1}{a_2(S)} \geq \frac{1}{a_2(T)}$, it follows that $y_2 \leq U_2(S)$ and therefore $y_1 \geq U_1(S)$, since S is comprehensive.

If $a_2(S) = a_2(T)$, $\quad y = U(S)$.

Now we show the uniqueness of the characterization. Let F and G be solutions satisfying the four axioms.

Step 1: Let S be an arbitrary situation in $\sum_1^2$. S has the ideal point $(a_1(S), a_2(S))$. We transform the situation S by linear transformations $t_2(x) = \frac{1}{a_2(S)} \cdot x \; \forall x \in \mathbb{R}$, applied to the first coordinate of points in S and $t_2(x) = \frac{1}{a_2(S)} \cdot x \; \forall x \in \mathbb{R}$, applied to the second coordinate. We denote $(t_1, t_2)(S)$ by S^1. Then $a_1(S^1) = 1$, $a_2(S^1) = 1$, and $F(S^1) = (t_1, t_2)(F(S))$ and $G(S^1) = (t_1, t_2)(G(S))$ by Scale Invariance.

Step 2: For every permutation π of $\{1,2\}$ we define a situation $S^1_\pi := \pi(S^1)$. There are two permutations. One of these is the identity on $\{1,2\}$ which we denote by π_1. Therefore $\pi_1 S^1 = S^1$. We denote $\pi_2 S^1$ by S^2. Anonymity implies $F(S^2) = \pi_2 F(S^1)$.

Step 3: Let $F(S^1)$ be a point (y_1, y_2). We assume w.l.o.g. $y_1 \geq y_2$. We change the situation S^1 by applying a linear transformation r_1 with $r_1(x) = \frac{y_2}{y_1} \cdot x$ for all $x \in \mathbb{R}$ to the first coordinate and change situation S^2 by applying r_2 with $r_2(x) = \frac{y_2}{y_1} \cdot x$ for all $x \in \mathbb{R}$ to the second coordinate.

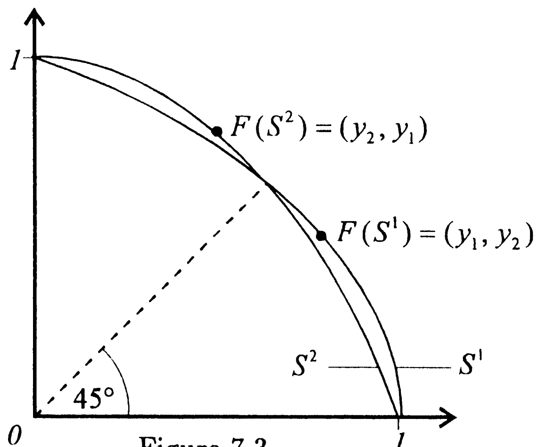

Figure 7.3

These changes lead to the new situations
$T^2 = (r_1, \mathrm{id})(S^1)$ and $T^2 = (\mathrm{id}, r_2)(S^2)$
with $a_1(T^1) = \frac{y_2}{y_1} \leq 1, \quad a_2(T^1) = 1, \quad F(T^1) = (y_2, y_2)$
and $a_1(T^2) = 1, \quad a_2(T^2) = \frac{y_2}{y_1}, \quad F(T^2) = (y_2, y_2)$.

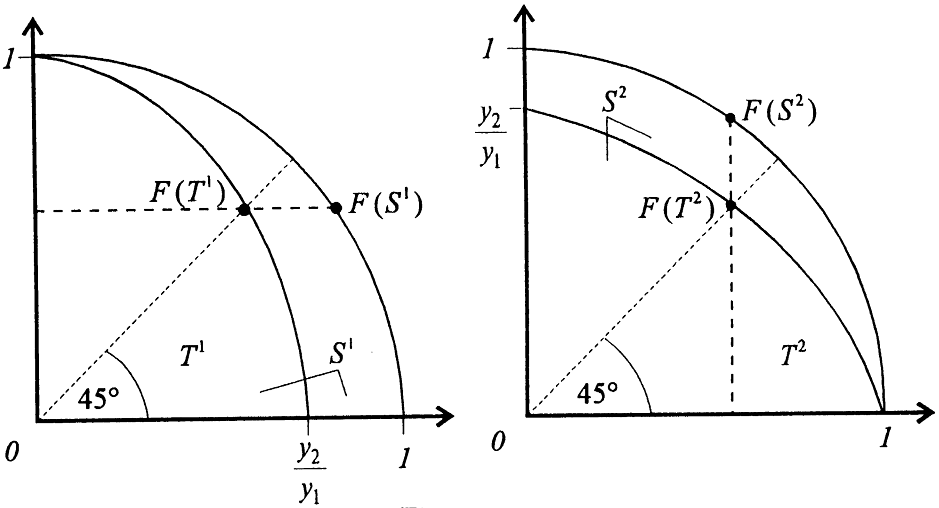

Figure 7.4

Step 4: We take the intersection of the feasible sets T^1 and T^2. $T^1 \cap T^2$ is a symmetric situation in $\sum_1^2$. Weak Pareto Optimality and Symmetry imply $F(T^1 \cap T^2) = (y_2, y_2)$ and $G(T^1 \cap T^2) = (y_2, y_2)$.

Step 5: Let $\overline{r_1}$ denote the inverse transformation to r_1.
Consider $R^1 := (\overline{r_1}, \mathrm{id})(T^1 \cap T^2)$.
Scale Invariance implies $F(R^1) = G(R^1) = (y_1, y_2) = F(S^1)$.

Step 6: We compare the situations R^1 and S^1.
$a_1(R^1) = a_1(S^1) = 1, \quad a_2(R^1) = \frac{y_2}{y_1} \leq a_2(S^1) = 1.$
$R^1 \subseteq S^1$, because $T^1 \cap T^2 \subset T^1$.
If x is an element in $S^1 \setminus R^1$, then $x_1 < G_1(R^1) = y_1$.
Therefore we can apply Weak Perverse Adding to S^1 and R^1.
This implies $G_1(S^1) \geq G_1(R^1) = y_1 = F_1(R^1) = F_1(S^1)$.

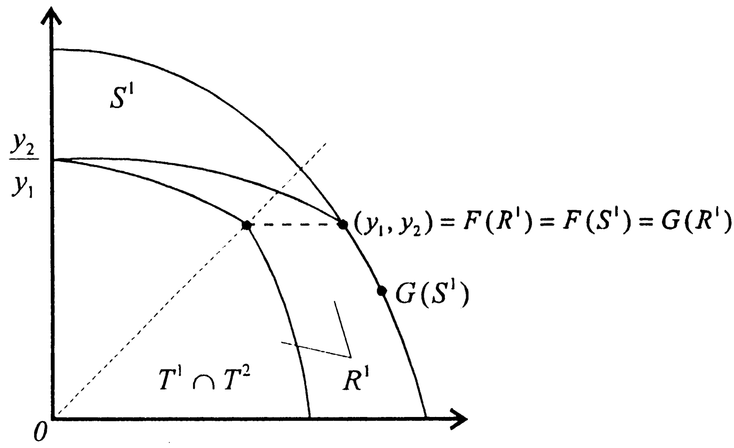

Figure 7.5

Step 7: Now we repeat steps 2 trough 6 with G instead of F using that both solutions have the same properties and that $G_1(S^1) \geq y_1 \geq y_2 \geq G_2(S^1)$.
We can conclude $F_1(S^1) \geq G_1(S^1)$.

Figure 7.6 shows that U does not fulfill Weak Perverse Adding on the domain $\sum_0^2$.

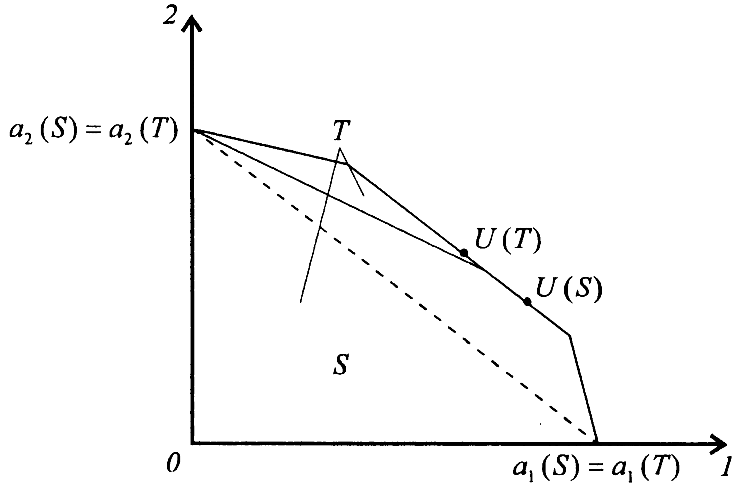

Figure 7.6

Steps 6 and 7 together with step 1 imply $F_1(S) = G_1(S)$ and therefore $F(S) = G(S)$. □

7.5 Concluding Remarks

The solution we characterize in this chapter is defined for bargaining situations with two persons where all feasible points are individually rational, where free disposal of utilities is allowed and the Pareto optimal boundary is strictly concave. Of course, one can raise the question how the normalized utilitarian solution and its characterization can be generalized to the case of n persons or to a larger domain of bargaining situations. In order to answer this question some decisions have to be made. Should the reference point in the general case be the ideal point? Or should the reference point consist of the maximal utility values in the whole feasible set? This might lead to non–individually rational outcomes (c.f. Thomson, 1992). Which point should replace the term "the middle of the segment of maximizers" in the n-person case, if the maximizer is not unique? Perhaps the "center of gravity" is an appropriate candidate. Or do we have to reduce the domain to situations with unique maximizers again? These decisions depend on the type of property of the solution we want to use in a characterization. Those questions lead to some further technical research problems.

The question we would like to deal with is a non technical one. Does the transfer of the utilitarian concept into the bargaining model lead to an appealing bargaining solution? Summing up normalized utility values of persons and maximizing this sum is a collective procedure. The normalizing coefficients capture one aspect of the bargaining positions of the persons. Therefore, some individual information, namely on the ideal utilities, influences the definition of the solution. But the influence of the normalization leads to perverse effects in the bargaining solution. Changing the set of feasible points in the way specified in the axiom of Weak Perverse Adding, may raise the ideal value of a person, but leads to a worse outcome for this person. One can think of a social distributive problem where this redistributive effect is desirable. This might serve as an argument in favor of this solution as a social decision rule. However, it is necessary to have more information on the economic or ethical background of the situation in order to judge the appropriateness of the redistributive norms captured by this solution. From the point of view of a bargaining problem, the effect which is described in the axiom seems to be a non-intuitive property of the solution. Changing the bargaining situation in the described way strengthens the bargaining position of one of the persons. This should lead to a better outcome for this person, and not conversely.

The subjects in the experiments distinguish between these two aspects of the same distributive principle. They say, that maximizing the sum of the payoffs is attractive for the whole group of players. Because they are not allowed to redistribute after they reached an agreement in the game, they analyse the distribution of the payoffs belonging to the "social" maximum. In cases where this distribution is too far away from equal or proportional payoffs, they reject the principle of the maximal sum. The players for which this principle would lead to a good result feel, that it is not applicable because of its effects on the payoffs of their opponents. Therefore, they do not choose their bargaining goals according to this principle.

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Chapter 8

Bargaining When Redistribution is Possible

8.1 Introduction

In many of the bargaining experiments we have run, players discuss the principle to maximize the sum of the payoffs of all agents. In most of the cases the players reject this principle and do not use it to formulate their aspirations. The reason for this is, that in the given sets of feasible payoff allocations this principle would lead to rather unequal payoffs for the players. Often the players express their regret for the fact, that they are not allowed to talk to their opponents in order to choose a common strategy. This strategy would have been to exploit the given situation as far as possible, what means maximization of the sums of the payoffs, and to bargain about a division of this amount afterwards. This procedure would lead to a greater payoff for all agents than a "typical" bargaining agreement would grant them. In our experiments, this procedure is not intended by the rules of the game. Nevertheless, there are groups of players, knowing each other, who apply this strategy. Within the given game, they agree to a payoff allocation that maximizes the sums of the payoffs, though the payoffs for the players are very different. They find it fair to distribute this sum equally among them afterwards.

If we consider the example of several subsidiary companies or departments of a firm having a common set of productive surpluses, then the procedure to maximize the collective gain in a given set of productive possibilities and to redistribute afterwards can be a rational strategy.

Nash (1950, p.161,162) describes a bartering problem with the use of money. The players first choose a barter deal that maximizes the sum of the payoffs in a given set of feasible alternatives. Afterwards they bargain about the division of the total amount. Our model generalizes Nash's representation of the problem. In contrast to Nash, we will not assume that the utility functions of the agents are linear in money. Therefore, the outcome of the procedure will not always be "equal payoffs".

In the following sections we define a new class of bargaining problems where redistribution is possible. We can interpret this model as a description of a new type of economic environment. We investigate how far bargaining solutions, that are defined on domains of regular bargaining situations, can be applied to this model. We will show that, if redistribution is possible, a reasonable solution concept has to involve distributive principles that depend on the game without redistribution and on the game with redistribution. The main part of this paper deals with characterizations of a generalized Kalai–Smorodinsky solution on three different domains of bargaining situations where redistribution is possible.

8.2 Notations, Definitions and Axioms

Definition 8.1
An *n–person bargaining situation* is a triple (S, E, d) with $S, E \subseteq \mathbb{R}^n$, $d \in \mathbb{R}^n$ such that
$d \in S$,
S, E are convex and compact,
$S \subseteq E$,
$\forall x \in E, y \in \mathbb{R}^n$ such that $d \leqq y \leqq x$, $\quad y \in E$ (i.e. E is comprehensive),
$\exists x \in S$ with $x > d$ (bargaining incentive).

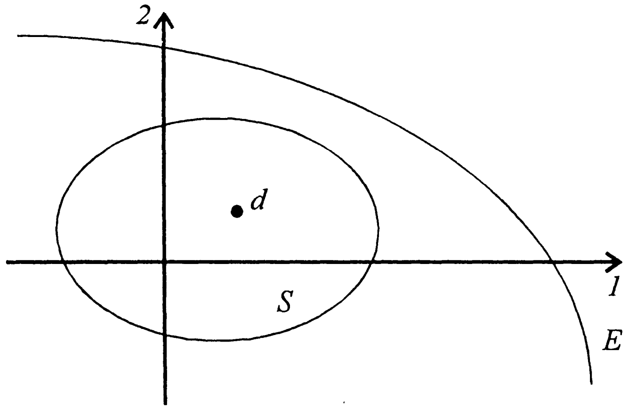

Figure 8.1

S is the set of given alternatives, E is called envelope, d is the disagreement point or status quo in an n–dimensional utility space.

The interpretation of the situation is as follows. The agents apply a collective choice procedure m to the set of given alternatives. This leads to a weakly Pareto optimal outcome x_m in S. Then we allow redistributions of x_m and free disposal of utilities. This leads to a set E_m, such that the point x_m is a point on the boundary of E_m. The set of feasible outcomes E is the convex hull of the union of the sets E_m for all choice procedures, the agents consider to be appropriate. The convex hull is chosen, because the agents have the possibility to choose lotteries between finite numbers of outcomes. The boundary of the envelope E does not necessarily have a common point with the boundary of S.

D^n denotes the set of all n–person bargaining situations. Let A^n be any specifically characterized subset of D^n. We call A^n a domain.

Definition 8.2

A *bargaining solution on the domain* A^n is a mapping
$F : A^n \to \mathbb{R}^n$ such that
$F(S, E, d) \in E$ for all $(S, E, d) \in A^n$.

We define the following axioms for a bargaining solution F on any domain A^n.

Axiom 8.1: Scale Invariance

For all n–tupels λ of positive affine transformations on $\mathbb{R}$ and all situations $(S, E, d) \in A^n$,
if $(\lambda S, \lambda E, \lambda d) \in A^n$, then $F(\lambda S, \lambda E, \lambda d) = \lambda F(S, E, d)$.

If we apply this axiom, it means that we assume the persons to have cardinal v. Neumann–Morgenstern utility functions and that we do not assume interpersonal comparisons of the utilities of the persons.

Axiom 8.2: Symmetry

For all (S, E, d) in A^n, if S, E, and d are symmetric, then $F(S, E, d)$ is symmetric.

This axiom means, that if it is not possible to distinguish between the persons in the situation, the utility value of the bargaining agreement should be equal for all persons.

Axiom 8.3: Weak Pareto Optimality

For all $(S, E, d) \in A^n$, $F(S, E, d) \in \mathrm{WPO}(E)$.

This axiom requires the solution of a given situation to be a weakly Pareto optimal point of the envelope.

Axiom 8.4: Individual Rationality

For all $(S, E, d) \in A^n$, $F(S, E, d) \geqq d$.

Individual rationality means, that the solution of a given situation is weakly preferred to the status quo of this situation by all persons.

Axiom 8.5: Extended Individual Rationality
For all $(S, E, d) \in A^n$, if $(S, S, d) \in A^n$, then $F(S, E, d) \geqq F(S, S, d)$.

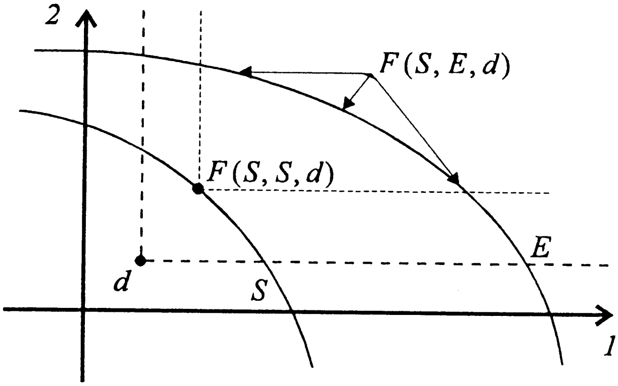

Figure 8.2

Assume that a bargaining situation $(S, E, d) \in A^n$ is given, and that the situation (S, S, d) with the reduced envelope $S \subseteq E$ is also in the domain. Then each person weakly prefers the situation with the envelope E. This means that the redistribution procedure leads to a bargaining outcome that is favorable to all persons in comparison to the bargaining agreement without using a redistribution.

Axiom 8.6: Monotonicity in Envelopes
For all (S, E, d) and (S', E', d') in A^n such that $S = S'$, $d = d'$ and $E \subseteq E'$,
$F(S', E', d') \geqq F(S, E, d)$.

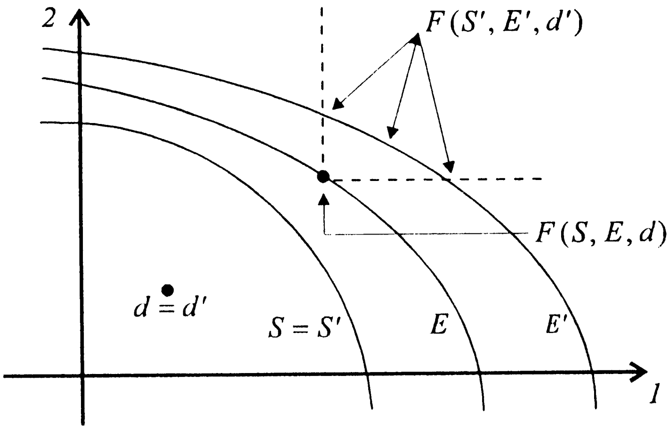

Figure 8.3

If the envelope is enlarged and the set of given alternatives and the status quo remain the same, no agent should receive a lower utility value in the bargaining outcome.

It is easy to see, that Monotonicity in Envelopes implies Extended Individual Rationality, because Extended Individual Rationality deals with the case $E = S$.

Axiom 8.7: Restricted Independence of Extensions of the Set of Alternatives
For all (S, E, d) and $(S', E', d',)$ in A^n,
if $E = E'$, $d = d'$, $S \subseteq S'$, and $a(S, d) = a(S', d')$,
then $F(S, E, d) = F(S', E', d')$.

In this axiom we consider two situations, where the envelope, the status quo, and the ideal point of the regular bargaining situations, defined by set of given alternatives and the status quo, are identical. If we enlarge the set of given alternatives under these restrictions, then the outcome should not change.

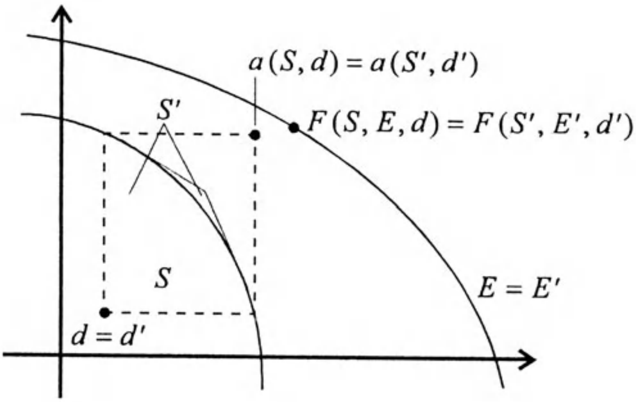

Figure 8.4

Now we consider a smaller domain of bargaining situations than D^n.

Definition 8.3

D_0^n is the set of all situations (S, E, d) in D^n with the additional properties

$d = 0$, $x \geqq 0$ for all $x \in E$, $\mathrm{WPO}(S) \cap \mathrm{WPO}(E) \neq \emptyset$, E is strictly comprehensive, i.e. for any point in the strongly individual rational part of the Pareto frontier of E, there exist utility transfers between every pair of persons.

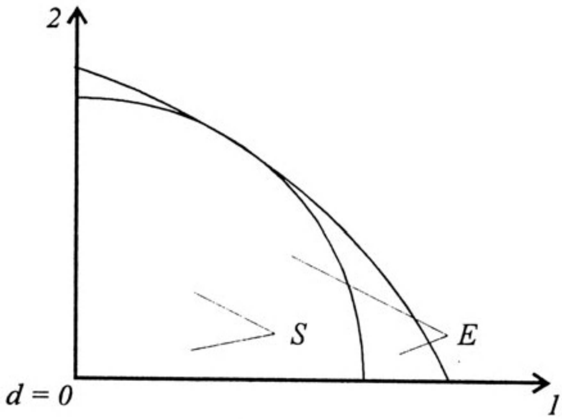

Figure 8.5

Situations in D_0^n can be interpreted as models of certain types of economic decision problems. A set of feasible allocations of a commodity (or money) to the n agents is given. This set is mapped onto the set

S by v. Neumann–Morgenstern utility functions of the persons. The persons choose a commodity allocation y that maximizes the sum of the amounts of the n persons. The allocation y is mapped on a point x_y in S. Afterwards they negotiate about the division of this total amount. All possible divisions of the whole amount or less are also mapped into the utility space. The image is denoted by E. It is assumed that the utility functions are normalized in a way that the image of the status quo is $0 \in \mathbb{R}^n$. If we assume the utility functions to depend only on the amount of the respective person and to be strictly increasing, then S and E contain only individually rational points, and x_y is weakly pareto optimal in S and in E. We can also conclude that E is strictly comprehensive.

Since $d = 0$ for all $(S, E, d) \in D_0^n$, we will denote (S, E, d) by (S, E) on this domain.

For the case of two players we define an axiom, which is a generalization of the axiom of individual monotonicity, being a property of the Kalai–Smorodinsky solution.

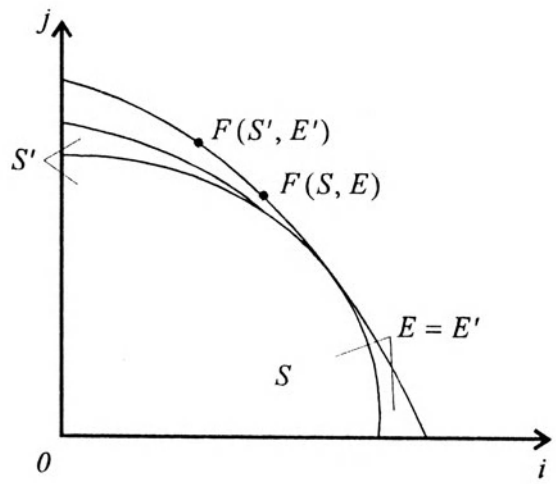

Figure 8.6

Axiom 8.8: Individual Monotonicity
For all (S, E) and (S', E') in D_0^2, if $E = E'$, $S \subseteq S'$, and $a_i(S) = a_i(S')$ for a person $i \in \{1, 2\}$, then $F_j(S', E') \geq F_j(S, E)$ for the person $j \neq i$.

In the assumptions of this axiom, the envelope and the maximal utility value of one person are not changed. Under these restrictions, an extension of the set of given alternatives should not make the other person worse off in the bargaining result.

Lemma 8.1
On the domain D_0^2, the axioms Individual Monotonicity and Weak Pareto Optimality imply Restricted Independence of Extensions of the Set of Alternatives.

Proof: We prove this by applying the axiom of Individual Monotonicity twice, once for each person, to a pair of situations as it is assumed in the axiom of Restricted Independence of Extensions of the Set of Alternatives. This leads to $F_1(S', E') \geq F_1(S, E)$ and $F_2(S', E') \geq F_2(S, E)$ with $E' = E$. Since F fulfills the axiom of Weak Pareto Optimality and all weakly Pareto optimal points in E are strongly Pareto optimal, $F(S', E') = F(S, E)$ holds. $\square$

8.3 Solutions

Let Σ^n denote the regular set of n–person bargaining situations, and let G be a bargaining solution on Σ^n. It is possible to extend G to a solution on the domain D^n.

Definition 8.4
For a given bargaining solution G on Σ^n, we define the *extension* F_G *of* G *on* D^n by
$F_G(S,E,d) = G(E,d)$ for all $(S,E,d) \in D^n$.

If (S,E,d) is a situation in D^n, then (E,d) is a situation in Σ^n. F_G ignores the information provided by the set of given alternatives S. The solution depends only on the envelope and the status quo.

It is obvious, that if G satisfies the axioms of Scale Invariance, Symmetry and Weak Pareto Optimality defined for solutions on the domain Σ^n, then F_G satisfies these axioms defined for solutions on D^n.

Lemma 8.2
Let G be a scale invariant, symmetric, and weakly Pareto optimal solution on Σ^n. Then F_G does not fulfill the axiom of Extended Individual Rationality on D_0^n.

Proof: We construct an n–person bargaining situation in D_0^n, with
$d = 0$,
$S = \text{convhull}(\{(0,\ldots,0),(\frac{2}{2n-1},\ldots\frac{2}{2n-1})\} \cup \{x \in \mathbb{R}^n \mid \exists i \in \{1,\ldots,n\}$ such
that $x_i = 1$ and $x_j = 0$ for all $j \neq i,\ j \in \{1,\ldots,n\}\})$,
$E = \text{convhull}(\{(0,\ldots,0),(2,0,\ldots,0)\} \cup \{x \in \mathbb{R}^n \mid \exists i \in \{2,\ldots,n\}$ such
that $x_i = 1$ and $x_j = 0$ for all $j \neq i,\ j \in \{1,\ldots,n\}\})$.

S is a subset of E, because $(1,0,\ldots,0) \in E$ and $(\frac{2}{2n-1},\ldots,\frac{2}{2n-1}) \in E$. The point $(0,1,0,\ldots,0)$ is in WPO(S) and in WPO(E). Therefore it is easy to see, that (S,S) and (S,E) are in D_0^n. Since S is symmetric, the properties of G imply $F_G(S,S) = (\frac{2}{2n-1},\ldots,\frac{2}{2n-1})$. E is obtained by a scale transformation of person 1 from a symmetric situation.

This transformation is a multiplication by 2. The properties of G imply $F_G(S,E) = (\frac{2}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$. For persons $2, \ldots n$ the outcome in $F_G(S,E)$ is smaller than the outcome in $F_G(S,S)$. □

The lemma implies, that any F_G, where G is scale invariant, symmetric, and weakly Pareto optimal, does not fulfill the axiom of Extended Individual Rationality on D^n. Since Monotonicity in Envelopes implies Extended Individual Rationality, F_G does not fulfill Monotonicity in Envelopes on D_0^n and on D^n. This means, that extensions of the Nash solution or the Kalai–Smorodinsky solution (1975) do not satisfy Extended Individual Rationality or Monotonicity in Envelopes on these domains. A solution fulfilling Extended Individual Rationality and the other three axioms of Scale Invariance, Symmetry and Weak Pareto Optimality on D^n has to depend on S. Therefore, this cannot be a solution, that uses only local information on the boundary of E. In the following definition, we present a solution on D^n which depends on d, S and E.

Definition 8.5

The *generalized Kalai–Smorodinsky solution* K is a solution on D^n such that for a given $(S,E,d) \in D^n$
$K(S,E,d)$ is the point $x \in E$ such that

$$(i) \qquad \forall i,j \in N \qquad \frac{x_i - d_i}{a_i(S,d) - d_i} = \frac{x_j - d_j}{a_j(S,d) - d_j}$$

holds and such that these ratios are maximal in E.

The generalized Kalai–Smorodinsky solution maximizes equal relative utility gains of the n persons in E. The relative utility gain of a person is defined by the ratio of the utility gain in a considered point of E in comparison to the status quo and the maximal utility gain of that person in S in comparison to the status quo. If $E = S$, we receive the Kalai–Smorodinsky solution outcome of the situation (S,d) in Σ^n.

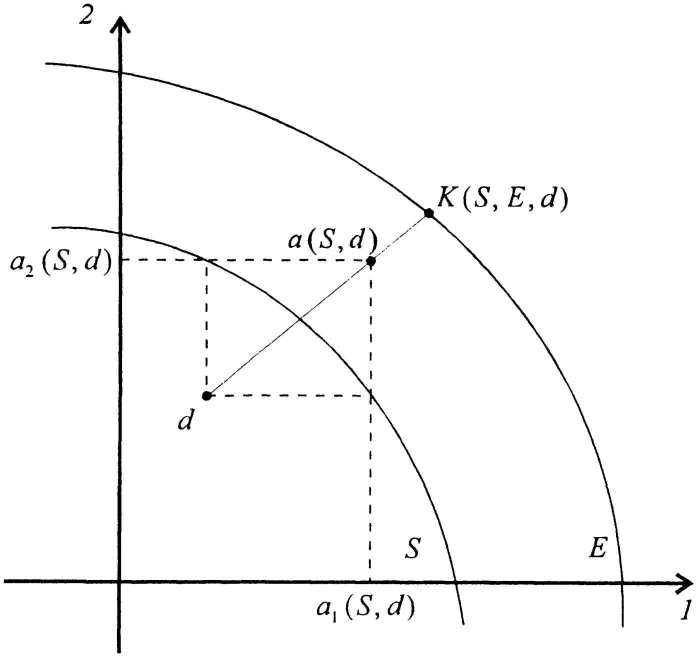

Figure 8.7

Lemma 8.3

The generalized Kalai–Smorodinsky solution fulfills the axioms Scale Invariance, Symmetry, Weak Pareto Optimality, Monotonicity in Envelopes and Restricted Independence of Extensions of the Set of Alternatives.

Proof: Scale Invariance, Symmetry, and Weak Pareto Optimality are directly implied by the definition of K.

In order to show, that K fulfills Monotonicity in Envelopes, we compare situations (S, E, d) and (S', E', d') in D^n with $d = d'$,$S = S'$, and $E \subseteq E'$. Since $d = d'$ and $S = S'$, $a(S, d) = a(S', d')$. This implies, that if for a point $x \in E$ (i) holds for the situation (S, E, d), then $x \in E'$ and (i) holds for the situation (S', E', d') too. Therefore, the maximal values of (i) in (S', E', d') are greater or equal to the maximal values of (i) in (S, E, d). This means, that $K(S', E', d') \geqq K(S, E, d)$.

We now show, that K satisfies Restricted Independence of Extensions of the Set of Alternatives. For situations (S, E, d) and (S', E', d') in D^n with $d = d'$, $E = E'$, $S \subseteq S'$, and $a(S, d) = a(S', d')$ the maximizations of (i) are equivalent, because they are independent of other information on S or S' than the ideal points. □

8.4 A Characterization of the Generalized Kalai–Smorodinsky solution on D^n

Theorem 8.1

The generalized Kalai–Smorodinsky solution K is the unique solution on D^n satisfying the axioms Scale Invariance, Symmetry, Weak Pareto Optimality, Monotonicity in Envelopes and Restricted Independence of Extensions of the Set of Alternatives.

Proof: We know already, that K fulfills the axioms named in the theorem. In the following we prove the uniqueness of the characterization.

Let F be a solution on D^n fulfilling the axioms of the theorem. Let $(S, E, d,)$ be a situation in D^n.

We apply positive affine transformations λ to (S, E, d) such that $\lambda d = (0, \ldots, 0)$ and $a(\lambda S, \lambda d) = (1, \ldots, 1)$. We define $E' = \lambda E$, $S' = \lambda S$, and $d' = \lambda d$.

From $a(S', d') = (1, \ldots, 1)$ and $d' = (0, \ldots, 0)$ it follows, that $K(S', E', d')$ is a point $x \in \mathbb{R}^n$ with $x_i = x_j$ for all $i, j \in N$ such that the components of x are maximal in E'. The axiom of Scale Invariance applied to F and K implies $F(S', E', d') = \lambda F(S, E, d)$ and $K(S', E', d') = \lambda K(S, E, d)$.

We denote the set of all permutations π on N by Π^n and define $\bar{S} = \bigcap \pi S'$ and $\bar{E} = \bigcap \pi E'$ where the intersections are taken for all $\pi \in \Pi^n$.

The situation $(\bar{S}, \bar{E}, d')$ is symmetric and in D^n and therefore Symme-

try and Weak Pareto Optimality imply $F(\bar{S},\bar{E},d') = K(\bar{S},\bar{E},d') = K(S',E',d')$. Monotonicity in Enevelopes and Restricted Independence of Extensions of the Set of Alternatives imply
$F(S',E',d') \geqq F(S',\bar{E},d') = F(\bar{S},\bar{E},d') = K(S',E',d')$.

Case 1: $K(S',E',d') \in \mathrm{PO}(E')$.
In this case it follows, that $F(S',E',d') = K(S',E',d')$.

Case 2: $K(S',E',d') \notin \mathrm{PO}(E')$.
Assume that $F(S',E',d') \geq K(S',E',d')$. $K(S',E',d')$ has identical coordinates. We denote $K(S',E',d')$ by $(x,\ldots,x) \in \mathbb{R}^n_+$. Then there is an $i \in N$ such that $F_i(S',E',d') > x$. We define a point $(y,\ldots,y) \in \mathbb{R}^n_+$ by $y = \frac{1}{2}(F_i(S',E',d') + x)$.
Consider $\tilde{E} = \mathrm{convhull}(E' \cup \{(y,\ldots,y)\})$.
It is easy to see, that $K(S',\tilde{E},d') = (y,\ldots,y)$ and that $(y,\ldots,y)$ is strongly Pareto optimal in $\tilde{E}$.

Therefore, we can apply the result of case 1 to the situation $(S',\tilde{E},d') \in D^n$ and know, that $F(S',\tilde{E},d') = K(S',\tilde{E},d') = (y,\ldots,y)$.

Monotonicity in Envelopes then implies $F(S',\tilde{E},d') \geqq F(S',E',d')$. This is a contradiction to $F_i(S',E',d') > y = F_i(S',\tilde{E},d')$.

This shows, that also in case 2 $F(S',E',d') = K(S',E',d')$ holds. In both cases Scale Invariance implies $K(S,E,d) = F(S,E,d)$. □

8.5 A Characterization of the Generalized Kalai–Smorodinsky Solution on D^n_0

In this section we show, that the generalized Kalai–Smorodinsky solution can uniquely be characterized on the smaller domain D^n_0 by the same set of axioms we used on the domain D^n. In this case, the smaller domain does not lead to a larger number of solutions fulfilling the axioms.

Theorem 8.2
K is the unique solution on D_0^n satisfying the axioms of Scale Invariance, Symmetry, Weak Pareto Optimality, Monotonicity in Envelopes, and Restricted Independence of Extensions of the Set of Alternatives.

Proof: Since K fulfills the axioms on the larger domain D^n, K also fulfills them on the smaller domain D_0^n.

In order to prove the uniqueness of the characterization, let F be a solution on D_0^n fulfilling the axioms and let (S,E) be a situation in D_0^n. We apply an n-tupel of positive affine transformations λ to S and E such that $a(\lambda S) = (1,\dots,1) \in \mathbb{R}^n$. We define $S' = \lambda S$ and $E' = \lambda E$. Then $K(S',E')$ is a symmetric point in E'.

We construct a symmetric situation $(\tilde{S},\tilde{E}) \in D_0^n$ by
$\tilde{S} = \text{convhull}(\{K(S',E'),(0,\dots,0)\} \cup \{(x_1,\dots,x_n) \mid \exists i \in N$ such that $x_i = 1$
and $x_j = 0$ for all $j \neq i, j \in N\})$.
Then $a(\tilde{S}) = (1,\dots,1)$ holds.
$\tilde{E} = \bigcap \pi E'$ where the intersection is taken over all permutations $\pi \in \Pi^n$.
$(\tilde{S},\tilde{E})$ has all the properties of a situation in D_0^n.

If we apply the axiom of Symmetry, we receive $F(\tilde{S},\tilde{E}) = K(\tilde{S},\tilde{E}) = K(S',E')$.

The situation $(\tilde{S},E')$ is an element of D_0^n, because $K(S',E')$ is weakly Pareto optimal in E' and in $\tilde{S}$.

The axiom of Monotonicity in Envelopes implies $F(\tilde{S},E') \geqq K(S',E')$. Since $K(S',E') \in \text{PO}(E')$, $F(\tilde{S},E') = K(S',E')$.

We define $\tilde{S}' = \text{convhull}(\{K(S',E')\} \cup S')$. Then $(\tilde{S}',E')$ is an element of D_0^n. Since $\tilde{S} \subseteq \tilde{S}'$, $S' \subseteq \tilde{S}'$ and $a(\tilde{S}) = a(\tilde{S}') = a(S')$, Restricted Independence of Extensions of the Set of Alternatives implies $F(S',E') = F(\tilde{S}',E') = F(\tilde{S},E') = K(S',E')$.

From the axiom of Scale Invariance, it follows that $F(S,E) = K(S,E)$ holds. □

8.6 A Characterization of the Generalized Kalai–Smorodinsky solution on D_0^2

For the case of two persons we can replace the axiom of Restricted Independence of Extensions of the Set of Alternatives in the characterization of K on D_0^n by the axiom of Individual Monotonicity.

Theorem 8.3
K is the unique solution on D_0^2 satisfying the axioms of Scale Invariance, Symmetry, Weak Pareto Optimality, Monotonicity in Envelopes and Individual Monotonicity.

Proof: First we show, that K satisfies Individual Monotonicity. Let us assume that situations (S, E) and (S', E) in D_0^2 are given with $S \subseteq S'$, $a_1(S') \geq a_1(S)$, and $a_2(S') = a_2(S)$.
This implies $\frac{K_1(S,E)}{a_1(S')} \leq \frac{K_1(S,E)}{a_1(S)} = \frac{K_2(S,E)}{a_2(S)}$.
We also know, that $\frac{K_1(S',E)}{a_1(S')} = \frac{K_2(S',E)}{a_2(S')}$ holds and that this value is maximal in E. If $K_1(S, E)$ would be greater than $K_1(S', E)$, then $K_2(S, E) > K_2(S', E)$ would hold too. This would be a contradiction to the definition of $K(S', E)$.

Since we have already proved, that Weak Pareto Optimality and Individual Monotonicity imply Restricted Independence of Extensions of the Set of Alternatives on the domain D_0^2, the uniqueness part of the proof follows from the uniqueness of the characterization of K on D_0^n. □

8.7 Concluding Remarks

In this chapter we have described a new bargaining model in which the agents are allowed to redistribute economically or socially defined feasible allocations. We have proved that regular bargaining solutions like the Nash solution or the Kalai–Smorodinsky solution do not have the property of Extended Individual Rationality. Applying these solutions to situations that include the alternatives after redistributions may imply that there is a person who does not have an incentive to enter the redistribution process. Since this effect violates a fundamental property of solutions in the new model, it is necessary to develop a solution concept that uses information on both sets, the given set of alternatives and the set including the alternatives after redistribution. We have axiomatically characterized a solution on three domains which is a generalized version of the Kalai–Smorodinsky solution. This concept uses the information on the ideal utility values of the persons in the set of given alternatives to formulate a proportionality principle which is applied to the set of feasible utility vectors after redistribution. For the case of two persons we have given a characterization using an axiom of Individual Monotonicity. This shows how our characterization generalizes the original characterization of Kalai and Smorodinsky (1975) in our model.

The idea of bargaining solutions with goal functions can also be applied to the model with redistribution. If we define goal functions dependent on the given set of alternatives and on the set of alternatives after redistribution this leads to proportional solutions that fulfill the axiom of extended individual rationality. The formal description of this concept, especially the definition of the goal functions are problems of further research.

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Chapter 9

A Combination of Part I and Part II

In this chapter we estimate the planned goals and the agreement payoffs for the bargaining situations of the experiments we presented in Part I. We choose the basic situations 1 through 9 and the situations $1 * 2.5$ through $9 * 2.5$, where the payoff tables are multiplied by the factor 2.5 . The procedure to estimate the variables is derived from the theory of bargaining solutions with goals of Part II.

To every given payoff situation we apply four distributive principles: The equal payoff principle, the two proportionality principles applied to the individually rational part of the payoff table or to the total payoff table, and the allocation that is defined by the Nash solution in payoff space. These four principles lead to four payoff allocations for all players. As the goal of each group we choose the allocation which is the best for this group. Since sometimes the corresponding payoffs are amounts of DM and Pf that would not be named by the players, we do with these theoretical allocations what the subjects use to do: we round them. For the basic situations we round by choosing the smallest interval around this allocation that has amounts of money in full DM or 50 Pf. For the situations with the multiplied payoff, we take the smallest intervall containing the calculated allocation such that the boundaries of the interval are points on the payoff table with assigned payoffs. These are focal payoff vectors. We choose this method because the subjects avoided to calculate the other feasible payoff vectors by interpolation.

In this way we determine the goals of each group in a given situation as a point or an interval defined by the prominence of allocations close to an allocation that is implied by a distributive principle. Then we know the goal point or goal set in the payoff space. The goal may be a goal set, if one or both goals are intervals. In this case, we have to consider all combinations of goals of Group A and B as possible goal points. Goal points usually are not feasible. Since we have no information on utilities we apply the concert of the proportional solution to the goal point and the status quo (0|0) in the payoff space. The result is a payoff vector, if the goal is a point, or it is a set of vectors, if the goals are given by a set. Now we have to apply the same rounding procedure to the payoff point or set to find an interval with prominent boundaries. The point or the interval defines our estimate for the agreement payoffs of the given situation. We call this estimate proportional agreement with best principle goals.

In the following tables, we compare our estimates for goals and agreement payoffs with the average values of the corresponding variables observed in the experiments. The purpose is to find out, whether the estimated values and the average observed values are close to each other. The payoff pairs mean
(payoff of each person in Group$A$ | payoff of each person in Group B).

Situation	1	2	3
Est. Goal A	(8|8)	(8|8)	(9|6)
Ø Goal A	(8.125|7.775)	(8|8)	(10|4)
Est. Goal B	(5|14)-(5.50|13)	(6|12)	(6|12)
Ø Goal B	(4.75|14.50)	(6.375|11.25)	(6|12)
Est. Agreem.	(6|12)-(6.50|11)	(6.50|11)-(7|10)	(7|10)-(7.50|9)
Ø Agreem.	($7.58\bar{3}$|$8.8\bar{3}$)	(6.50|11)	(7.50|9)

Situation	4	5	6
Est. Goal *A*	(9.50|5)	(10|4)	(14|5)-(15|4.50)
Ø Goal *A*	(10.50|3)	(9.50|5)	(12|6)
Est. Goal *B*	(8|8)	(?|6)	(8|8)
Ø Goal *B*	(8|8)	(7.50|6)	(9|7.50)
Est. Agreem.	(8|8)-(8.50|7)	(9.50|5)-(?|6)	(11|6.50)-(12|6)
Ø Agreem.	(8.50|7)	(8.375|6)	(11|6.50)

Situation	7	8	9
Est. Goal *A*	(15|4.50)-(16|4)	(17|3.50)-(16|4)	(18|3)
Ø Goal *A*	(13|5.50)	no data	(13.50|5.25)
Est. Goal *B*	(8|8)	(8|8)	(?|6)
Ø Goal *B*	($8.\bar{3}$|$7.8\bar{3}$)	(8|8)	(?|6)
Est. Agreem.	(11|6.50)-(12|6)	(12|6)-(13|5.50)	(14|5)-(15|4.5)
Ø Agreem.	($11.0\bar{6}$|$6.4\bar{6}$)	(11.50|6.25)	(14|5)

In this table for a given situation "Est. Goal *A*" and "Est. Goal *B*" are the goal points or goal intervals estimated by the procedure described above. These estimates are compared with the average values of planned bargaining goals expressed by the subjects in the games with the respective payoff situation. "Est. Agreem." are the payoff allocations derived from the estimated goals by application of the proportional solution in the payoff space. "Ø Agreem." is the average result of the games with the given situation in our experiments. The question marks "?" in the estimated data of Situations 5 and 9 mean that, dependent on the fact whether the strong Pareto principle is applied or not, different payoffs for Group *A* have to be estimated. If the strong Pareto principle is applicable as a collective norm, we have to estimate the kink of the situation. In the other case the equal payoff principle dominates and ? = 6 holds. Whether the strong Pareto principle in payoffs belongs to the set of applicable norms or not, depends on personality aspects of the players. The "?" in the variable "Ø Goal *B*" of Situation 9 means that these subjects formulate their goal by saying that they want to get 6 DM each. They do not say how much their opponents should get.

We observe that the estimation of goals and agreement payoffs is very good in Situations 1 through 5. In Situations 6 through 9 we overestimate the goals of the stronger group, Group A. Here the goals are derived from the "Prop_T" principle. It seems that the "Prop_IR" principle would fit better for the goals of this group. This means that the stronger group does not use its best principle to formulate the goal. The estimates for the agreement payoffs of these situations are good. Therefore, we feel justified to predict the bargaining outcome with the best principle rule and the proportional solution. One possible explanation could be that the stronger group does not dare to formulate a very demanding goal, but negotiates in the process relatively tough to get a payoff close to the formulated goal.

Now we come to the situations with multiplied payoff tables. The estimated goals and agreements may be different from the estimated values of the standard situations multiplied by 2.5 because of the different rounding procedure.

Situation	$1 * 2.5$	$2 * 2.5$	$3 * 2.5$
Est. Goal A	(20|20)	(20|20)	(22.50|15)
Ø Goal A	(20|20)	(16.25|27.50)	(22.50|15)
Est. Goal B	(12.50|35)-(15|30)	(15|30)	(15|30)
Ø Goal B	(12.50|35)	no data	(15|30)
Est. Agr.	(15|30)-(17.5|25)	(15|30)-(17.5|25)	(17.5|25)-(20|20)
Ø Agreem.	(17.25|25.50)	(15.75|28.50)	(18.75|22.50)

Situation	$4 * 2.5$	$5 * 2.5$	$6 * 2.5$
Est. Goal A	(22.5|15)-(25|10)	(25|10)	(35|12.5)-(40|10)
Ø Goal A	(21.50|17.50)	(23.50|13)	(36.25|11.875)
Est. Goal B	(20|20)	(?|15)	(20|20)
Ø Goal B	(20|20)	(22.50|15)	(28.75|15.625)
Est. Agr.	(20|20)-(22.5|15)	(22|10)-(22.5|15)	(25|17.5)-(30|15)
Ø Agreem.	(20|20)	(22.50|15)	(32|14)

Situation	$7 * 2.5$	$8 * 2.5$	$9 * 2.5$
Est. Goal A	(35|12.5)-(40|10)	(40|10)-(45|7.5)	(45|7.5)
Ø Goal A	(35|12.50)	(40|10)	(43.75|8.125)
Est. Goal B	(20|20)	(20|20)	(?|15)
Ø Goal B	(30|15)	(20|20)	(?|15)
Est. Agr.	(25|17.5)-(30|15)	(30|15)-(35|12.5)	(35|12.5)-(40|10)
Ø Agreem.	(35|12.50)	(36.25|11.875)	(37.50|11.25)

The explanation of "?" is the same as in the former table. We have two possibilites for these goals. However, the estimation of the agreement payoffs does not depend on this decision, because we only have to define the payoff, Group B wants to get for itself.

In general, the estimation fits very well to the average values. In this environment we do not overestimate the goals of Group A in Situations $6 * 2.5$ through $9 * 2.5$. The goals of Group A can be derived from the best principle "Prop_T". There is a difference between our estimates of the goals of Group B and the average values in Situations $6 * 2.5$ and $7 * 2.5$. "Equal" is the best principle for B. Nevertheless, in the experiments there is one game for each of these situations, where Group B does not have this goal. The subjects in these groups pronounce rather low goals for themselves and in addition do not negotiate very toughly. Therefore, in these situations we overestimate the agreement payoffs for Group B.

For the set of situations we use in our experiments, the distributive principles that are applied by the players are determined. The only question is whether the collective principle of strong Pareto optimality should be considered. Envy does not always play a role, but sometimes, dependent on the personality of the players. Since we have no information on personality data, we cannot decide this question. However, the estimation of the proportional agreement with best principle goals is independent of that decision in our situations.

The procedure to estimate agreement payoffs, we propose and apply in this chapter, can be generalized. If the set of principles is known, a player applies to a given payoff situation in a certain economic or ethical environment, we define the goal of that player by selecting his best principle. The set of principles depends on the environment of the bargaining problem and also on the personality of the person. The best principle depends on the preferences of the person, for example if she envies her opponent or not. The estimate defined by the best principle rule has to be corrected by choosing prominent or focal allocations close to the allocation defined by this principle. In the last step the proportional solution has to be applied to the payoff situation, connecting the disagreement point and the goal point or set of goal points. The estimate of the agreement payoff is corrected analogously to the estimates of the goals.

Of course, this estimation does not consider the bargaining process and the sizes of concessions. This part of the estimation is captured by applying the proportional solution. This means that the sizes of the total concession have the same proportion as the payoffs, the players want for themselves. For the situation in our experiments this simple assumption seems to be good enough. It is possible that for more complicated shapes of the set of payoff pairs the negotiation process has to be handled in a different way. This would need a theory of concessions on economic and ethical environments, we mentioned in our outlook on a more general bargaining theory at the end of Chapter 6.

One of the next steps of our further research will be to compare the predictive power of our estimation procedure to that of estimates that are generated by other solutions. This will be possible, when we have the data of enough repetitions of the games for at least some of the situations.

Notations

$\mathbb{N}$	the positive integers
$\mathbb{R}$	the real numbers
$\mathbb{R}_0^+$	the non–negative real numbers
$\mathbb{R}^+$	the positive real numbers
$\mathbb{R}^n$, ${\mathbb{R}_0^+}^n$, ${\mathbb{R}^+}^n$	the n–fold cartesian product $\mathbb{R}$, $\mathbb{R}_0^+$, $\mathbb{R}^+$
$\exists$	there exists
$\forall$	for all
$x \geqq y$	for $x, y \in \mathbb{R}^n$: $x_i \geq y_i$ for all $i \in \{1, \ldots, n\}$
$x \geq y$	for $x, y \in \mathbb{R}^n$: $x \geqq y$ and $x \neq y$
$x > y$	for $x, y \in \mathbb{R}^n$: $x_i > y_i$ for all $i \in \{1, \ldots, n\}$
$A \subseteq B$	A is a subset of B
$A \cup B$	union of A and B
$A \cap B$	intersection of A and B
$A \setminus B$	$\{x \in A \mid x \notin B\}$
∂A for $A \subseteq \mathbb{R}^n$	the boundary of A
convhull(A) for $A \subseteq \mathbb{R}^n$	the convex hull of A
WPO(A) for $A \subseteq \mathbb{R}^n$	the set of weakly Pareto optimal points of A
PO(A) for $A \subseteq \mathbb{R}^n$	the set of strongly Pareto optimal points of A
S^+ for a situation (S, d)	$\{x \in S \mid x \geqq d\}$
$u(\bar{X})^+$ for a situation (X, x_0, u)	$\{x \in u(\bar{X}) \mid x \geq u(x_0)\}$
$\square$	end of the proof

Appendix

Nonparametric Monotonic Regression

Payoff Ratio B/A, Sit. 12345

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
36	1.56	12	12	13.76	-
36	1.0	12	8	8.35	13.28
36	1.0	12	8	8.35	13.28
15	2.8	9.5	13	15.11	-
15	1.0	9.5	8	8.35	13.28
12	1.0	7.5	8	8.35	13.28
12	1.43	7.5	11	12.41	-
8	0.67	5.5	2	0.24	-
8	1.0	5.5	8	8.35	13.28
6	0.8	2.5	5	4.3	7.20
6	0.75	2.5	4	2.95	6.30
6	0.67	2.5	2	0.24	-
6	0.67	2.5	2	0.24	-

X_i :	truncation level of experiment $i = 1, 2, \ldots, 13$
Y_i :	agreement payoff ratio B/A
$R(X_i)$:	rank of X_i
$R(Y_i)$:	rank of Y_i
$\hat{R}(X_i)$:	estimated rank of X_i
$\hat{X}_i$:	estimated truncation level
$\hat{E}(Y\|X = 6) =$	0.74
$\hat{E}(Y\|X = 36) =$	1.39

Nonparametric Monotonic Regression

Planned Bargaining Goal Group A, Sit. 12345

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
36	0.8236	10.5	7	7.0863	13.1726
36	1.0	10.5	10	10.6041	-
36	1.0	10.5	10	10.6041	-
36	1.0	10.5	10	10.6041	-
15	1.0	8	10	10.6041	-
12	0.0	6.5	1	0.0507	-
12	1.0	6.5	10	10.6041	-
8	0.2857	5	2	1.2233	-
6	0.4	2.5	3.5	2.9822	6.38576
6	0.4	2.5	3.5	2.9822	6.38576
6	0.6374	2.5	5	4.7411	7.79288
6	0.6667	2.5	6	5.9137	10.43653

X_i : truncation level of experiment $i = 1, 2, \ldots, 12$

Y_i : bargaining goal of group A in payoff ratios B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 0.3126

$\hat{E}(Y|X = 36) =$ 0.99478

Nonparametric Monotonic Regression

Planned Bargaining Goal Group B, Sit. 12345

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
36	4.0000	12.5	14	14.3827	-
36	2.3636	12.5	11	11.2061	26.9427
36	2.8000	12.5	12	12.2649	34.3543
36	3.3333	12.5	13	13.3238	-
15	2.0	9.5	9	9.0883	14.38245
15	1.2	9.5	7	6.9706	10.9412
12	2.0	7.5	9	9.0883	14.38245
12	2.0	7.5	9	9.0883	14.38245
8	1.0	5.5	4.5	4.3234	7.2156
8	1.0	5.5	4.5	4.3234	7.2156
6	1.0	2.5	4.5	4.3234	7.2156
6	1.0	2.5	4.5	4.3234	7.2156
6	0.6667	2.5	1.5	1.1468	-
6	0.6667	2.5	1.5	1.1468	-

X_i : truncation level of experiment $i = 1, 2, \ldots, 14$

Y_i : bargaining goal of group B in payoff ratios B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 0.8090

$\hat{E}(Y|X = 36) =$ 2.9183

Nonparametric Monotonic Regression

Payoff Ratio B/A, Sit. 12345*2.5

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
36	3.33	9.5	10	10.33	-
36	1.53	9.5	8	8.18	22.14
15	2.8	7.5	9	9.26	33.48
15	1.16	7.5	6	6.03	12.80
12	1.0	5.5	4	3.88	8.76
12	1.43	5.5	7	7.11	14.42
8	1.0	3.5	4	3.88	8.76
8	1.0	3.5	4	3.88	8.76
6	0.67	1.5	1.5	1.19	-
6	0.67	1.5	1.5	1.19	-

X_i : truncation level of experiment $i = 1, 2, \ldots, 10$

Y_i : agreement payoff ratio B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 0.71

$\hat{E}(Y|X = 36) =$ 2.92

Nonparametric Monotonic Regression

Planned Bargaining Goal Group A, Sit. 12345*2.5

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
36	1.0	8.5	6.5	7.1481	21.805
36	1.0	8.5	6.5	7.1481	21.805
15	3.05	6.5	9	10.7282	-
15	1.0	6.5	6.5	7.1481	21.805
12	0.6667	5	3	2.1359	6.6359
8	1.0	3.5	6.5	7.1481	21.805
8	0.6667	3.5	3	2.1359	6.6359
6	0.4490	1.5	1	-	-
6	0.6667	1.5	3	2.1359	6.6359

X_i : truncation level of experiment $i = 1, 2, \ldots, 9$

Y_i : bargaining goal of group A in payoff ratios B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 0.6178

$\hat{E}(Y|X = 36) =$ 1.7742

Nonparametric Monotonic Regression

Planned Bargaining Goal Group B, Sit. 12345*2.5

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
36	4.0	7.5	8	8.1842	-
36	2.0	7.5	6	6.0789	18.9468
12	2.0	5.5	6	6.0789	18.9468
12	2.0	5.5	6	6.0789	18.9468
8	1.0	3.5	3.5	3.4479	7.9473
8	1.0	3.5	3.5	3.4479	7.9473
6	0.6667	1.5	1.5	1.3421	-
6	0.6667	1.5	1.5	1.3421	-

X_i : truncation level of experiment $i = 1, 2, \ldots, 8$

Y_i : bargaining goal of group B in payoff ratios B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 0.6911

$\hat{E}(Y|X = 36) =$ 3.35

Nonparametric Monotonic Regression

Payoff Ratio B/A, Sit. 16789

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
15	0.64	11	8	8.55	12.06
15	1.0	11	11.5	13.34	-
15	1.0	11	11.5	13.34	-
12	0.5	8.5	4	3.07	7.57
12	0.7	8.5	9	9.92	13.70
10	0.59	6	6.5	6.49	10.39
10	0.36	6	2	0.33	-
10	0.96	6	10	11.29	-
8	0.5	3.5	4	3.07	7.57
8	0.59	3.5	6.5	6.49	10.39
6	0.5	1.5	4	3.07	7.57
6	0.25	1.5	1	-	-

X_i :	truncation level of experiment $i = 1, 2, \ldots, 12$
Y_i :	agreement payoff ratio B/A
$R(X_i)$:	rank of X_i
$R(Y_i)$:	rank of Y_i
$\hat{R}(X_i)$:	estimated rank of X_i
$\hat{X}_i$:	estimated truncation level
$\hat{E}(Y\|X = 6) =$	0.42
$\hat{E}(Y\|X = 15) =$	0.90

Nonparametric Monotonic Regression

Planned Bargaining Goal Group A, Sit. 16789

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
15	0.25	8.5	1	17.7150	-
15	0.4231	8.5	6	4.1428	10.6428
15	0.3571	8.5	4.5	8.2144	14.7144
15	0.3	8.5	2.5	13.6433	-
12	0.5	5.5	8.5	-	-
12	0.5	5.5	8.5	-	-
10	0.3571	3.5	4.5	8.2144	14.7144
10	0.5	3.5	8.5	-	-
6	0.5	1.5	8.5	-	-
6	0.3	1.5	2.5	13.6433	-

X_i : truncation level of experiment $i = 1, 2, \ldots, 10$

Y_i : bargaining goal of group A in payoff ratios B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 0.4530

$\hat{E}(Y|X = 15) =$ 0.3541

Nonparametric Monotonic Regression

Planned Bargaining Goal Group B, Sit. 16789

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
15	1.2142	10.5	12	15.5555	-
15	1.0	10.5	8	8.9695	13.4695
15	1.0	10.5	8	8.9695	13.4695
15	1.0	10.5	8	8.9695	13.4695
12	1.0	7.5	8	8.9695	13.4695
12	0.7	7.5	3	0.7378	-
10	1.0	5	8	8.3695	13.4695
10	1.0	5	8	8.9695	13.4695
10	0.8333	5	4	2.3841	7.1788
8	1.0	3	8	8.9695	13.4695
6	0.5	1.5	1.5	-	-
6	0.5	1.5	1.5	-	-

X_i :	truncation level of experiment $i = 1, 2, \ldots, 12$
Y_i :	bargaining goal of group B in payoff ratios B/A
$R(X_i)$:	rank of X_i
$R(Y_i)$:	rank of Y_i
$\hat{R}(X_i)$:	estimated rank of X_i
$\hat{X}_i$:	estimated truncation level
$\hat{E}(Y \mid X = 6) =$	0.7614
$\hat{E}(Y \mid X = 15) =$	1.0498

Nonparametric Monotonic Regression

Payoff Ratio B/A, Sit. 16789*2.5

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
15	0.3	9.5	5	4.27	8.77
15	0.65	9.5	9	14.02	-
12	0.75	7.5	10	16.46	-
12	0.25	7.5	3.5	0.61	-
10	0.5	5.5	7	9.15	14.48
10	0.25	5.5	3.5	0.61	-
8	0.21	3.5	2	-	-
8	0.5	3.5	7	9.15	14.48
6	0.5	1.5	7	9.15	14.48
6	0.17	1.5	1	-	-

X_i : truncation level of experiment $i = 1, 2, \ldots, 10$

Y_i : agreement payoff ratio B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 0.26

$\hat{E}(Y|X = 15) =$ 0.51

Nonparametric Monotonic Regression

Planned Bargaining Goal Group A, Sit. 16789*2.5

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
15	0.25	8.5	4	2.3636	6.8636
15	0.5	8.5	8	12.9093	-
12	0.7	6.5	9	15.5457	-
12	0.1316	6.5	2	-	-
10	0.3571	5	7	10.2729	-
8	0.25	3.5	4	2.3636	6.8636
8	0.25	3.5	4	2.3636	6.8636
6	0.3	1.5	6	7.6364	13.7046
6	0.1	1.5	1	-	-

X_i : truncation level of experiment $i = 1, 2, \ldots, 9$

Y_i : bargaining goal of group A in payoff ratios B/A

$R(X_i)$: rank of X_i

$R(Y_i)$: rank of Y_i

$\hat{R}(X_i)$: estimated rank of X_i

$\hat{X}_i$: estimated truncation level

$\hat{E}(Y|X = 6) =$ 3.6725

$\hat{E}(Y|X = 15) =$ 0.3331

Nonparametric Monotonic Regression

Planned Bargaining Goal Group B, Sit. 16789*2.5

X_i	Y_i	$R(X_i)$	$R(Y_i)$	$\hat{R}(X_i)$	$\hat{X}_i$
15	1.0	8.5	7	10.2729	-
15	1.0	8.5	7	10.2729	-
12	1.0	6.5	7	10.2729	-
12	0.3	6.5	2	-	-
10	1.0	4.5	7	10.2729	-
10	0.25	4.5	1	-	-
8	1.0	3	7	10.2729	-
6	0.5	1.5	3.5	1.0453	-
6	0.5	1.5	3.5	1.0453	-

X_i :	truncation level of experiment $i = 1, 2, \ldots, 9$
Y_i :	bargaining goal of group B in payoff ratios B/A
$R(X_i)$:	rank of X_i
$R(Y_i)$:	rank of Y_i
$\hat{R}(X_i)$:	estimated rank of X_i
$\hat{X}_i$:	estimated truncation level
$\hat{E}(Y\|X = 6) =$	0.5246
$\hat{E}(Y\|X = 15) =$	0.9039

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