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Identifying Students' Mathematical Argumentation Competence in Solving Cubes and Pyramid Problems

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Identifying Students' Mathematical Argumentation Competence in Solving Cubes and Pyramid Problems

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Abstract. This article aimed to describe students' mathematical argumentation competence in solving cubes and pyramid problems. The kind of problem to be solved in this study referred to proving. The students' mathematical arguments were thoroughly analyzed using Toulmin scheme. It used qualitative method. The data was collected by asking 40 students to do argumentation task by thinking out loud strategy. The subject's works would be compared to the problem structure based on the Toulmin scheme. From data analysis, it found three kinds of mathematical arguments the students had made to solve cubes and pyramid problems. Those three mathematical arguments were inductive-incomplete, inductive-complete, and deductive arguments.

1. Introduction

Mathematical problems are classified into two categories including to find and to prove [1]. Solving to prove problems is vital since proof is the core of mathematical thinking [2], [3]. By proving things, students may interpret any symbols and logical statements, as well as relating definitions to theorem, to train their logical thinking.

The proofing is commonly in the form of mathematical statement " $p(x) \Rightarrow q(x)$ ", therefore, problem solver should understand logical implications [4], [5], [6]. When he proves implication truth "if $p(x)$ then $q(x)$ ", he should describe a reason behind his argumentation in order to ensure the truth he had just addressed, in addition to defining the truth "if $p(x)$ then $q(x)$ ". Furthermore, he should also understand the rules of proving the statement.

When solving to-prove problems, the problem solver needs supporting arguments [7]-[14]. Developing such competence of making argumentation is vital for the problem solver to define, reveal, and support reasonable solutions. Through argumentation, he may give descriptions to either support or refuse a premise, standpoint, or idea. When he has argumentation competence, he may correct his solution and action, and undoubtedly leave his hesitancy in solving a problem. He is more independent to select or even propose a reasonable solution.

The arguer (those revealing arguments) constructs his arguments using Toulmin scheme [15] in addition to relating between premises and conclusions to get more complex argument analysis. The scheme consists of data, claim, warrant, backing, rebuttal, and qualifier. Data contains actual facts to support claims. Claim is a statement that refers to the truth. Warrant is a hypothetical proposition to bridge and correct the steps. Backing presents further evidence such as legal basis for the warrant. Rebuttal refers to any exceptions for arguments. Qualifier shows the level of power the data gives to the claim by warrant.



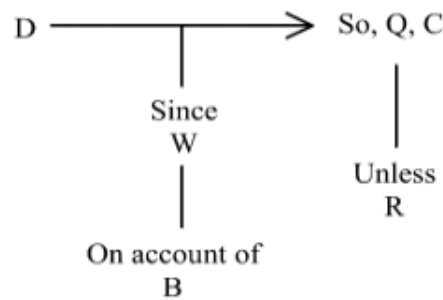


Figure 1. Toulmin Scheme

Arguments may reveal in terms of either dialogue or non-dialogue [16]. For instance, non-dialogue argument refers to problem solving. This activity evokes both self-interaction and self-debate. This non-dialogue argument is fundamental for an individual to show and ensure that his viewpoint is right through self-arguing [17]. Therefore, this study aims to identify students' mathematical argumentation competence in solving cubes and pyramid problems. The students' mathematical argumentation is thoroughly analyzed using Toulmin scheme. Four components including data, backing, warrant, and conclusion are all based on students' written works, while rebuttal and qualifier are seen from the result of interview.

2. Methodology

The subject of this study is The Students of STKIP PGRI Jombang. They were asked to solve argumentation problems individually. The instrument to be used in this study is classified into two categories, including main and supporting instruments. The main instrument refers to the researcher self who acts as the planner, data collector, data analyst, data interpreter, and reporter. The supporting instrument refers to argumentation problem-solving task and guidelines of interview.

2.1. Problem-Solving Task of Cubes and Pyramid

It aims to prove a theorem that shows the correlation between cubes and pyramid. The following presents the problem-solving task of cube and pyramid the students should complete.

Prove the Truth of the Following Statement!

If area of the base of the cube = area of the base of the pyramid and the length of cube = $2 \times$ the height of pyramid, then the volume of the cube = $6 \times$ Pyramid volume

Figure 2. Argumentation Problem-Solving Task

The structure of mathematical argumentation in solving cube and pyramid problem are presented as follow.

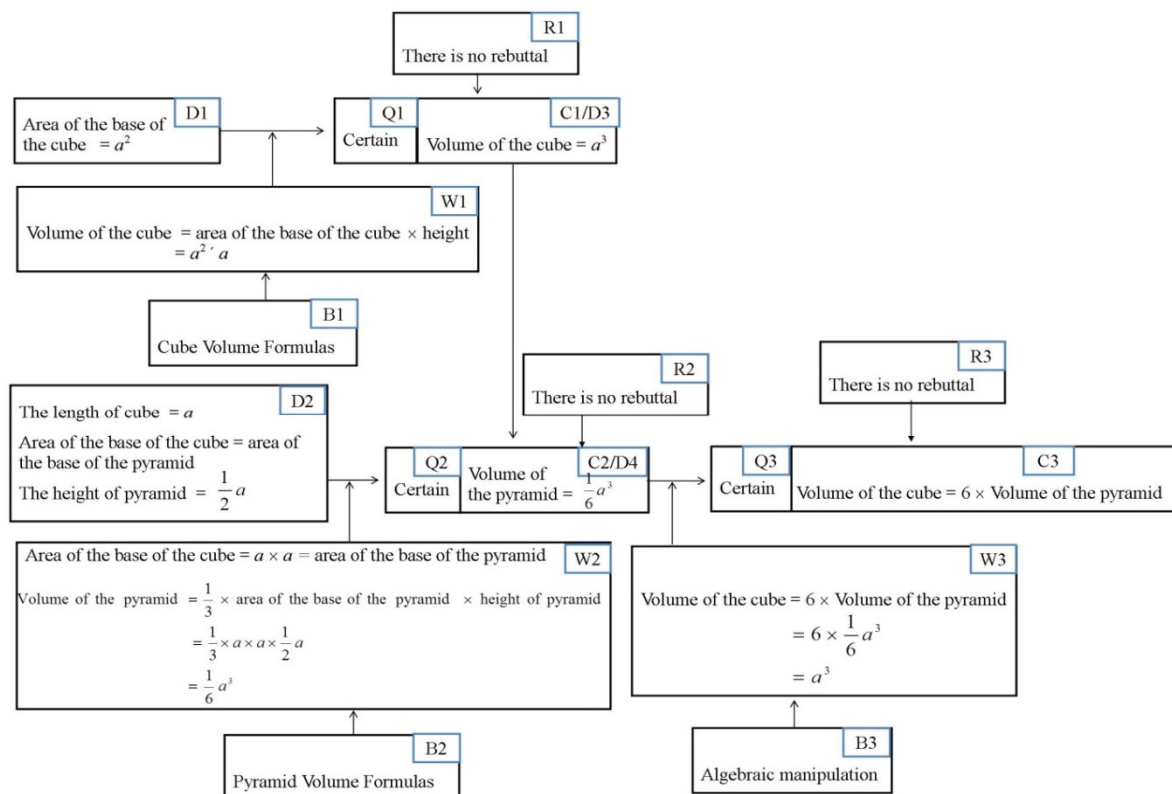


Figure 3. The Problem Structure Based on the Toulmin Scheme

2.2. Guidelines of Interview

The guideline of interview is used as reference of the researcher in interviewing the subject. It aims to confirm the results of think out loud strategy. It applies unstructured interview which questions are adjusted with the students' mathematical arguments. The interview session begins after the students complete the given task. The research process includes: First, the students solve the problem individually through think-out-loud strategy, while the researcher records the process. Second, the researcher has a task-based interview with the students to confirm their mathematical arguments in solving the problem.

In this stage data analysis, the researcher should make (1) transcription on the data collected from think-out-loud strategy and interview, (2) data reduction that involves describing, selecting main aspects, focusing on fundamental aspects, eliminating any unnecessary things, and organizing any raw data, (3) data codification that involves written data collection, data categorization, category labeling with specific terms, (4) description on students' mathematical argumentation competence in solving argumentation (i.e., *to-prove*) problems, and (5) conclusion.

3. Result and Discussion

The result of this study on Students of STKIP PGRI Jombang reveals three kinds of mathematical arguments, including inductive-incomplete, inductive-complete, and deductive arguments. The following describes each of those in more detail.

3.1. Inductive-Incomplete Argument

Inductive-incomplete argument shows that the subject uses particular examples to verify the truth of a statement (inductive argument), and the argument does not show all the components of Toulmin scheme. 20 students are found using inductive-incomplete arguments. An example of student's written work that uses inductive-incomplete argument is as follows.

$L \cdot \text{Kubus} : 16 \text{ cm}^2 = \text{L. alas piramis}$
~~Panjang sisi kubus = 4 cm~~
 $\text{Tinggi piramis} = 2 \text{ cm}$
 $V \cdot \text{Kubus} = \text{L. alas} \times t$
 $= 16 \times 4$
 $= 64$
 $V \cdot \text{Piramis} = \frac{1}{3} \cdot \text{L. alas} \times t$
 $= \frac{1}{3} \cdot 16 \cdot 2$
 $= 10,67$
 $V \cdot \text{Kubus} = 6 \times V \cdot \text{Piramis}$
 $= 6 \times 10,67$
 $= 64,02$
 $= 64$

Figure 4. S1 Written Work

Based on Figure 4 it appears that S1 uses a special example in proving the truth of a statement. S1 assume the area of the base of the cube = 16 cm^2 = area of the base of the pyramid, the length of cube = 4 cm and the height of pyramid = 2 cm. Volume of the cube = area of the base of the cube \times height of cube = $16 \times 4 = 64$. Volume of the pyramid = $\frac{1}{3} \times$ area of the base of the pyramid \times height of pyramid = $\frac{1}{3} \times 16 \times 2 = 10.67$. Finished volume of the cube = $6 \times$ Volume of the pyramid = $6 \times 10.67 = 64$. According to the S1 mathematical argumentation, it can be presented in the Toulmin scheme, as follows.

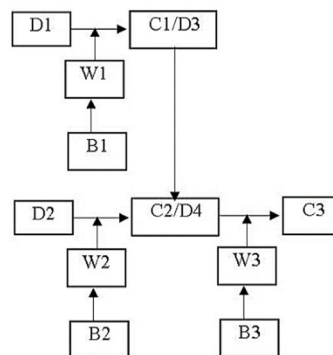


Figure 5. Mathematical Argumentation Scheme of S1

However, the subject fails to correctly reveal the qualifier (Q) of the conclusion on his mathematical argument. Qualifier (Q) should actually be a probability, as conclusion may only apply on specific examples the subject gives, and it may never apply generally. In addition, the subject may not think about rebuttal (R) in his mathematical argument. It is according to the result of interview between the researcher and S1, as follow.

	Qualifier (Q)	Conclusion (C)
P	: What about the conclusion level?	
S1	: It is confirmed that the cube volume = 6 \times pyramid volume as it clearly shown from this way. If we want to take the length of the cube side which is not 6 cm, it confirms that the cube volume = 6 \times pyramid volume	
P	: Is there any rebuttal for this conclusion?	
S1	: I did not think of it	

3.2. Inductive-Complete Argument

Inductive-complete Argument shows that the subject uses particular examples to reveal the truth of a statement (i.e., inductive argument), and the argument contains all the components of Toulmin

Scheme. 15 students are found applying this argument. One example that shows a student's written work with such argument (S2) is presented in the following.

Panjang sisi kubus = 6 cm → tinggi Limas = 3 cm
 Luas alas kubus = $6 \times 6 = 36 \text{ cm}^2$ → L. alas Limas = 36 cm^2
 $V. \text{Kubus} = s^3 = 6^3 = 216$
 $V. \text{Limas} = \frac{1}{3} \cdot \text{L. alas} \times \text{tinggi} = \frac{1}{3} \cdot 36 \times 3 = 36$
 $\therefore \text{Volume kubus} = 6 \times \text{Volume Limas}$

Figure 6. S2 Written Work

Based on Figure 6 it appears that S2 uses a special example in proving the truth of a statement. S2 assume the length of cube = 6 cm, so the height of pyramid = 3 cm. The area of the base of the cube = $s \times s = 36 \text{ cm}^2$, so area of the base of the pyramid = 36 cm^2 . Volume of the cube = $s^3 = 6^3 = 216$. Volume of the pyramid = $\frac{1}{3} \times \text{area of the base of the pyramid} \times \text{height of pyramid} = \frac{1}{3} \times 36 \times 3 = 36$. Finished volume of the cube = $6 \times \text{Volume of the pyramid}$. An interview with a student regarding to his written work (S2) shows this kind of argument, as follow.

		Qualifier (Q)	Conclusion (C)
P	: what about the conclusion level?		
S2	: probably, the cube volume = 6 × pyramid volume since I did not look into the length of the cube side, the height of the pyramid, the base area of the cube, and the base area of the pyramid thoroughly. I just saw that the length of the cube side = 6 cm, and thus, the height of the pyramid = 3 cm, and the base area of the cube = the base area of the pyramid = 36 cm^2		
P	: is there any rebuttal on this conclusion?		
S2	: No, I don't think so , as no matter how much the length of the cube side, the pyramid height, and the base area of those two buildings, the cube volume remains = $6 \times \text{the pyramid volume}$		
		Rebuttal (R)	

According to the S2 mathematical argumentation and from the result of interview between the researcher and S2, it can be presented in the Toulmin scheme, as follows.

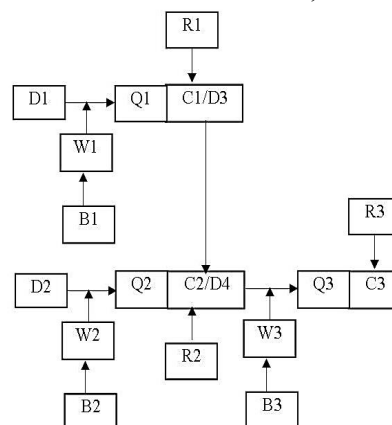


Figure 7. Mathematical Argumentation Scheme of S2

3.3. Deductive-Complete Argument

Deductive-complete argument shows that the subject applies axiom reduction, algebra manipulation, or rebuttal example to reveal the truth of a statement (i.e., deductive argument), and it shows each component of Toulmin Scheme. 5 students are found applying deductive-complete arguments. One example that shows a student's written work with such argument (S3) is presented in the following.

Misal: panjang sisi kubus = a
 L alas kubus = $a^2 = L$ alas Limas
 tinggi Limas = $\frac{1}{2}a$
 $V_{\text{kubus}} = L \text{ alas} \cdot t = a^2 \cdot a = a^3$
 $V_{\text{limas}} = \frac{1}{3} L \text{ alas} \cdot t = \frac{1}{3} a^2 \cdot \frac{1}{2} a = \frac{1}{6} a^3$
 $V_{\text{kubus}} = 6 V_{\text{limas}} = 6 \cdot \frac{1}{6} a^3 = a^3$
 ☺ $V_{\text{kubus}} = 6 \cdot V_{\text{limas}}$

Figure 8. S3 Written Work

Based on Figure 8 it appears that S3 assume the length of cube = a , the area of the base of the cube = a^2 = area of the base of the pyramid, the height of pyramid = $\frac{1}{2}a$. Volume of the cube = area of the base of the cube \times height of cube = $a^2 \times a = a^3$. Volume of the pyramid = $\frac{1}{3} \times$ area of the base of the pyramid \times height of pyramid = $\frac{1}{3} \times a^2 \times \frac{1}{2}a = \frac{1}{6}a^3$. Finished volume of the cube = $6 \times$ Volume of the pyramid = $6 \times \frac{1}{6}a^3 = a^3$. An interview with a student regarding to his written work (S3) shows this kind of argument, as follow.

	Qualifier (Q)	Conclusion (C)
P	: what about the conclusion level?	
S2	: it is confirmed that the cube volume = $6 \times$ the pyramid volume as I showed the length of the cube side, the pyramid height, and the base area of both buildings generally, which is a for instance, I also used the formula of volume and algebraic operation to solve the problem.	
P	: Is there any rebuttal for this conclusion?	
S2	: No, there is no, as the conclusion has already applied generally	
	Rebuttal (R)	

According to figure 8 and from the interview between the research and S3, it can be presented in Toulmin Scheme, as follow.

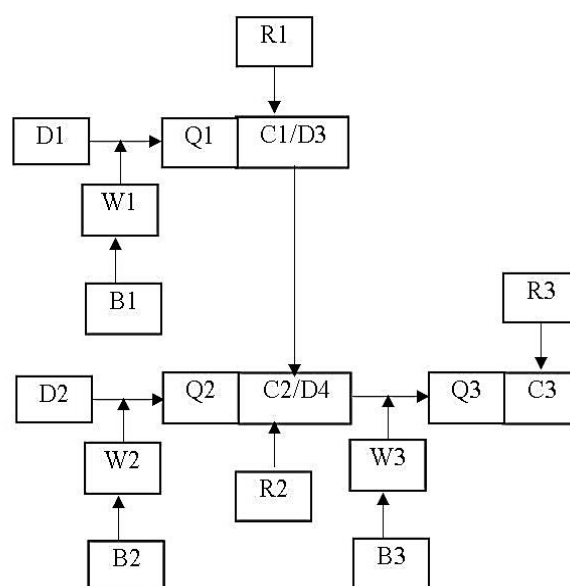


Figure 9. Mathematical Argumentation Scheme of S3

At the college level, students should use deductive arguments. But the results of this study indicate that there are students who still use non-deductive arguments (ie inductive arguments). This is in accordance with the results of research [18], [19]. Therefore it is necessary to have an evaluation in the learning process. One that must be evaluated is the learning model. The applied learning model must develop students' mathematical argumentation skills.

Inductive-incomplete argument shows that the students have less capability to reveal the qualifier (Q) and rebuttal (R). They did not see that the conclusion they made inductively was not absolute in nature. Hence, *qualifier* (Q) is fundamental to thoroughly see how mathematicians contended [19]. The students who use inductive arguments show that they have no capability to construct a valid scheme of argument. It is similar to [20-24] that students are unable to distinguish between valid and invalid evidence. Those with inductive arguments have less competence to formally construct evidence/proof, and thus, they do not conduct any evidence validity process. Another factor is that they get confused to begin and have no clue to construct evidence in formal way [25].

Furthermore, there are some implications of inductive arguments (either incomplete or complete one) to students. One of those is making them difficult to manipulate mathematical expression that is equal to another form. They have verbal knowledge, but they cannot write in formal symbols. Such condition makes discrepancies on arguments that make them fail constructing formal evidence.

Finding the process of mathematical argumentation is very fundamental for students, as it may give them pictures about the process of constructing success and failure in proving things. Additionally, it is also helpful to give them guidelines on how evidence/proof should be taught in such a way and how concepts should be constructed in their mind. The scheme of inductive argument indicates that students learn neither mathematical concept nor deductive argument yet. Or, otherwise, they have already learned it but the concept is not that meaningful for them to be embedded and applied. Therefore, in developing their students' mathematical argumentation competence, lecturers should shift the scheme from inductive to deductive one [27]. In developing students' mathematical argument skills, lecturers can apply the infusion learning strategy [28]. Three types of mathematical arguments made by students to solve cube and pyramid problems include inductive-incomplete, inductive-complete, and deductive. Apart from the three types of mathematical arguments, there may be other types of arguments that are related to the intuitive structural as well as the research results [20], [29].

4. Conclusion

There are three kinds of arguments. Those are inductive-incomplete, inductive-complete, and deductive arguments. Inductive-incomplete argument shows that the subject uses particular examples

to verify the truth of a statement (inductive argument), however, the argument does not show all the components of Toulmin scheme. Inductive-complete Argument shows that the subject uses particular examples to reveal the truth of a statement (i.e., inductive argument), and the argument contains all the components of Toulmin Scheme. Finally, Deductive-complete argument shows that the subject applies axiom reduction, algebra manipulation, or rebuttal examples to reveal the truth of a statement (i.e., deductive argument), and it shows each component of Toulmin Scheme.

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References

- [1] G. Polya, "How to Solve it, 2nd ed". Amerika: Princeton University Press. ISBN 0-691-08097-6. 1973
- [2] G. Hanna, "Proof, Explanation and Exploration: an Overview". *Educational Studies in Mathematics*, 44, 5-23. 2000.
- [3] M. Kamaruddin, N. Kamariah, & Z. Md Amin, "Dilemma in Teaching Mathematics". *Journal for Research in Mathematics Education*, 19, 371–384. US-China Education Review B 2 (2012) 145-149. Earlier title: US-China Education Review, ISSN 1548-6613. 2012.
- [4] V. Deloustal-Jorrand, "Implication and Mathematical Reasoning". *26th Conference for the Psychology of Mathematics Education* (Vol. 2, pp. 2-281). Norwich. 2002.
- [5] D. Küchemann, & C. Hoyles, "Students' Understanding of a Logical Implication and its Converse". In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th International Conference on the Psychology of Mathematics Education* (Vol. 3, pp. 3-241). Norwich, UK: IGPM. 2002.
- [6] M.M. Rodd, "On Mathematical Warrants: Proof Does Not Always Warrant, and a Warrant May Be Other Than a Proof". *Mathematical Thinking and Learning*, 2:3, 221-244. 2000.
- [7] B. Cerbin, "The Nature and Development of Informal Reasoning Skills in College Students". *ERIC Document Reproduction Service* No. ED 298 805. 1988.
- [8] G. Krummheuer, "The Narrative Character of Argumentative Mathematics Classroom Interaction in Primary Education". *European Research in Mathematics Education I: Group 4*, pp. 331-341. 1999.
- [9] J.M.P. Alexandre, P. Munoz, Cristina, A. Cuadrado, and Virginia. "Expertise, Argumentation and Scientific Practice: A Case Study about Environmental Education in the 11th Grade". *The National Association for Research in Science Teaching (NARST) annual meeting, New Orleans, April-May 2000*.
- [10] K.L. Cho, & D.H. Jonassen, "The Effects of Argumentation Scaffolds on Argumentation and Problem Solving". *ETR&D*, Vol. 50, No. 3, 2002, pp. 5–22 ISSN 1042–1629. 2002.
- [11] C. Hoyles, & D. Küchemann, "Students' Understandings of Logical Implication". *Educational Studies in Mathematics*, 51(3), 193-223. 2002.
- [13] D. Kuhn, & W. Udell. "The Development of Argument Skills". *Child Development*, 74(5), 1245-1260. 2003.
- [14] H.B. Verheij, "Evaluating Arguments Based on Toulmin's Scheme". *Argumentation* (2005) 19: 347–371. Springer 2006. DOI 10.1007/s10503-005-4421-z. 2005.
- [15] L.B. Trisanti, A. Sutawidjaja, A.R. As' ari, & M. Muskar. "The Construction of Deductive Warrant Derived from Inductive Warrant in Preservice-Teacher Mathematical Argumentations". *Educational Research and Reviews*, 11(17), 1696-1708. 2016.
- [16] S. Toulmin. *The Uses of Argument*. UK: Cambridge University Press. 2003
- [17] N.D. Walton, "What is Reasoning? What is an Argument?". *The Journal of Philosophy*. Vol. 87, Issue 8 (Aug., 1990), 399-419. 1990.
- [18] L.B. Trisanti, "The process of thinking by prospective teachers of mathematics in making

- arguments". *Journal of Education and Learning (EduLearn)*, 13 (1), 17-24. 2019.
- [19] L.B. Trisanti, A. Sutawidjaja, A.R. As' ari, & M. Muskar. "Types of Warrant in Mathematical Argumentations of Prospective-Teacher". *International Journal of Science and Engineering Investigations*. 6 (68), 96-101. 2017.
 - [20] M. Inglis, J.P. Mejia-Ramos, & A. Simpson, "Modelling Mathematical Argumentation: The Importance of Qualification". *Educational Studies in Mathematics*, 66(1), 3-21. DOI 10.1007/s10649-006-9059-8. Published online: 5 April 2007 © Springer Science + Business Media B.V. 2007.
 - [21] A. Selden, & J. Selden, "Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem?". *Journal for research in mathematics education*, 34(1), 4-36. 2003.
 - [22] K. Weber, "Mathematics majors' perceptions of conviction, validity, and proof". *Mathematical thinking and learning*, 12(4), 306-336. 2010.
 - [23] M. Inglis, & L. Alcock, "Expert and novice approaches to reading mathematical proofs". *Journal for Research in Mathematics Education*, 43(4), 358-390. 2012.
 - [24] M. Hodds, L. Alcock, & M. Inglis, "Self-explanation training improves proof comprehension". *Journal for Research in Mathematics Education*, 45(1), 62-101. 2014.
 - [25] Fuat., T. Nusantara, E. Hidayanto, & S. Irawati, "The Exploration Of Argument Scheme Expression In Students' Proof Construction". *International Journal Of Scientific & Technology Research*, 9 (01), 2369–2372. 2020.
 - [26] L. Alcock, & K. Weber, "Referential and syntactic approaches to proving: Case studies from a transition-to-proof course". *Research in collegiate mathematics education VII*, vol. 16, 93-114. 2010.
 - [27] G. Harel, & L. Sowder, "Students' Proof Schemes: Results from Exploratory Studies". *CBMS Issue in Mathematics Education*. Volume 7. 234-283. 1998
 - [28] L.B. Trisanti, L. B., & T. Nusantara, "Improving Students' Mathematical Argumentation Skill Through Infusion Learning Strategy". In *Journal of Physics: Conference Series*, Vol. 1783, No. 1, p. 012103. IOP Publishing. 2021.
 - [29] L.B. Trisanti, A. Sutawidjaja, A. R. As'ari, & M. Muksar, "Modelling student mathematical argumentation with structural-intuitive and deductive warrant to solve mathematics problem". *Proceeding of ICERD*, 130-139. 2015.