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The aspects of reversible thinking in solving algebraic problems by an elementary student winning National Olympiad medals in science

Karya :

(1) Syarifatul Maf'ulah;
(2) Dwi Juniati;
(3) Tatag Yuli Eko Siswono

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Results	Query	Domains (original links)
Unique	Hence, Lamon asked researchers in the education field to study students' reversibility	-
Unique	In fact, most primary graders found themselves having difficulties to solve arithmetical problems [4]	-
1 results	Effective algebraic thinking sometimes involves reversibility (i	insidemathematics.org
Unique	being able to undo mathematical processes, as well as to do them)	-
Unique	Algebra taught to elementary students was termed by Kaput as early-algebra [8]	-
3 results	This could be seen from the strategy the subject used in making the equation	researchgate.net researchgate.net wiete.com.au
Unique	Reciprocity was identified when the subject squared the both sides of the equation	-
Unique	190 reversible thinking in algebra	-
Unique	The focus of the study mentioned earlier was on algebra taught to elementary students	-
Unique	The algebra taught to them was referred to by the term early-algebra [8]	-
Unique	They could represent unknown elements, identify quantities that vary and generalise the properties	-
Unique	Two equations were defined as equivalent, if both had similar solutions	-
Unique	This study selected an elementary student who had won national Olympiad medals in science	-
Unique	in this phase, the researchers examined the theories of reversible thinking	-

Unique	in this phase, the researchers selected the subject for this study	-
Unique	in this phase, the researchers conducted data analysis and wrote a report	-
Unique	Analysis was conducted after the interview had ended	-
13,200 results	The test provided for the subject is presented in Figure	dtic.mil gpo.gov instagram.com researchgate.net ufdc.ufl.edu ascelibrary.org ascelibrary.org dtic.mil patents.google.com google.com
1 results	The intended form of the test was an equation	wiete.com.au
Unique	Figure 2: One of the equations made by the subject	-
Unique	SJ1 : Yes, the initial form is 24 added to a equals 16	-
Unique	Then, I divide both sides of this initial form by	-
Unique	The result is 24 added to a divided by 4 equals	-
Unique	P2 : Would you please write it down	-
Unique	SJ2 : Of course (and, then, writing down the following form): , then	-
Unique	(and writing down the following): , then	-
Unique	P7 : Why it is not allowed for both sides to be different	-
Unique	SJ8 : There is an equal sign (what the subject indicated was …=…)	-
Unique	Table 3: Aspects of reversible thinking in making equations by the subject	-
Unique	Reciprocity, since the subject used compensation, squaring both sides of	-
Unique	Reciprocity, since the subject used compensation, adding the both sides of with	-
Unique	But, would you please first, write down the new procedures before giving some explanation	-
Unique	SJ9 : Yes, of course (and, then, the subject wrote the following)	-
Unique	P10 : Now, would you please explain it to me	-
Unique	The result was 4 (what he intended was	-
Unique	Table 4: Aspects of reversible thinking in reversing equations by the subject	-
Unique	- LK6 : Multiplying both sides of by 4, which resulted in	-

Unique	Reciprocity, since the subject used compensation, multiplying both sides of by	-
Unique	Those were on the steps with codes LM2, LM4 and LM6	-
Unique	Those were the steps with codes LK2, LK4 and LK6	-
1,420 results	In order to facilitate understanding, these codes are described in Table	patents.google.com isca-speech.org thefreelibrary.com onlinelibrary.wiley.com docplayer.net researchgate.net scribd.com pubs.rsc.org federalregister.gov epdf.tips
Unique	LK2 Moving 8 from one side to the other side	-
Unique	LM4 Squaring both sides of the equation	-
Unique	LK4 Squaring both sides of the equation	-
Unique	LM6 Adding 8 to both sides of the initial equation	-
Unique	LK6 Multiplying both sides of the new equation by	-
Unique	The subject fully understood the equal sign (=)	-
Unique	Students often defined the equal sign (=) as the context of an answer	-
Unique	They seldom defined it as a link of two sides (right and left sides)	-
4 results	When making an equation, the only aspect of reversible thinking identified was reciprocity	researchgate.net researchgate.net
Unique	This could be seen from the strategy the subject used in making the equation	-
Unique	and Piaget, J., The Growth of Logical Thinking from Childhood to Adolescence	-
Unique	New York: Basic Books, 4-11, 272-274 (1958)	-
Unique	J., Rational Numbers and Proportional Reasoning: towards a Theoretical Framework for Research	-
Unique	Charlotte, NC: Information Age Publishing, 629-667 (2007)	-
Unique	A., The Psychology of Mathematical Abilities in School Children	-
Unique	Chicago: The University of Chicago Press, 287-289 (1976)	-
Unique	E., Pupils ’ error on the concept of reversibility in solving arithmetic problems	-
Unique	Educational Research and Reviews, 18, 11, 1775-1784 (2016)	-
Unique	and Olive, J., Reversibility of thought: an instance in multiplicative tasks	-

Unique	of Mathematical Behavior, 27, 138-151 (2008)	-
Unique	Greenes, C., Algebra: It’s Elementary (2004), 7 August 2016, www	-
Unique	M., Developing Algebra-’Rithmetic in the Elementary Grades (2007), 7 August 2016, www	-
Unique	Kaput, J., Teaching and Learning a New Algebra with Understanding	-
Unique	Dartmouth, MA: NCISLA, 34-36 (1998)	-
Unique	E., Educational Psychology: Theory and Practice	-
Unique	Boston: Allyn &Bacon, 43-51 (2006)	-
Unique	Adi, H., Intellectual development and reversibility of thought in equation solving	-
Unique	for Research in Mathematics Educ., 3, 9, 204-213 (1978)	-
Unique	J., Students’ reasoning with reversible multiplicative relationships	-
Unique	Cognition and Instruction, 4, 28, 383-432 (2010)	-
Unique	A., Problem Solving As a Goal, Process, and Basic Skill	-
Unique	(Ed), Problem Solving in School Mathematics	-
Unique	Reston, VA: NCTM, 103-108 (1980)	-
Unique	(2nd Edn), Princeton: Princeton University Press, 80-83 (1973)	-
Unique	R., Equations and the equal sign in elementary mathematics textbooks	-
Unique	of Educational Psychology, 4, 112, 627-648 (2012)	-
Unique	K., Qualitative Research for Education: An Introduction to Theory and Methods	-
Unique	Boston: Allyn & Bacon, 96-97 (1998)	-
2 results	E., Middle- school students’ understanding of the equal sign: the books they read can’t help	link.springer.com link.springer.com
Unique	Cognition and Instruction, 3, 24, 367-385 (2006)	-
Unique	aged seven up to 11 years of age was the development of a capacity for	-
Unique	The researchers were also motivated by Lamon [2] and that the attention on reversibility	-
Unique	Krutetskii identified the mathematics skill related to the successfulness in solving problems, that is,	-

Unique	of problems in mathematics [1], meaning that reversibility could be considered to be a primary	-
Unique	Ramful stated that reversibility was related to mathematical operations, fractions, comparisons, algebra, and some	-
Unique	a goal, but also to be able to understand the process well enough to work	-
Unique	This definition indicated that it involved reversibility in solving algebraic problems, that is, an	-
Unique	Reversibility did not merely involve a process of achieving objectives, but also a process	-
Unique	of achieving objectives or a result, and a process of reversing the objectives or result	-
Unique	As Greenes said, algebra is sometimes referred to as generalised arithmetic, because it formalises	-
Unique	Its power lies in the ways it allows one to represent relationships between quantities,	-
Unique	Algebra provides rules for manipulating symbols or signs, such as simplifying an expression and,	-
Unique	Suh stated the importance of algebra for elementary students, and asserting on algebra-arithmetic could	-
Unique	also helped to develop their mathematics concepts in a deeper and more complex manner since	-
Unique	to grow in a concrete operational phase in children from seven up to 11 years	-
Unique	b) the student who won National Olympiad medals in science was an asset of	-
14 results	thinking in solving algebraic problems by an elementary student winning National Olympiad medals in science	researchgate.net researchgate.net wiete.com.au wiete.com.au erfanyudianto.com mafiadoc.com mafiadoc.com erfanyudianto.com
Unique	Indonesia‡ ABSTRACT: This study aimed to identify the aspects of reversible thinking of an elementary	-
Unique	In this qualitative research, data were collected by a reversible thinking task contained	-
Unique	The results showed that when making an equation, the only aspect of reversible thinking	-
Unique	It was done by dividing both sides of the equation by the same element,	-
Unique	When reversing the new equation to its starting point, the aspects of reversible thinking	-
Unique	It could be seen from the strategy the subject used in reversing the equation	-
Unique	in this case, the subject used a subtraction operation, which was the inversion of addition	-
Unique	Therefore, this study aimed to identify the aspects of reversible thinking in solving particular	-
Unique	REVERSIBLE THINKING IN SOLVING ALGEBRAIC PROBLEMS There were two important matters within reversible thinking,	-

Unique	goal, whereas, reverse was a mental process from the expected goal moving to its starting		-
1 results	The aspects of reversible thinking could be identified as follow: Table 1: The aspects	wiete.com.au	
Unique	Aspect of reversible thinking Explanation Negation It was when a subject used inversion towards		-
Unique	Reciprocity It was when a subject used compensation or any other equivalent relationships with		-
Unique	Capability to return to the initial data after obtaining the result It was when		-
Unique	Solving the problem was an activity of seeking solution for particular situation, using the		-
Unique	it also helped in developing the conception of students' mathematics in a deeper and more		-
Unique	Greenes stated that in teaching mathematics for elementary graders, variables were used in three		-
Unique	(=) was used to show the equivalency between a number or expression on the left-hand		-
Unique	The expression itself was defined as a combination of operant numbers and arithmetic operations		-
Unique	The problem of this study consisted of an initial equation containing one variable as		-
Unique	The instruction provided was that the subject was asked to make as many equations		-
Unique	make as many equivalent equations as possible with its initial as a solution of the		-
Unique	To identify the student's reversible thinking in solving algebraic problems, this study applied		-
Unique	Then, the student was asked to make other equivalent equations based on the initial		-
Unique	Thus, the indicators of reversible thinking that could be identified in algebra are included		-
Unique	Table 2: Indicators of the aspects of reversible thinking that could be identified in		-
Unique	the subject made other equations equivalent with its initial) Negation When the subject used inversion		-
1 results	Reciprocity When the subject used compensation or any other relationships equivalent with a given	wiete.com.au	
Unique	the initial one) Negation When the subject used inversion towards the related operation in his		-
Unique	Reciprocity When the subject used compensation or any other relationships equivalent with a given		-
Unique	Capability to return to initial data after obtaining the result When the subject could		-
Unique	METHOD Bogdan and Biklen explained the characteristics of qualitative research, and the present study		-
Unique	These are: a) naturalistic in nature, since it was conducted using the real situation		-

Unique	b) descriptive, since the data collected were qualitative, such as a set of words		-
Unique	and c) inductive, since it did not aim to prove any hypothesis, but merely		-
Unique	The researchers provided a test for the subject, and conducted an interview later to		-
Unique	191 The procedures conducted in this study consisted of three primary phases as follows:		-
Unique	Subsequently, the researchers gave a test to the subject and, then, conducted an interview		-
Unique	Subsequently, the researchers analysed the data entirely referring to the framework of reversible thinking,		-
Unique	RESULTS AND DISCUSSION The researchers initially engaged a group of elementary school students from		-
Unique	Subsequently, the researchers conducted the research and analysed the data relating to the research		-
Unique	The researchers used the term form since it was assumed that elementary students would		-
Unique	The work showed that the subject had successfully made 34 equations equivalent with the		-
Unique	However, in this case, the researchers only analysed one of the 34 equations, as		-
Unique	In order to identify reversible thinking, the researchers presented the procedures the subject applied		-
Unique	i th answer by the subject P1 : Explain the procedures you used in making		-
Unique	SJ3 : Then, I take the square root of both sides of this (pointing		-
Unique	SJ5 : I add 8 to both sides of this (pointing out)		-
1 results	TEST Given the following form: Make as many forms as possible based on the	wiete.com.au	
Unique	192 P5 : Is it OK, if I divide the right-hand side of the		-
Unique	SJ6 : Because the arithmetical result of the right-hand side is different from the		-
Unique	Based on the interview above, the aspects of reversible thinking were revealed in the		-
Unique	of the initial equation (that was) by 4, so that it resulted in Reciprocity,		-
Unique	LM3 : Determining the result of operation of , that was 4, so that		-
Unique	- LM4 : The subject squared the both sides of so that it resulted		-
Unique	LM5 : Determining the result of , which was , so that it resulted		-
Unique	- LM6 : The subject added the both sides of with 8, so that		-

Unique	LM7 : Determining the result of , which was , so that it resulted	-
Unique	its starting point, the researchers initially presented the procedures the subject applied to reverse the	-
Unique	P9 : Now, please explain the procedure you used to reverse all these forms	-
Unique	the left-hand side), so that it resulted in 10 minus 8, which is equal to	-
Unique	SJ11 : Then, I squared both sides resulting in 24, added by a and	-
Unique	SJ12 : I multiplied both sides of this (by pointing out) by 4,	-
Unique	procedures the subject used to reverse the equations and constructed the initial one or went	-
Unique	LKi Activity of reversing equation Aspects of reversible thinking revealed LK1 : The subject	-
Unique	- LK2 : The subject moved the side of element 8, so that it	-
Unique	8 (that was + 8) by subtracting 8 (that was) in which the subtraction	-
Unique	it resulted in - LK4 : Squaring both sides of so that it resulted in	-
Unique	LK5 : Determining the result of , which was , so that it resulted	-
Unique	starting point Based on Table 3 and Table 4, there were three matters the researchers	-
Unique	There were other three matters the researchers considered when the subject reversed the new	-
Unique	Table 5: Interesting matters on the steps of making and reversing a new equation	-
Unique	reversed a new equation to its starting point LM2 Dividing both sides of an initial	-
Unique	The explanation of Table 5: In the 2 nd step of making the equation	-
Unique	however, when reversing the equation on 6 th step (coded as LK6), the subject	-
Unique	when reversing the equation in the 4 th step (coded as LK4), the subject took	-
Unique	In the 6 th step in making the equation (coded as LM6), the subject	-
Unique	subject moved 8 from one side of the equation to the other, so that it	-
Unique	The result showed that the reversible thinking of an elementary student who had won	-
Unique	It is not in accordance with the stage of cognitive development presented by Piaget	-
Unique	In accordance to the 194 subject, the equal sign indicated that ...the both sides	-

Unique	Other students in the same developmental stage as the subject assumed that the equal	-
Unique	McNeil et al argued ...equal signs were often presented in standard operations-equals- answer contexts	-
Unique	g., 3 + 4 = 7) and were rarely presented in nonstandard operations on both	-
Unique	CONCLUSIONS The aspects of reversible thinking in solving algebraic problems by an elementary student	-
Unique	It was by dividing both sides of equation with the same element, subject took	-
Unique	Whereas, when reversing the new equation to its starting point, the aspects of reversible	-
Unique	It could be seen from the strategy the subject used in reversing the equation	-
Unique	in this case, the subject used a subtraction operation, which was the inversion of an	-
Unique	On the other hand, reciprocity was identified when the subject took the square root	-
Unique	(Ed), Second Handbook of Research on Mathematics Teaching and Learning: a Project of the	-

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World Transactions on Engineering and Technology Education 2017 WIETE Vol.15, No.2, 2017 189 INTRODUCTION This study was inspired by Piaget's theory on reversibility, that the primary characteristic of children aged seven up to 11 years of age was the development of a capacity for reversibility [1]. The researchers were also motivated by Lamon [2] and that the attention on reversibility was still limited. Hence, Lamon asked researchers in the education field to study students' reversibility. Krutetskii identified the mathematics skill related to the successfulness in solving problems, that is, reversibility and flexibility [3]. In fact, most primary graders found themselves having difficulties to solve arithmetical problems [4]. Inhelder and Piaget stated …reversibility can be considered a key requirement in a number of problems in mathematics [1], meaning that reversibility could be considered to be a primary requirement for solving a number of mathematics problems. Ramful stated that reversibility was related to mathematical operations, fractions, comparisons, algebra, and some other cases [5]. Effective algebraic thinking sometimes involves reversibility (i.e. being able to undo mathematical processes, as well as to do them). In effect, it is the capacity not only to use a process to reach a goal, but also to be able to understand the process well enough to work backwards from the answer to the starting point. This definition indicated that it involved reversibility in solving algebraic problems, that is, an ability to reverse the mathematical process. Reversibility did not merely involve a process of achieving objectives, but also a process of reversing into the initial state with the generated answer. It suggested that there were two processes in reversibility: those that were a process of achieving objectives or a result, and a process of reversing the objectives or result to the initial state. As Greenes said, algebra is sometimes referred to as generalised arithmetic, because it formalises arithmetic relationships [6]. Its power lies in the ways it allows one to represent relationships between quantities, to describe properties of operations (such as commutative and distributive), and to describe patterns. Algebra provides rules for manipulating symbols or signs, such as simplifying an expression and, then, solving the unknown. Suh stated the importance of algebra for elementary students, and asserting on algebra-arithmetic could be used to learn algebra concepts in elementary grades [7]. Algebra taught to elementary students was termed by Kaput as early-algebra [8]. It did not only facilitate the learning of the subsequent levels of algebra, but also helped to develop their mathematics concepts in a deeper and more complex manner since the early stage. The subject of this present study was an elementary student who won National Olympiad medals in science, because a) based on Piaget's theory, the ability of reversible thinking begins to grow in a concrete operational phase in children from seven up to 11 years old [9]; b) the student who won National Olympiad medals in science was an asset of the nation that must be well-preserved, concerned and developed from early on; and c) the student who won National Olympiad medals in science had better mathematics skills than their peers, assuming that the subject could figure out The aspects of reversible thinking in solving algebraic problems by an elementary student winning National Olympiad medals in science Syarifatul Maf'ulah†, Dwi Juniati‡ & Tatag Y.E. Siswono‡ STKIP PGRI Jombang, Surabaya, East Java, Indonesia† Universitas Negeri Surabaya, Surabaya, East Java, Indonesia‡ ABSTRACT: This study aimed to identify the aspects of reversible thinking of an elementary student who had won national Olympiad medals in science in solving particular algebraic problems. In this qualitative research, data were collected by a reversible thinking task contained a simple equation and an interview. The results showed that when making an equation, the only aspect of reversible thinking identified was reciprocity. This could be seen from the strategy the subject used in making the equation. It was done by dividing both sides of the equation by the same element, taking the square root of those two sides, and adding them with the same element. When reversing the new equation to its starting point, the aspects of reversible thinking identified were negation and reciprocity. It could be seen from the strategy the subject used in reversing the equation to its starting point. Negation was identified when the subject moved the parts of a known element, and in this case, the subject used a subtraction operation, which was the inversion of addition operation within the initial equation. Reciprocity was identified when the subject squared the both sides of the equation. 190 reversible thinking in algebra. Therefore, this study aimed to identify the aspects of reversible thinking in solving particular algebraic problems by an elementary student who won the national Olympiad in science. REVERSIBLE THINKING IN SOLVING ALGEBRAIC PROBLEMS There were two important matters within reversible thinking, described as forward or reverse [1][3][10][11]. Forward was a mental process from a starting point that moved into the expected goal, whereas, reverse was a mental process from the expected goal moving to its starting point. The aspects of reversible thinking could be identified as follow: Table 1: The aspects of reversible thinking that could be identified. Aspect of reversible thinking Explanation Negation It was when a subject used inversion towards the related operation [1][10][11]. Reciprocity It was when a subject used compensation or any other equivalent relationships with a given equation [1][3][11]. Capability to return to the initial data after obtaining the result It was when a subject could return the equation to its starting point using correct procedures [1][3]. Solving the problem was an activity of seeking solution for particular situation, using the insight previously obtained [12-14]. The focus of the study mentioned earlier was on algebra taught to elementary students. The algebra taught to them was referred to by the term early-algebra [8]. Early algebra was not merely a bridge to learning algebra at subsequent levels, but it also helped in developing the conception of students' mathematics in a deeper and more complex manner than earlier [6][8]. Greenes stated that in teaching mathematics for elementary graders, variables were used in three ways [6]. They could represent unknown elements, identify quantities that vary and generalise the properties. Powel stated that an equation is a mathematical statement in which the equal sign (=) was used to show the equivalency between a number or expression on the left-hand side and a number or expression on the right-hand side [14]. The expression itself was defined as a combination of operant numbers and arithmetic operations without any equal sign (=). The problem of this study consisted of an initial equation containing one variable as an unknown element. The instruction provided was that the subject was asked to make as many equations as possible that were equivalent to the initial one. Thus, solving an algebraic problem was an activity the subject undertook in order to make as many equivalent equations as possible with its initial as a solution of the given problem using his/her insight. To identify the student's reversible thinking in solving algebraic problems, this study applied a test containing an equation. Then, the student was asked to make other equivalent equations based on the initial one. Two equations were defined as equivalent, if both had similar solutions. Thus, the indicators of reversible thinking that could be identified in algebra are included in Table 2. Table 2: Indicators of the aspects of reversible thinking that could be identified in algebra. Process of reversible thinking Aspects of reversible thinking Indicators Forward (a process in which the subject made other equations equivalent with its initial) Negation When the subject used inversion towards the related operation in making equations. Reciprocity When the subject used compensation or any other relationships equivalent with a given equation in making equations. Reverse (a process in which the subject reversed the equations he just made into the initial one) Negation When the subject used inversion towards the related operation in his way reversing the equations. Reciprocity When the subject used compensation or any other relationships equivalent with a given equation in reversing the equations. Capability to return to initial data after obtaining the result When the subject could return the equation made to the initial one using correct procedures. METHOD Bogdan and Biklen explained the characteristics of qualitative research, and the present study is in accordance with these characteristics [15]. These are: a) naturalistic in nature, since it was conducted using the real situation as the data source and the researchers are its primary instrument; b) descriptive, since the data collected were qualitative, such as a set of words or writing, in cases when the data were in the form of the subject's work; and c) inductive, since it did not aim to prove any hypothesis, but merely to describe a phenomenon. The researchers provided a test for the subject, and conducted an interview later to reveal any other uncovered material relating to the test result. This study selected an elementary student who had won national Olympiad medals in science. 191 The procedures conducted in this study consisted of three primary phases as follows: 1. Preparation; in this phase, the researchers examined the theories of reversible thinking. 2. Implementation; in this phase, the researchers selected the subject for this study. Subsequently, the researchers gave a test to the subject and, then, conducted an interview based on their work. 3. Analysis; in this phase, the researchers conducted data analysis and wrote a report. Analysis was conducted after the interview had ended. Subsequently, the researchers analysed the data entirely referring to the framework of reversible thinking, which is described in Table 2 using the following steps: 1) data reduction; 2) data presentation; and 3) conclusion making. RESULTS AND DISCUSSION The researchers initially engaged a group of elementary school students from which they selected a student who had won medals in the National Kuark Science Olympiad. Subsequently, the researchers conducted the research and analysed the data relating to the research results. The test provided for the subject is presented in Figure 1. Figure 1: Test instrument. The intended form of the test was an equation. The researchers used the term form since it was assumed that elementary students would not yet recognise the term equation. The work showed that the subject had successfully made 34 equations equivalent with the initial one. However, in this case, the researchers only analysed one of the 34 equations, as shown in Figure 2. Figure 2: One of the equations made by the subject. In order to identify reversible thinking, the researchers presented the procedures the subject applied in making equations based on the results of the interview. However, before presenting the result of the interview, the researchers initially described the code of the interview as follows: PPi = i th statement by the researchers SJi = i th answer by the subject P1 : Explain the procedures you used in making this form! (by pointing out this). SJ1 : Yes, the initial form is 24 added to a equals 16. Then, I divide both sides of this initial form by 4. The result is 24 added to a divided by 4 equals 4. P2 : Would you please write it down? SJ2 : Of course (and, then, writing down the following form): , then . P3 : And, then? SJ3 : Then, I take the square root of both sides of this (pointing out that) and, the result will become... (and writing down the following): , then . P4 : After that? SJ5 : I add 8 to both sides of this (pointing out) , so that it equals this... (pointing out). TEST Given the following form: Make as many forms as possible based on the form above! 192 P5 : Is it OK, if I divide the right-hand side of the initial equation by 4 and the left-hand side by 2? SJ5 : No, it is not allowed. P6 : How come? SJ6 : Because the arithmetical result of the right-hand side is different from the left-hand one. P7 : Why it is not allowed for both sides to be different? SJ7 : It is not allowed. P8 : Why? SJ8 : There is an equal sign (what the subject indicated was …=…). Based on the interview above, the aspects of reversible thinking were revealed in the procedures used to make equations equivalent with the initial one are presented in Table 3. Table 3: Aspects of reversible thinking in making equations by the subject. LMi Activity of making equation Aspects of reversible thinking revealed LM1 : The subject payed attention to the initial equation provided - LM2 : The subject divided both sides of the initial equation (that was) by 4, so that it resulted in Reciprocity, since the subject used compensation, dividing both sides of the initial equation with 4. LM3 : Determining the result of operation of , that was 4, so that the result would be . - LM4 : The subject squared the both sides of so that it resulted in . Reciprocity, since the subject used compensation, squaring both sides of . LM5 : Determining the result of , which was , so that it resulted in . - LM6 : The subject added the both sides of with 8, so that it resulted in . Reciprocity, since the subject used compensation, adding the both sides of with 8. LM7 : Determining the result of , which was , so that it resulted in = 10. - *) LMi is i th step in making equation Subsequently, in order to identify the subject's reversible thinking in reversing the equations made into the initial one or its starting point, the researchers initially presented the procedures the subject applied to reverse the equations made into its starting point based on the following interview result. P9 : Now, please explain the procedure you used to reverse all these forms you had made (by pointing out) into its initial one or its starting point! But, would you please first, write down the new procedures before giving some explanation? SJ9 : Yes, of course (and, then, the subject wrote the following). , then . P10 : Now, would you please explain it to me! SJ10 : OK, I moved this 8 into here (what the subject intended was the left-hand side), so that it resulted in 10 minus 8, which is equal to 2 (by pointing out). P11 : And, then? SJ11 : Then, I squared both sides resulting in 24, added by a and divided by 4. The result was 4 (what he intended was). P12 : And, then? SJ12 : I multiplied both sides of this (by pointing out) by 4, which was equal to 24, added a, which then equalled 16. 193 Based on the interview above, the aspects of reversible thinking, revealed in the procedures the subject used to reverse the equations and constructed the initial one or went back to its starting point, are presented in Table 4. Table 4: Aspects of reversible thinking in reversing equations by the subject. LKi Activity of reversing equation Aspects of reversible thinking revealed LK1 : The subject paid attention to the equations made, that was . - LK2 : The subject moved the side of element 8, so that it resulted in , then . Negation, since the subject used inversion of the related operation, cancelling the addition of 8 (that was + 8) by subtracting 8 (that was) in which the subtraction operation was the inversion of the addition operation. LK3 : The subject determined the result of operation which was 2, so that it resulted in - LK4 : Squaring both sides of so that it resulted in Reciprocity, since the subject used compensation, squaring both sides of . LK5 : Determining the result of , which was , so that it resulted in . - LK6 : Multiplying both sides of by 4, which resulted in . Reciprocity, since the subject used compensation, multiplying both sides of by 4. LKi is i th step in reversing the equation the subject made into its starting point Based on Table 3 and Table 4, there were three matters the researchers were concerned with relating to reversible thinking when the subject made the equation. Those were on the steps with codes LM2, LM4 and LM6. There were other three matters the researchers considered when the subject reversed the new equation. Those were the steps with codes LK2, LK4 and LK6. In order to facilitate understanding, these codes are described in Table 5. Table 5: Interesting matters on the steps of making and reversing a new equation into its initial form. Code Interesting matter when the subject made equations Code Interesting matter when the subject reversed a new equation to its starting point LM2 Dividing both sides of an initial equation by 4. LK2 Moving 8 from one side to the other side. LM4 Squaring both sides of the equation. LK4 Squaring both sides of the equation. LM6 Adding 8 to both sides of the initial equation. LK6 Multiplying both sides of the new equation by 4. The explanation of Table 5: In the 2 nd step of making the equation (coded as LM2), the subject divided the sides of the initial equation by 4; however, when reversing the equation on 6 th step (coded as LK6), the subject multiplied both sides of the new equation by 4. In the 4 th step in making the equation (coded as LM4), the subject took the square root of both sides of so that it resulted in , however, when reversing the equation in the 4 th step (coded as LK4), the subject took the square root of both sides of ; hence, it resulted in . In the 6 th step in making the equation (coded as LM6), the subject added 8 to both sides of the equation ; hence, it resulted in ; however, when reversing the equation in the 2 nd step (coded as LK2), the subject moved 8 from one side of the equation to the other, so that it resulted in . The result showed that the reversible thinking of an elementary student who had won national Olympiad medals in science in solving particular algebraic problem had progressed beyond their age. It is not in accordance with the stage of cognitive development presented by Piaget [1]. The subject fully understood the equal sign (=). In accordance to the 194 subject, the equal sign indicated that …the both sides are equal or the right-hand side equals the left one. Other students in the same developmental stage as the subject assumed that the equal sign (=) indicated the answer or the result of an operation. McNeil et al argued …equal signs were often presented in standard operations-equals- answer contexts (e.g., 3 + 4 = 7) and were rarely presented in nonstandard operations on both sides contexts (e.g. 3 + 4 = 5 + 2) [16]. Students often defined the equal sign (=) as the context of an answer. They seldom defined it as a link of two sides (right and left sides). CONCLUSIONS The aspects of reversible thinking in solving algebraic problems by an elementary student who had won National Olympiad medals in science were negation and reciprocity. When making an equation, the only aspect of reversible thinking identified was reciprocity. This could be seen from the strategy the subject used in making the equation. It was by dividing both sides of equation with the same element, subject took the square root of those two sides, and adding to both the same element. Whereas, when reversing the new equation to its starting point, the aspects of reversible thinking identified were negation and reciprocity. It could be seen from the strategy the subject used in reversing the equation that it was made into the starting point. Negation was identified when the subject moved the parts of the known element and, in this case, the subject used a subtraction operation, which was the inversion of an addition operation within the initial equation. On the other hand, reciprocity was identified when the subject took the square root of squared both sides of the equation and multiplied them. REFERENCES 1. Inhelder, B. and Piaget, J., The Growth of Logical Thinking from Childhood to Adolescence. New York: Basic Books, 4-11, 272-274 (1958). 2. Lamon, S.J., Rational Numbers and Proportional Reasoning: towards a Theoretical Framework for Research. In: Lester, F.K. (Ed), Second Handbook of Research on Mathematics Teaching and Learning: a Project of the National Council of Teachers of Mathematics. Charlotte, NC: Information Age Publishing, 629-667 (2007). 3. Krutetskii, V.A., The Psychology of Mathematical Abilities in School Children. Chicago: The University of Chicago Press, 287-289 (1976). 4. Maf'ulah, S., Juniati, D. and Siswono, T.Y.E., Pupils ' error on the concept of reversibility in solving arithmetic problems. Educational Research and Reviews, 18, 11, 1775-1784 (2016). 5. 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